Markovian Pricing with Price Guarantees

Jianghua Wu
School of Business, Renmin University of China, Beijing 100872, China
jwu@ruc.edu.cn

Dan Zhang
Leeds School of Business, University of Colorado Boulder, CO 80309, USA
dan.zhang@colorado.edu

Yan Liu* (Corresponding author)
Department of Logistics and Maritime Studies, Faculty of Business
Hong Kong Polytechnic University, Kowloon, Hong Kong
yan.y.liu@polyu.edu.hk

A stream of recent research considers the practice of random price discounts (a.k.a randomized pricing) when selling to forward-looking customers and shows that such pricing strategies can mitigate strategic customer waiting and boost seller profit. In practice, random price discounts are often offered together with price guarantees, in which customers are refunded the price difference if the price is lowered within a given time window after purchase. This paper investigates the efficacy of price guarantees under randomized pricing. To that end, we consider a model in which a firm adopts Markovian pricing and interacts with customers over an infinite time horizon. The following results are obtained. First, while Markovian pricing allows firms to price discriminate customers based on their monitoring costs, price guarantees further allow firms to price discriminate customers based on their willingness to pay. Second, offering price guarantees under Markovian pricing can help retain customers effectively by inducing high-valuation customers to purchase early, regardless of their arrival time. Third, even with price guarantees, Markovian pricing can dominate static pricing only when high-valuation customers are more likely to have a high monitoring cost, which illustrates that customer composition plays a crucial role in the effectiveness of the firm's pricing strategy. Fourth, the optimal duration of price guarantees is closely related to customers' lifetime duration. Finally, perhaps surprisingly, offering price guarantees can decrease the aggregate customer surplus since the firm offers sale prices less often under price guarantees.

Key words: Markovian/Randomized Pricing, Customer Behavior, Price Guarantees
History: Received: January 2024; accepted: October 2024 by Haresh Gurnani after two revisions.

1. Introduction

A stream of recent research in operations management considers random price discounts (a.k.a randomized pricing), where prices are drawn randomly from pre-committed distributions (Wu et al. 2014, Moon et al. 2017, Chen et al. 2023). These papers offer several explanations for why randomized pricing works well, often dominating non-randomized pricing strategies. As an example, Figure 1 shows the historical prices of a robot vacuum cleaner on suning.com, a major online retailer in China. From October 1, 2019 to January 31, 2020, the price was usually 2,500 CNY;

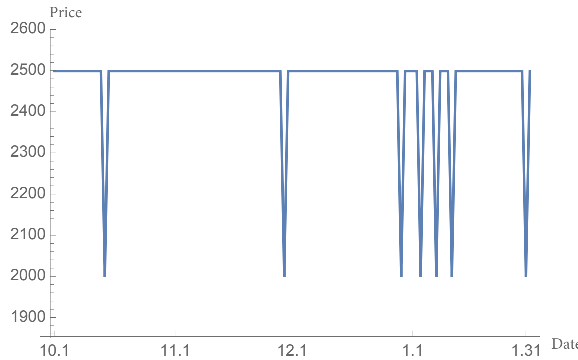


Figure 1 Price of a Robot Vacuum Cleaner on Suning.com from October 1, 2019 to January 31, 2020.

however, a sale price of 2,000 CNY was occasionally offered for a short duration.

In response to price variations over time, some customers may choose to wait for a sale price even if the current price is affordable. To mitigate such customer waiting behavior, price guarantees are often offered in practice (Aviv et al. 2009, Netessine and Tang 2009). Price guarantees can take many forms. A common one is the so-called posterior price guarantee, under which customers are offered a refund of the price difference if the price is marked down within a given time window after the purchase. In practice, the duration of price guarantees is usually finite and varies from seller to seller or even from product to product for the same seller. For example, a major online retailer in China, JD.com, specifies that its price guarantee lasts 30 days for household appliances and 7 days for a set of products including laptops and cameras. Due to refund claims made by customers in the event of a markdown, price guarantees can negate the power of varying prices over time. Then, what is the effect of price guarantees on customers' purchase behavior and firm profits? To the best of our knowledge, the efficacy of price guarantees under randomized pricing has not been investigated. In addition, given the variations in the duration of price guarantees, a question naturally arises: What is the optimal duration for a price guarantee? We investigate these issues in our work.

We consider a more general form of randomized pricing, called Markovian pricing, for a firm interacting with customers over an infinite time horizon. Under a Markovian pricing strategy, the firm switches between a regular price and a sale price following a continuous-time Markov chain. The firm offers a price guarantee that lasts for a fixed amount of time to induce customers to purchase. Customers arrive over time and differ in two dimensions: valuation (either high or low) and price monitoring cost (or simply "monitoring cost" for short), where type I customers have a zero monitoring cost, and type II customers have a non-zero monitoring cost. All customers are short-lived with a lifetime that is exponentially distributed. They can choose to purchase and leave immediately, purchase and monitor for price refund, or wait for a better price until their lifetime ends.

An important feature of our model is that customers have a limited and uncertain lifetime that is exponentially distributed. That customers have a limited lifetime is implicitly assumed in all finite-horizon models in the literature. Customer models with limited and random lifetime are considered in the marketing literature (Bemmaor and Glady 2012, Abe 2009), albeit under different settings. By modeling customer lifetime as a random quantity, we assume that customers cannot predict perfectly when they might exit the market. Such a model is quite natural if we interpret a customer's lifetime as the duration of interest for a product. For example, a customer's need for a product may vanish while waiting for a sale. Her tastes might change over time and she may no longer be interested in the product.

We formulate a customer's decision problem as a continuous-time Markov decision process and derive her optimal purchase decisions. Then, based on customers' optimal responses, we analyze the firm's optimal Markovian pricing strategy. We find that the optimal pricing strategy, with and without price guarantees, is either static pricing or high/low pricing with flash sales, where the firm charges a high price all the time, except for occasional price drops. Then, we compare the firm's optimal pricing strategy and customers' purchase behavior with and without price guarantees to investigate how price guarantees affect the firm's profit and customer welfare.

Our main results can be summarized as follows. First, without price guarantees, Markovian pricing can price discriminate customers based on their monitoring costs: All customers with a low monitoring cost would monitor the prices and only purchase at low prices, while those with a high monitoring cost would purchase immediately if the current price is below their valuations. With price guarantees, the high-valuation customers with a low monitoring cost may choose to buy at high prices and try to take advantage of price guarantees. That is, on top of the discrimination based on customers' monitoring cost, price guarantees entail another type of price discrimination based on customers' valuation. Overall, our result establishes price guarantees as an additional lever to boost profitability under Markovian pricing.

Second, price guarantees under Markovian pricing can help boost customer demand by retaining short-lived customers. Without price guarantees, some customers may choose to wait for a lower price when the price is high upon their arrival. However, these waiting customers may exit the market before purchase when they reach the end of their lifetime. Offering a price guarantee can induce these customers to purchase immediately at a high price, regardless of their arrival time. This is because, by injecting randomness into the pricing strategy, the expected surplus does not depend on a customer's arrival timing in case the customer chooses to take advantage of the price guarantee.

Third, we compare Markovian pricing with static pricing and find that even with price guarantees, high/low pricing with flash sales can dominate static pricing only when high-valuation customers are more likely to have a high monitoring cost. Substantial variations in customers' monitoring costs are established empirically in Moon et al. (2017), albeit in a different setup. It is plausible that high-valuation customers are more likely to have a high monitoring cost, given that high-valuation customers may also assign a higher value to their time and effort. Our research therefore offers an explanation for the widespread use of high/low randomized pricing in online retail.

Interestingly, the optimal duration for the price guarantee is closely related to customers' lifetime duration. On the one hand, as the guarantee duration decreases, it is less likely for customers to claim the refund, which increases the firm's profit. On the other hand, as the guarantee duration decreases, it becomes less appealing to customers and results in fewer customers to purchase at a high price. The optimal guarantee duration balances the two counteracting forces. In practice, one would expect that customers have different lifetime duration for different products. Our results therefore offer an explanation for the observed heterogeneity in guarantee duration for different products in practice.

Finally, offering price guarantees can either increase or decrease the aggregate customer surplus, which is somewhat surprising because price guarantees are often viewed favorably by customers. We find that when the firm switches from a high static price to high/low pricing with price guarantees, the aggregate customer surplus increases and thus a win-win outcome occurs. However, when the firm switches from either a low static price or high/low pricing without price guarantees to high/low pricing with price guarantees, the aggregate customer surplus is reduced, because price guarantees allow the firm to better discriminate customers and thus charge higher effective prices.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model setup and describes the Markovian pricing strategy with price guarantees. Section 4 analyzes customers' decision problem with price guarantees. Based on customers' strategy, Section 5 analyzes the firm's decision problem with price guarantees. Section 6 first derives the optimal Markovian pricing strategy without price guarantees and then investigates the impact of price guarantees by comparing the Markovian pricing strategy and customer behavior with and without price guarantees. Section 7 considers an alternative assumption on customers' monitoring behavior as a robustness check for our main results. Section 8 concludes. Technical proofs and supplemental materials are relegated to e-companion and online supplement.

2. Literature Review

Our literature review focuses on several closely related streams: dynamic pricing focusing on randomized/Markovian pricing, revenue management with strategic customers, and price guarantees.

Unlike much of the literature on dynamic pricing (Elmaghraby and Keskinocak 2003, Caldentey and Vulcano 2007, Elmaghraby et al. 2008, Board and Skrzypacz 2016), our model does not incorporate inventory considerations and the demand is deterministic. Furthermore, we consider Markovian pricing in an infinite-horizon setting and the form of the optimal pricing strategy is also quite different from literature. A salient feature of our optimal pricing strategy is that it may involve flash sales from time to time, which do not occur under inventory-driven dynamic pricing. One exception is the recent work of Dilmé and Li (2019), where flash sales are used to clear inventory early and charge higher prices later. Their driver of flash sales is completely different from ours and they do not consider price guarantees.

When customers are forward-looking, firms may vary prices over time to price discriminate them. In response, customers may choose to time their purchases. Such customers are called strategic customers and have been studied extensively (Besanko and Winston 1990, Su 2007, Liu and van Ryzin 2008, Besbes and Lobel 2015, Wang and Sahin 2018). More recent literature considers many variations of such customer behavior. As an example, a stream of literature considers the so-called patient customers who do not know the price path and purchase as soon as the price falls below their valuation (Liu and Cooper 2015, Lobel 2019, Zhang and Jasin 2022). It is shown that the optimal price path is non-monotonic and may involve price cycles. Such patient customers are closely related to the ones considered in Conlisk et al. (1984), who show that the optimal price path involves price cycles where the firm holds periodic sales. All the aforementioned papers consider finite-horizon models, where the optimal pricing strategy is deterministic. Implicitly, they all assume that customers have a finite, limited lifetime.

Injecting randomness in pricing is shown to be effective under certain conditions to mitigate strategic customer behavior. Wu et al. (2014) analyze the impact of randomized pricing on a firm's profitability, where high-valuation customers wait for at most one period and low-valuation customers wait for multiple periods. Using data from a North American specialty retail brand, Moon et al. (2017) empirically show that a randomized markdown policy performs better than a contingent state-dependent markdown policy. Randomized pricing and a version of the Markovian pricing strategy were considered in Chen et al. (2023), albeit in a discrete-time model. They study the intertemporal price discrimination effect of randomized pricing and show that randomized pricing (weakly) dominates the optimal deterministic cyclic policy in their setup. Collectively, Wu et al.

(2014), Moon et al. (2017), and Chen et al. (2023) establish the effectiveness and rationale for randomized pricing. When price guarantees are not considered, our results closely mirror theirs, even though our modeling framework is somewhat different. However, there are several key differences between these papers and our work. First, the most significant difference is that all these papers do not consider price guarantees, whereas the effectiveness of price guarantees under Markovian pricing is our focus. Second, our customer model is somewhat different from Chen et al. (2023) and Moon et al. (2017). We consider the price monitoring cost, instead of the waiting cost as in Chen et al. (2023) or the opportunity cost for a visit as in Moon et al. (2017). This modeling choice allows for more parsimonious modeling for price guarantees. In particular, the interpretation of price monitoring cost is different from the opportunity cost for a visit in Moon et al. (2017) in which customers need to decide how often to monitor the product. Finally, another major distinction of our work is the consideration of a continuous time model. Note that randomized pricing in its basic form means that the seller pre-commits to a distribution of prices. Such a pricing strategy is quite natural in a discrete-time model that does not allow price changes within each period, but would lead to prices bouncing around in a continuous time model. However, the Markovian pricing strategy is quite natural in a continuous-time model and substantially simplifies some of the analysis compared to a discrete-time model, such as the one in Wu et al. (2014). This analytical tractability is crucial since it allows us to analyze the more complicated cases involving price guarantees.

There is a substantial stream of literature on price guarantees. Much of the research in marketing explores the underlying mechanisms of price guarantees, such as signaling (Jain and Srivastava 2000, Moorthy and Winter 2006, Mamadehussene 2019), consumer reservation price boosting (Janssen and Parakhonyak 2013), collusion facilitation (Hay 1982, Salop 1986, Hviid and Shaffer 1999), and price discrimination (Png and Hirshleifer 1987, Corts 1996, Hviid and Shaffer 2012, Janssen and Parakhonyak 2013). We refer readers to Hviid (2010) for a comprehensive review of the price discrimination effect of price guarantees. The price guarantees in our study also facilitate price discrimination. Little research considers the optimal guarantee duration. One exception is Xu (2011), who studies the price guarantee duration for a monopolist that drops prices once in a finite selling horizon. By contrast, we consider an infinite horizon model in which prices vary over time following a continuous-time Markov chain.

Modeling work on price guarantees in the operations management literature often emphasizes the role of inventory and demand uncertainty. Lai et al. (2010) analyze a two-period model and argue that a posterior pricing matching policy can substantially improve the firm's profit under certain conditions. Huang et al. (2017) study price guarantees when customers are boundedly rational.

Levin et al. (2007) incorporate price guarantees into a finite-horizon revenue management problem with limited inventory. Nalca et al. (2013) analyze a concurrent price guarantee mechanism that is contingent on the verification of product availability at the competitor's location. Our study differs from this line of research in four aspects. First, none of the aforementioned papers deal with short-lived customers. Second, inventory and demand uncertainty are not considered in our model. Third, none of these papers study the optimal duration of price guarantees. Finally, we consider an infinite-horizon continuous-time model, whereas most existing work considers finite-horizon discrete-time models. To summarize, we study the impact of price guarantees in the context of Markovian pricing, which has never been studied in the literature.

3. Model Setup

3.1 Market Composition

We consider a monopolist selling a product to a market over an infinite time horizon. Without loss of generality, we assume that the unit production cost is zero. We considered a fluid model where infinitesimal customers flow in at a fixed rate of one per unit time. Customers are short-lived with a lifetime that is exponentially distributed with rate λ.[1] Customers' lifetime can be interpreted as their interest in the product, the randomness of which is determined by many factors. For example, a customer's taste or preference might change while waiting to buy a product, or another product is purchased as a substitute elsewhere. Note that customers may not be able to perfectly predict when they will lose interest in a product. In marketing literature, it is quite common to model customers' lifetime as a random variable, although in a different model setup (Bemmaor and Glady 2012, Abe 2009).

Upon arrival, a customer observes the current price and decides whether to purchase immediately. When a price guarantee is not offered, the customer leaves the market if she decides to purchase immediately. She can also choose to wait in the market and monitor the price in the hope of a price drop until her lifetime ends. When a price guarantee is offered, the customer may also choose to purchase immediately and then monitor the price for price refund.

Customers differ in their product valuation. A proportion α of customers are high-valuation customers with valuation V_H, whereas the rest are low-valuation customers with valuation $V_L < V_H$.

[1] The exponential lifetime assumption is made for tractability. It is a reasonable assumption when the seller does not monitor customers constantly; under a non-exponential lifetime distribution, the seller needs to know where each customer is in her lifetime cycle, which is nearly impossible in practice. There are many precedence in the operations literature on this type of approximation, such as the exponential arrival and service times in the queueing literature.

	Type I 0 (β)	Type II c ($1-\beta$)
v_H (α)	HI (γ)	HII ($\alpha-\gamma$)
v_L ($1-\alpha$)	LI ($\beta-\gamma$)	LII ($1-\alpha-\beta+\gamma$)

Figure 2 The Four Customer Segments.

In case a customer chooses to wait and monitor the price, a monitoring cost is incurred over time.[2] Different customers may have different monitoring costs. Given that automated price monitoring services (e.g., email notifications) can make it almost costless for some customers to learn about price changes in practice, we assume that a proportion β are type I customers with a zero monitoring cost, while the rest are type II customers with a monitoring cost $c > 0$. Note that customers differ in two dimensions: product valuation and monitoring cost. To model the correlation between these two dimensions, we assume that a proportion γ of all customers are high-valuation type I customers. Consequently, $\alpha - \gamma$ ($\beta - \gamma$, $1 - \alpha - \beta + \gamma$) proportion are high-valuation type II (low-valuation type I, low-valuation type II) customers. Figure 2 illustrates the four customer segments. It can be shown that the market model described here can incorporate arbitrary correlations between the two dimensions. Let ρ denote the correlation between having a high valuation and being type I. Then, $\rho = \frac{\gamma - \alpha\beta}{\sqrt{\alpha(1-\alpha)}\sqrt{\beta(1-\beta)}}$. When $\gamma = \alpha\beta$, the two customer characteristics are independent. When $\gamma > (<) \alpha\beta$, the two characteristics are positively (negatively) correlated.

3.2 Markovian Pricing Strategy

We assume that the firm adopts Markovian pricing, special cases of which include static pricing, randomized pricing, and high/low pricing. A Markovian pricing strategy can be described as follows. Suppose the price process $\{r(t) : t \geq 0\}$ follows a *regular* continuous-time Markov chain. A continuous-time Markov chain is said to be regular if, with probability one, the number of transitions in any finite length of time is finite (Ross 1996). The price process consists of two prices, r_H and r_L, which are chosen by the firm. If the price is r_H, the time before switching to the price r_L follows an exponential distribution with rate μ_H. Similarly, if the price is r_L, the time before

[2] Alternatively, one may consider a model with customer waiting cost. An undesirable consequence of a waiting cost model is that it requires a separate assumption on customers' price monitoring behavior after purchase when a price guarantee is offered (customers who already purchased are not waiting anymore). In contrast, we are able to use a single parameter (the price monitoring cost c) to capture consumers' price monitoring behavior before and after purchase, resulting in a more parsimonious model.

switching to the price r_H follows an exponential distribution with rate μ_L. For $i, j \in \{H, L\}$, let $P_{ij}(t)$ denote the probability that the Markov chain, presently in state i, will be in state j after an additional time t. Then, it can be shown that (Ross 1996)

$$P_{HH}(t) = 1 - P_{HL}(t) = \frac{\mu_L}{\mu_H + \mu_L} + \frac{\mu_H}{\mu_H + \mu_L} e^{-(\mu_H + \mu_L)t},$$

$$P_{LL}(t) = 1 - P_{LH}(t) = \frac{\mu_H}{\mu_H + \mu_L} + \frac{\mu_L}{\mu_H + \mu_L} e^{-(\mu_H + \mu_L)t}.$$

With a slight abuse of the notation, let P_i be the limiting probability of state $i \in \{H, L\}$ as $t \to \infty$. Then,

$$P_H = \frac{\mu_L}{\mu_H + \mu_L}, \quad P_L = \frac{\mu_H}{\mu_H + \mu_L}. \tag{1}$$

Here, P_i can also be interpreted as the probability that the price is r_i for $i \in \{H, L\}$. We also assume that a cost m is incurred each time the price changes. Price change cost is well recognized in the marketing and economics literature; see, e.g., Slade (1998), Levy et al. (2010), and related references. There is also a strand of the operations management literature that studies dynamic pricing with price change cost; see, e.g., Çelik et al. (2009) and Netessine (2006). In online retail, price changes can be made simply by changing the display on a webpage, the nominal cost of which is minimal. Even in this situation, however, frequent price changes are rarely desirable because they may create customer confusion and increase customers' search costs. Therefore, there is often a significant *implicit* cost of price changes. The firm's objective is to maximize the long-run average profit per customer per unit time.

Several commonly used pricing strategies can be viewed as special cases of Markovian pricing:

• Static pricing: The firm charges a fixed price over time. Static pricing is a special case of Markovian pricing where $r_H = r_L$.

• Randomized pricing: The firm chooses either high or low prices with a given probability at each moment. Let the probabilities of high and low prices be ϕ and $1 - \phi$ for any t, respectively.

Randomized pricing can be viewed as a special case of Markovian pricing where

$$P_{HH}(t) = 1 - P_{HL}(t) = \phi, \quad P_{LL}(t) = 1 - P_{LH}(t) = 1 - \phi, \quad P_H = \phi, \quad P_L = 1 - \phi.$$

Note that the transition probabilities for randomized pricing do not depend on t. Randomized pricing as specified above can be obtained as a limit of Markovian pricing by setting $\mu_L = \frac{\phi}{1-\phi}\mu_H$ and taking μ_H to infinity. That is, the transitions between prices are instantaneous with infinite transition rates; however, some transitions occur more frequently than others. Furthermore, the Markov chain associated with randomized pricing is not regular because the firm can switch prices at any moment and there is no guarantee that the number of price changes is finite within a finite length of time.

- High/low pricing with flash sales: The firm usually offers high prices but chooses to reduce the price occasionally. This pricing strategy can be viewed as a special case of Markovian pricing where μ_H is finite and μ_L approaches infinity.

3.3 Price Guarantees under the Markovian Pricing Strategy

Under Markovian pricing, high-valuation customers may choose to wait for the sale price, hurting the firm's profit. To encourage customers to purchase at the regular price, the firm can offer a price guarantee so that customers who purchase at a high price are refunded the price difference in case of a price drop within a time window after purchase. Of course, customers need to monitor the price after purchase in order to obtain a refund. Specifically, we assume that the firm refunds customers the price difference if a lower price is offered within T periods after purchase, where T is a constant chosen by the firm. Such price guarantees are often called posterior price matching in the literature and are widely observed in practice.

If the pricing strategy (price levels and switching rates) is fixed, a price guarantee should benefit customers, as they may receive refunds in the event of a price drop. However, because customers are more likely to purchase at the regular price when a price guarantee is offered, the firm has incentives to alter the pricing strategy. The overall effect of offering price guarantees on the firm and customers can be rather complicated due to these strategic interactions. Therefore, the objective of this work is to investigate the impact of price guarantees on the firm's profit and customer welfare in the context of Markovian pricing strategy.

4. Customers' Decision Problem

To explore the effects of price guarantees on the firm's pricing strategy and customer behavior, we first analyze customers' optimal purchase strategy under Markovian pricing.

We analyze the optimal purchase strategy for a type II customer with valuation $v \in \{V_H, V_L\}$ and monitoring cost c. Note that a type I customer's optimal purchase strategy can be obtained by taking $c = 0$. We assume that the firm uses a Markovian pricing strategy with parameters $(r_H, r_L, \mu_H, \mu_L, T)$, where $r_L \leq r_H$. As a tie-breaking rule, we assume that the customer always purchases if she is indifferent between purchasing and not purchasing.

Upon arrival, the customer observes the current price $r \in \{r_H, r_L\}$. If the observed price is r_L and the customer's valuation is above r_L, the customer purchases immediately and leaves. If the observed price is r_H, then the customer has four options. First, she can leave immediately without a purchase. Second, she can purchase immediately and leave. Third, she can purchase immediately and then monitor the price. A decision that the customer needs to make is when to stop monitoring for price refund. Lemma 1 below shows that it is optimal for the customer to keep monitoring until the price guarantee expires/applies. Hence, if the price drops before the price guarantee expires, she will be refunded the price difference $r_H - r_L$. Finally, she can monitor the price and purchase only when there is a price drop. Note that it is possible that she does not make a purchase if the price does not drop before her lifetime ends (i.e., she loses interest in the product).

The customer's decision problem can be formulated as a continuous-time Markov decision process. The state is the current price r. We consider the discrete-time Markov chain embedded in the continuous-time semi-Markov process and apply uniformization by taking the maximum transition rate to be $\nu = \lambda + \mu_H + \mu_L$ (Puterman 1994). Uniformization is a well-known technique used to formulate continuous-time dynamic programs and simplify the analysis of the resulting dynamic programming model. The transition rates in different states for a continuous-time Markov chain usually differ. For the continuous-time Markov chain we consider here, there are two possible transitions in state r_L: either the price changes from r_L to r_H (with rate μ_L) or the customer leaves with rate λ. Hence, the total transition rate is $\mu_L + \lambda$. Similarly, when the state is r_H, the total transition rate is $\mu_H + \lambda$. The key idea of uniformization is to add fictitious transitions such that the total transition rate is the same in both states. Here, by taking the total transition rate ν, we add fictitious transitions with rate μ_H in state r_L and rate μ_L in state r_H. Note that these fictitious transitions return to the same states.

Before writing down the decision problem for the customer as a dynamic program, we first derive the expected surplus if the customer chooses to purchase immediately at price r_H and then monitor the price, as shown in the following lemma.

LEMMA 1. *If the customer who purchased at price r_H chooses to monitor the price, then it is optimal for her to keep monitoring until the price guarantee expires/applies, and her expected surplus is $v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H})$.*

Lemma 1 shows that if the customer chooses to monitor the price after purchase, then it is optimal for her to keep monitoring until the price guarantee expires/applies, instead of stopping earlier. The main reason is that at any given time point during the monitoring, the expected benefit of monitoring until the end dominates the expected cost.

Let $G(\cdot)$ be the value function, which denotes the maximum surplus earned by the customer. Then, the dynamic program can be formulated as

$$G(r_H) = \max\left\{v - r_H, v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}), \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}, 0\right\}, \quad (2)$$

$$G(r_L) = \max\left\{v - r_L, \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}, 0\right\}. \quad (3)$$

The right-hand side of equation (2) is the maximum of four terms, corresponding to purchasing and then leaving, purchasing and monitoring for a refund, waiting and monitoring for a price drop, and leaving immediately without a purchase. If the customer purchases immediately and leaves, she obtains a surplus $v - r_H$. If the customer purchases and then monitors the price, she obtains an expected surplus $v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H})$; see Lemma 1. If the customer chooses to wait, the state becomes r_L at the next transition with probability μ_H/ν, whereas the next transition is a fictitious transition back to state r_H with probability μ_L/ν. These fictitious transitions are the result of uniformization. Note that the customer may also lose interest in the product while waiting, the probability of which is λ/ν and the surplus of which is zero. Thus, the term $\lambda/\nu \cdot 0$ is not necessary. Because the customer incurs a monitoring cost of c per unit time and the time until the next transition has a mean $1/\nu$, the expected monitoring cost until the next transition is c/ν. Therefore, the expected surplus of waiting for a sale is $\frac{\mu_L}{\nu} G(r_H) + \frac{\mu_H}{\nu} G(r_L) - \frac{c}{\nu}$. Of course, the customer also has the option to leave immediately without a purchase with zero surplus, which is reflected in the last term within the maximization.

Equation (3) can be explained similarly. When $r = r_L$, the price guarantee is never used. Hence, there are only three terms in equation (3). Intuitively, if $v \geq r_L$, the customer will always purchase immediately and equation (3) can be simplified to $G(r_L) = v - r_L$.

Lemmas 2 and 3 provide the optimal solution to the dynamic program in equations (2) and (3) and characterize the customer's optimal purchase strategy for a type II customer with parameters (v, c). The two lemmas consider cases with a high and a low price monitoring cost, respectively.

LEMMA 2 **(Optimal purchase decisions when the price monitoring cost is high)**. *Consider a type II customer with parameters (v, c) where $c > \mu_H(r_H - r_L)$. The optimal solution to equations (2)–(3) and the optimal purchase strategy of the customer are as follows:*

(a) If $v < r_L$, then $G(r_H) = G(r_L) = 0$ and the customer never purchases;

(b) If $r_L \leq v < r_H$, then $G(r_H) = 0$, $G(r_L) = v - r_L$, and the customer purchases upon arrival when the price is r_L, but leaves immediately without a purchase when the price is r_H;

(c) If $v \geq r_H$, then $G(r_H) = v - r_H$ and $G(r_L) = v - r_L$. The customer purchases immediately upon arrival.

LEMMA 3 **(Optimal purchase decisions when the price monitoring cost is low)**. *Consider the purchase decisions for a type II customer with parameters (v, c) where $c \leq \mu_H(r_H - r_L)$. The optimal solution to equations (2)–(3) and the optimal purchase decision are given as follows:*

(a) If $v < r_L$, then $G(r_H) = G(r_L) = 0$; the customer never purchases and the price guarantee is never used;

(b) If $r_L \le v < r_L + \frac{c}{\mu_H}$*, then* $G(r_H) = 0$*,* $G(r_L) = v - r_L$*; the customer purchases immediately upon arrival at the price* r_L*, but leaves without a purchase if the price is* r_H*. The price guarantee is never used;*

(c) If $r_L + \frac{c}{\mu_H} \le v < \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$*, then*

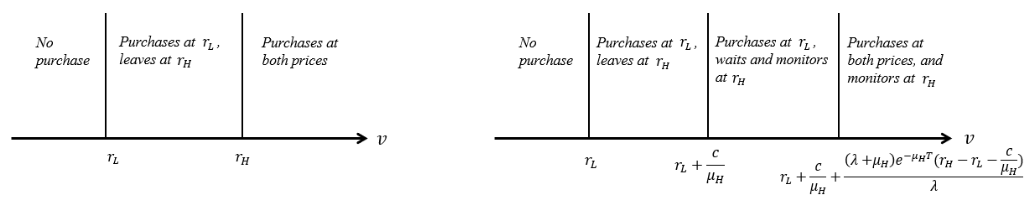
$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H}, \quad G(r_L) = v - r_L.$$

The customer purchases immediately upon arrival when the price is r_L*. When the price is* r_H*, the customer would wait for the price* r_L *until she leaves the market. The price guarantee is never used;*

(d) If $v \ge \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$*, then*

$$G(r_H) = v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}), \quad G(r_L) = v - r_L.$$

The customer purchases immediately upon arrival. If the purchase is made at the price r_H*, she would keep monitoring the price until the price guarantee is applied/expired.*



(a)When the monitoring cost is high (b)When the monitoring cost is low

Figure 3 A Type II Customer's Optimal Purchase Strategy with Price Guarantees.

Lemmas 2 and 3 show that a type II customer's purchase strategy depends critically on the magnitude of the monitoring cost. Lemma 2 characterizes a type II customer's purchase strategy when the monitoring cost is relatively high. The result is illustrated in Figure 3(a). In this case, the customer either purchases immediately at the current price or leaves. Therefore, her behavior is the same as a myopic customer. That a customer chooses not to wait and monitor the price at a high monitoring cost is quite intuitive. Lemma 3 considers the case where the monitoring cost is relatively low. The result is illustrated in Figure 3(b). If the customer's valuation is below r_L, she never purchases. If her valuation is at least r_L but below $r_L + \frac{c}{\mu_H}$, she purchases at the price r_L, but leaves at the price r_H; due to the price monitoring cost, it is not desirable to wait for a price drop if the current price is r_H. If her valuation is between $r_L + \frac{c}{\mu_H}$ and $\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$, she only purchases at the price r_L. When the price is r_H, the customer would wait and monitor for the price r_L rather than purchase immediately, and thus the price guarantee does not apply. If the customer's valuation is above $\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$, she would purchase immediately. Moreover, if the purchase is made at the price r_H, then she would keep monitoring the price until the price guarantee is applied/expired.

Observe that the upper end of the range in Lemma 3(c), $\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$, may be above r_H. For example, when there is no price guarantees (i.e., $T = 0$), the upper end reduces to $r_H + \frac{\mu_H(r_H - r_L) - c}{\lambda}$, which is greater than r_H because $c \le \mu_H(r_H - r_L)$; another example is when there is an intermediate expiration term for price guarantees such that $e^{-\mu_H T} \ge \frac{\lambda}{\lambda+\mu_H}$, in which case the upper end is also greater than r_H. This means that a type II customer with valuation between r_H and $\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$ may eventually leave without a purchase even though the price r_H is acceptable to her. One may wonder about the rationale behind this result. In our model, a customer calculates and compares the expected utility of each option when making a decision. Even if the current price is below their valuation, she may still choose to wait for a price drop due to a higher expected utility. However, during the waiting, she may lose interest in the product and thus exit the market. Customers cannot predict precisely when they may lose interest and exit the market. In our model, the waiting occurs because customers are not sure whether the price drop or the transition out of the market happens first. To our knowledge, that customers may not purchase even if the price is below their valuation is quite common in the literature on strategic customer behavior (Su 2007, Liu and van Ryzin 2008). In fact, whenever customers engage in strategic waiting, they risk the opportunity to purchase even if the price is below their willingness to pay.

For a relatively small T such that $\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H} \ge r_H$, Lemma 3(d) shows that a type II customer chooses to purchase at price r_H only when her valuation is substantially above r_H. If a type II customer's valuation is only slightly above r_H, she may prefer to wait in the market for a price drop. Such customer behavior is consistent with the so-called deal-proneness (Fortin 2000). Note that customers in our model are still rational in the sense that they take into account the possibility of leaving without a purchase when they choose to wait. On the other hand, for a large T such that $\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H} < r_H$, a type II customer may choose to purchase at price r_H even if her valuation is below r_H. This is because when the firm offers a price guarantee with a long duration, there is a high chance for a purchased customer to receive the price refund, encouraging those with valuations below r_H to purchase at the high price. When $c = 0$, Part (b) no longer exists, and a type I customer's behavior follows Parts (a), (c), and (d).

5. The Optimal Markovian Pricing Strategy with Price Guarantees

This section analyzes the firm's optimal Markovian pricing strategy with price guarantees. Given customers' optimal purchase decisions, the firm optimizes its Markovian pricing strategy by choosing the parameters r_H, r_L, μ_H, μ_L, and T. Note that based on the customer's decision, we can characterize the revenue contribution of a generic customer in different cases. Therefore, for any given pricing strategy, the revenue contribution from each of the four customer segments can be determined accordingly. Summing up these revenue contributions gives the firm's expected profit per customer per unit time. The optimal parameters balance the revenue received from all four customer segments.

5.1 A Low Monitoring Cost

Recall that our analysis in Section 4 indicates that type II customers monitor the price after they purchase at the high price if the monitoring cost is low, and do not monitor otherwise; type I customers always monitor after purchase at the high price because their monitoring cost is zero. We first analyze the firm's problem when the monitoring cost for type II customers is low. The result is summarized in Proposition 1.

Proposition 1. *If the monitoring cost c is low (i.e., $c \le \mu_H(r_H - r_L)$) such that type II customers who purchase at the high price choose to monitor the price after purchase, then high/low pricing with price guarantees cannot improve firm profit, compared to static pricing.*

Proposition 1 considers the situation where the monitor cost is low such that type II customers who purchase at the high price choose to monitor the price after purchase. Thus, high-valuation type I customers behave the same as high-valuation type II customers and pay the same price. Therefore, the price guarantee does not differentiate these two customer segments. However, offering price guarantees still enables Markovian pricing to discriminate customers based on their valuations; but such discrimination does not benefit the firm. To understand this result, note that if the price does (does not) switch to V_L before the price guarantee expires, the high-valuation customers who take advantage of price guarantees would pay an effective price V_L (V_H at most), whereas the low-valuation customers pay an effective price V_L (do not purchase). Consequently, the aggregate revenue is always dominated by V_L or αV_H, which is the revenue under static pricing at V_L and V_H, respectively. Hence, high/low pricing with price guarantees does not benefit the firm in this case.

5.2 A High Monitoring Cost

We then analyze the firm's pricing problem when the monitoring cost is high ($c > \mu_H(r_H - r_L)$) such that type II customers do not monitor the price after they make a purchase at the price r_H. That is, only type I customers take advantage of the price guarantee. The analysis in this case is somewhat complicated. On the one hand, the firm needs to decide on the duration of price guarantees T and the switch rates μ_H and μ_L to effectively manage the behavior of those who take advantage of price guarantees. On the other hand, inevitably, the decisions of T, μ_H, and μ_L also affect the purchase probability and thus revenue contribution of other customer segments, i.e., high-valuation type II and low-valuation type I. Therefore, the overall effect of price guarantees is not clear due to these interactions. The result is summarized in Proposition 2 below. To simplify notations, let

$$K = \beta V_L - \gamma V_H. \tag{4}$$

Note that K can be interpreted as the revenue difference between serving all type I customers at the price V_L and serving only high-valuation type I customers at the price V_H. Recall that m is the cost of each price change.

Proposition 2 **(The optimal Markovian pricing strategy for a high monitoring cost).** *Suppose the monitoring cost c is high (i.e., $c > \mu_H(r_H - r_L)$) such that type II customers do not monitor the price after they make a purchase at the price r_H. If $K \leq 2m\lambda$, then the firm's optimal pricing strategy reduces to that without price guarantees. If $K > 2m\lambda$, there are three possible outcomes for the firm's optimal pricing strategy:*

(i) ***Static pricing at V_H.*** *The firm prices at V_H. All high-valuation customers purchase immediately and all low-valuation customers leave without a purchase. The profit per unit time is αV_H;*

(ii) ***Static pricing at V_L.*** *The firm prices at V_L. All customers purchase immediately. The profit per unit time is V_L;*

(iii) ***High/low pricing with price guarantees.*** *Only if*

$$c > \lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right)(V_H - V_L), \tag{5}$$

the firm uses a Markovian pricing strategy with price guarantees where

$$r_H^{B,*} = V_H, \quad r_L^{B,*} = V_L, \quad T^* = \frac{\ln\sqrt{\frac{K}{2m\lambda}}}{\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right)}, \quad \mu_H^{B,*} = \lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right), \quad \mu_L^{B,*} = \infty.$$

The profit per unit time is

$$\Phi^{B,*} = (\alpha - \gamma)V_H + (\beta - \gamma)\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L + \gamma\left[\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L + \sqrt{\frac{2m\lambda}{K}}V_H\right]$$

$$-2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right). \tag{6}$$

All high-valuation customers purchase immediately at both prices; in particular, high-valuation type I customers try to take advantage of price guarantees by monitoring the price after purchase, while high-valuation type II customers leave immediately after purchase; low-valuation type I customers either purchase immediately at price V_L or wait for price V_L; and low-valuation type II customers purchases immediately at V_L and leave without a purchase at V_H.

Proposition 2 characterizes the firm's optimal pricing strategies with price guarantees when the monitoring cost is relatively high such that type II customers do not monitor after they make a purchase at the high price. If γV_H is relatively large, then the refund amount claimed by high-valuation type I customers who take advantage of price guarantees would be significant. As a result, when $K \leq 2m\lambda$, the firm does not offer price guarantees and the pricing strategy is the same as in Proposition 4. Therefore, we focus on the case $K > 2m\lambda$. Proposition 2 shows that the optimal pricing strategy is either static pricing or high/low pricing with flash sales. In static pricing, the firm sets the price at either V_H or V_L with no price guarantees, and the corresponding profit is αV_H or V_L. The high/low pricing strategy always charges a high price V_H, except for occasional price drops to V_L. The optimal pricing strategy can be obtained by comparing the profits in the three cases.[3]

The first three terms on the right-hand side of Equation (6) represent the revenues from high-valuation type II customers, low-valuation type I customers, and high-valuation type I customers, respectively. Because high-valuation type II customers purchase immediately upon arrival and leave, they pay a price V_H with a purchase probability one (as V_L is rarely offered). Low-valuation type I customers either purchase immediately at the price V_L or wait for the price V_L. They pay a price V_L with a purchase probability $\frac{\mu_H}{\mu_H+\lambda} = 1-\sqrt{\frac{2m\lambda}{K}}$, where $\frac{\mu_H}{\mu_H+\lambda}$ is also the probability that the price drops to V_L before customers' lifetime ends. High-valuation type I customers purchase immediately at both prices and monitor the price if the purchase is made at the high price to take advantage of the price guarantee. Because the low price is only offered occasionally, the revenue contribution at the low price is negligible. Suppose the purchase is made at the high price. Note that $e^{-\mu_H T^*}$ is the probability that the price guarantee expires before a sale is offered, in which

[3] If $K > 2m\lambda$ and condition (5) do not hold, then the high/low pricing is dominated by static pricing at either V_H or V_L. However, $K > 2m\lambda$ and condition (5) are necessary rather than sufficient conditions, because we did not compare $\Phi^{B,*}$ (the profit of high/low pricing) with $\max\{V_L, \alpha V_H\}$ (the profit of static pricing). We can establish the exact conditions under which $\Phi^{B,*} \geq \max\{V_L, \alpha V_H\}$, allowing us to state sufficient conditions for the firm to adopt a high/low pricing strategy. We choose not to state the sufficient conditions because they are long and tedious, although not difficult to derive.

case these customers pay a price V_H. With probability $1-e^{-\mu_H T^*}$, a sale is offered before the price guarantee expires, in which case these customers obtain a refund $V_H - V_L$, and the effective price paid is V_L. Therefore, the expected price paid by high-valuation type I customers is

$$e^{-\mu_H T^*} V_H + (1-e^{-\mu_H T^*}) V_L = \left(1-\sqrt{\frac{2m\lambda}{K}}\right) V_L + \sqrt{\frac{2m\lambda}{K}} V_H.$$

The switching rate $\mu_H^{B,*}$ characterizes the frequency of a markdown under the high/low pricing strategy. A larger $\mu_H^{B,*}$ implies that the sale price is offered more frequently. We make several observations. First, $\mu_H^{B,*}$ decreases in γV_H. This can be explained as follows. Because of the existence of price guarantees, high-valuation type I customers (who purchase only at the low price without price guarantees) purchase immediately at the high price and then keep monitoring with the hope of receiving a refund in the event of a price drop. Recall that γ is the proportion of high-valuation type I customers and V_H is the initial price they paid in the presence of price guarantees. Therefore, the larger the γV_H, the less frequently the firm should offer the sale price to avoid the loss from the refund claimed by this customer segment. Second, $\mu_H^{B,*}$ decreases in the price change cost m. The higher m, the less often the firm should offer the sale price. Finally, it is not immediately clear how $\mu_H^{B,*}$ changes with respect to λ, the rate of customers' lifetime distribution. Intuitively, as λ increases, customers wait in the market for a shorter duration; hence, the firm should offer the sale price more often to collect profit from low-valuation type I customers who only purchase at the low price. On the other hand, there is a cost associated with each price change. The more frequently the price changes, the more costs are incurred for the firm. Therefore, how $\mu_H^{B,*}$ changes with respect to λ depends on the relative magnitude of the two counteracting forces.

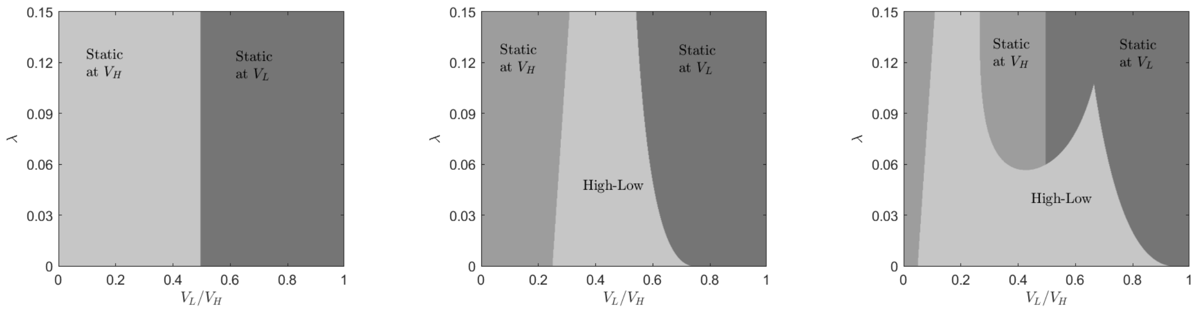
Interestingly, we show that the optimal guarantee duration is set such that the probability of getting the refund $(1-e^{-\mu_H T^*})$ is equal to the probability that the price switches to r_L before the customer's lifetime ends ($\frac{\mu_H}{\lambda+\mu_H}$). In other words, the optimal guarantee duration is closely related to the customers' expected lifetime. This optimal guarantee duration balances two counteracting forces. On the one hand, as the guarantee duration decreases, it is less likely for customers to claim the refund, which increases the firm's profit. On the other hand, as the guarantee duration decreases, it becomes less appealing to customers and results in fewer customers to purchase at a high price. Intuitively, customers are more "patient" when their lifetime duration increases, so they are more willing to wait for a sale rather than purchase at the high price, rendering the price guarantee unused. Therefore, in order to induce customers to purchase at a high price, the firm has to offer a more attractive price guarantee with a longer duration. In practice, one would expect that customers have different lifetime durations for different products. Therefore, this result may explain the variations in the duration of price guarantees for different products. In particular, it implies that price guarantees should be longer for products with a longer customer lifetime. Indeed, one often observes a long price guarantee duration for durable products (such as furniture and mattresses), while the price guarantee for less durable products (such as electronics) usually lasts for a much shorter time.

5.3 Negative Correlation

High/low pricing with price guarantees can be beneficial to the firm because it allows the firm to discriminate customers based on their monitoring cost and valuation, whereas static pricing discriminates customers only based on their valuation. Compared with static pricing at V_H where all high-valuation customers purchase at price V_H, under the high/low pricing strategy, high-valuation type I customers pay a lower price due to price refund. On the positive side, low-valuation type I customers, who do not purchase under static pricing at V_H, purchase under the high/low pricing strategy. Therefore, high/low pricing dominates static pricing at V_H only when there is a relatively small (large) fraction of high-valuation (low-valuation) type I customers. Compared with static pricing at V_L where all customers purchase at price V_L, under the high/low pricing strategy, low-valuation type II customers do not purchase given that the price V_L is rarely offered. On the positive side, high-valuation type II customers purchase at price V_H. Therefore, high/low pricing dominates static pricing at V_L only when there is a relatively small (large) fraction of low-valuation (high-valuation) type II customers. Indeed, as shown in Proposition 3 below, high/low pricing is only profitable when the correlation coefficient ρ is negative, which means that the high-valuation customers tend to have a higher monitoring cost and the low-valuation customers tend to have a lower monitoring cost.

PROPOSITION 3. *If $\gamma \geq \alpha\beta$, then the high/low pricing strategy with price guarantees is no more profitable than static pricing.*

To help explain Proposition 3, Figure 4 illustrates the optimal market outcomes when $\alpha = \beta = 0.5$. Here, $c = 0.1$, $m = 0.1$, $V_H = 1$, and we vary V_L from 0 to 1 and λ from 0 to 0.15. The three sub-figures correspond to the three levels of correlation between the valuation and monitoring cost among the customer population: 0 (no correlation), -0.5 (moderately negative correlation), and -0.9 (highly negative correlation). The horizontal axis is $\frac{V_L}{V_H}$ and the vertical axis is λ. The ratio $\frac{V_L}{V_H}$ can be interpreted as a measure of valuation homogeneity; a higher ratio indicates higher valuation homogeneity. Figure 4(a) shows that when the correlation coefficient ρ is zero, the firm does not benefit from high/low pricing. Instead, it serves either the entire market with a low price (when $\frac{V_L}{V_H}$



(a)$\rho = 0$ (b)$\rho = -0.5$ (c)$\rho = -0.9$

Figure 4 The Firm's Optimal Pricing Strategy with Price Guarantees for A High Monitoring Cost.

is high) or only the high-valuation segments (when $\frac{V_L}{V_H}$ is low). Figure 4(b) shows that when the correlation coefficient ρ is moderately negative at -0.5, there is an intermediate range of valuation homogeneity $\frac{V_L}{V_H}$ where high/low pricing is optimal. Figure 4(c) shows that when the correlation coefficient ρ becomes highly negative at -0.9, the firm is even more likely to offer high/low pricing. According to condition (5), the threshold on the monitoring cost varies with V_L and λ; therefore, the region where high-low pricing is optimal has an irregular shape.

6. The Effects of Price Guarantees

This section explores the effects of price guarantees on the firm's profit and customer welfare. To that end, Section 6.1 derives the optimal Markovian pricing without price guarantees, which will serve as a benchmark. Section 6.2 compares the optimal Markovian pricing with and without price guarantees to investigate how price guarantees affect the firm's profit and pricing strategy. Section 6.3 discusses the effects on customer surplus and social welfare.

6.1 The Optimal Markovian Pricing Strategy without Price Guarantees

We first discuss customers' optimal purchase strategy without price guarantees. Customers' optimal purchase strategy can be obtained straightforwardly by taking $T = 0$ in Lemmas 2 and 3. We choose not to repeat the result here.

Then, we analyze the firm's optimal Markovian pricing strategy without price guarantees. Similar to Proposition 1, we can show that if the monitoring cost c is low enough such that a type II customer behaves as stated in Lemma 3 (with $T = 0$), then the high/low pricing strategy is dominated by static pricing.[4] Hereafter, we focus on the case with a high monitoring cost. Moreover, we restrict our attention to the price pair (r_H, r_L) where $V_H \geq r_H$, because customers never purchase

[4] This result is summarized as Lemma E.5 and relegated to Section E.2 of the e-companion.

at $r_H > V_H$. It can be shown that it is never optimal to charge a low price r_L different from V_L and $V_L - \frac{c}{\mu_H}$. In the end, it turns out that the firm's profit when $r_L = V_L - \frac{c}{\mu_H}$ is always lower than static pricing at either V_H or V_L. Proposition 4 characterizes the firm's optimal pricing strategy without price guarantees.

PROPOSITION 4 **(The optimal Markovian pricing strategy without price guarantees)**. *Suppose the monitoring cost c is high (i.e., $c > \mu_H(r_H - r_L)$) such that type II customers never choose to wait and monitor at the high price. There are three possible outcomes for the firm's optimal Markovian pricing strategy:*

(i) ***Static pricing at** V_H**.*** *The firm prices at V_H. All high-valuation customers purchase immediately and all low-valuation customers leave without a purchase. The profit per unit time is αV_H;*

(ii) ***Static pricing at** V_L**.*** *The firm prices at V_L. All customers purchase immediately. The profit per unit time is V_L;*

(iii) ***High/low pricing with flash sales.*** *Only if*

$$\beta V_L > 2m\lambda, \tag{7}$$

$$c > \lambda\left(\sqrt{\frac{\beta V_L}{2m\lambda}} - 1\right)(V_H - V_L), \tag{8}$$

the firm uses a Markovian pricing strategy where

$$r_H^* = V_H, \quad r_L^* = V_L, \quad \mu_H^* = \lambda\left(\sqrt{\frac{\beta V_L}{2m\lambda}} - 1\right), \quad \mu_L^* = \infty.$$

The profit per unit time is

$$\Phi^* = (\alpha - \gamma)V_H + \beta\left(1 - \sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L - 2m\lambda\left(\sqrt{\frac{\beta V_L}{2m\lambda}} - 1\right). \tag{9}$$

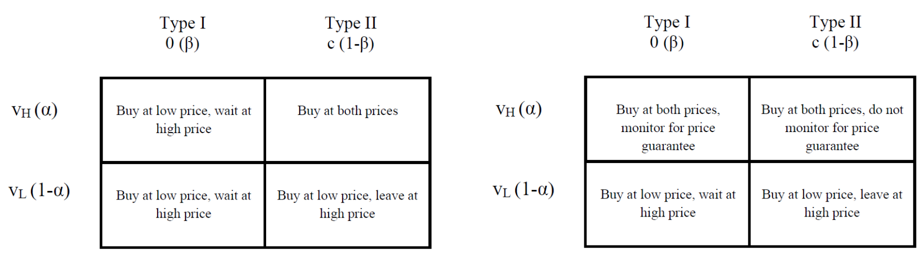
All type I customers either purchase immediately at the price V_L or wait for the price V_L; high-valuation type II customers purchase immediately at both prices and leave; and low-valuation type II customers purchase at the price V_L but leave without a purchase at the price V_H.

Proposition 4 shows that the optimal pricing strategy without price guarantees is still either static pricing or high/low pricing with flash sales. Under high/low pricing, all type I customers either purchase immediately at price V_L or wait for price V_L, so they pay a price V_L with a purchase probability $\frac{\mu_H}{\mu_H+\lambda} = 1 - \sqrt{\frac{2m\lambda}{\beta V_L}}$, where $\frac{\mu_H}{\mu_H+\lambda}$ is also the probability that the price drops to V_L before customers' lifetime ends. High-valuation type II customers purchase immediately at both prices and leave; as V_L is rarely offered, they pay a price V_H with a purchase probability one. Low-valuation type II customers purchase at V_L but leave without a purchase at V_H; as V_L is rarely offered, this segment of customers does not contribute revenue to the firm. Summing up the revenue contribution from each segment of customers gives the firm's profit in Equation (9). Again, the optimal pricing strategy can be obtained by comparing the profits in the three cases.

Note that high/low pricing is offered only when conditions (7) and (8) hold. Condition (7) requires a large value of βV_L (which is the potential revenue contribution from type I customers who wait for sale prices); it is not worthwhile to offer sales prices otherwise due to price change costs. Condition (8) can be equivalently stated as $c > \mu_H^*(V_H - V_L)$. Therefore, the high-low pricing strategy is applied only when the monitoring cost c is relatively high for type II customers so that they never choose to wait and monitor if the price is r_H. Recall that type I customers choose to wait and monitor when the price is r_H. If the monitoring cost c is relatively low such that high-valuation type II customers also choose to wait and monitor when the price is r_H, then no customers would purchase at r_H and the firm's optimal profit under the high-low pricing strategy is dominated by static pricing at V_L.

6.2 Effects on the Firm's Profit and Pricing Strategy

Proposition 1 shows that offering price guarantees cannot improve the firm's profit when type II customers have a low price monitoring cost. Therefore, we restrict our attention to situations where type II customers have a high price monitoring cost.



(a) Without Price Guarantees (b) With Price Guarantees

Figure 5 Customer Behavior under the Optimal Markovian Pricing Strategy with and without Price Guarantees.

It is instructive to compare customers' purchase strategies under the optimal Markovian pricing strategy with and without price guarantees, which are illustrated in Figure 5. When there are no price guarantees, the Markovian pricing strategy discriminates customers based on their price monitoring cost: type I customers (both high- and low-valuation) wait and monitor at the price r_H due to their zero monitoring cost, whereas type II customers behave myopically due to their high monitoring cost. Offering price guarantees also allows the firm to discriminate customers based on their valuation: high-valuation type I customers purchase at the price r_H due to the existence of

		Without Price Guarantees	v.s.	With Price Guarantees
Revenue Contribution	High-valuation type I	$\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L$	$<$	$\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_L+\sqrt{\frac{2m\lambda}{K}}V_H$
	High-valuation type II	V_H	$=$	V_H
	Low-valuation type I	$\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L$	$>$	$\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_L$
	Low-valuation type II	0	$=$	0
Price change cost		$2m\lambda\left(\sqrt{\frac{\beta V_L}{2m\lambda}}-1\right)$	$>$	$2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right)$

Table 1 Revenue Contribution of Each Customer Segment and the Cost of Price Changes.

price guarantees, whereas low-valuation type I customers wait and monitor when the price is r_H, leading to different effective prices for them. Therefore, offering price guarantees allows the firm to discriminate among customers based on their monitoring cost as well as their valuation.

It is worth noting that offering price guarantees can induce all high-valuation type I customers to purchase at a high price, regardless of their arrival time. By injecting randomness into the pricing strategy, the expected surplus for later-arriving high-valuation type I customers becomes the same as that of earlier-arriving ones if they choose to take advantage of price guarantees. This equalization of expected surplus incentivizes all high-valuation type I customers, regardless of their arrival time, to make an immediate purchase to take advantage of price guarantees. As a result, compared to the average purchase probability $1-\sqrt{\frac{2m\lambda}{\beta V_L}}$ without price guarantees, offering price guarantees raises high-valuation type I customers' average purchase probability to 1, as shown in Table 1 below. This means that offering price guarantees helps to retain high-valuation type I customers, because without price guarantees, those customers who do not purchase immediately may exit the market while waiting. Furthermore, optimizing the price guarantee enables the firm to collect an effective price higher than V_L from high-valuation type I customers, even after accounting for possible price refunds. To summarize, under Markovian pricing, offering price guarantees not only enables an additional layer of price discrimination (based on customers' valuation), but also helps to boost customer demand by retaining short-lived customers to the greatest extent possible.

To analyze the effect of price guarantees on the firm's profit, we first examine the revenue contribution by each customer segment. We have the following corollary that holds immediately by the firm's profit equations (6) and (9).

Corollary 1. *The revenue contribution of each customer segment, with and without price guarantees, is summarized in Table 1.*

Without price guarantees, high- and low-valuation type I customers purchase at the price r_L and wait for a price drop at the price r_H; the price paid is V_L, and the purchase probability is $1-\sqrt{\frac{2m\lambda}{\beta V_L}}$, yielding the revenue contribution $\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L$. Offering a price guarantee allows the firm to better price discriminate type I customers by charging different prices to high-valuation type I customers and low-valuation type I customers. While low-valuation type I customers still wait for price drops at the high price r_H, high-valuation type I customers would purchase immediately at the price r_H due to the price guarantee. According to Proposition 5(ii), the sale price is offered less frequently under price guarantees. Consequently, compared with the case without price guarantees, the purchase probability of low-valuation type I customers is lower; however, high-valuation type I customers pay a higher expected price with a higher purchase probability. Moreover, the price change cost is lower due to less frequent price changes.

Proposition 5 presents a few results regarding the effects of price guarantees on the firm's profit and pricing strategy.

PROPOSITION 5. *Suppose $K > 2m\lambda$ and condition* (8) *hold. We have the following results:*

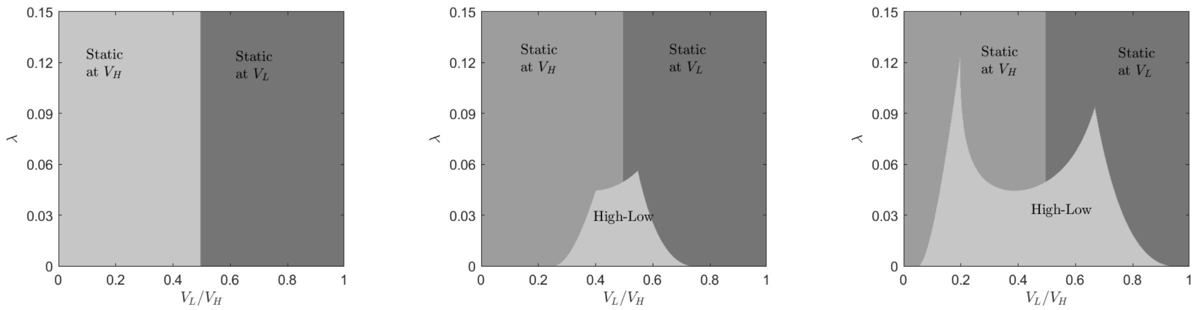
(i) $\Phi^{B,*} \geq \Phi^*$. That is, offering price guarantees improves the firm's profit when the high/low pricing strategy is offered. Furthermore, compared with the situation without price guarantees, the firm is more likely to offer high/low pricing;

(ii) $\mu_H^{B,*} \leq \mu_H^*$. That is, the firm offers the sale price V_L less frequently under price guarantees.

Table 1 shows that compared with the case without price guarantees, low-valuation type I customers contribute less revenue, while high-valuation type I customers contribute more revenue. Proposition 5(i) confirms that the net effect is a positive one and the firm's profit increases with a price guarantee under the high/low pricing. Not surprisingly, high/low pricing is more likely to be adopted with price guarantees than without, given that it is more likely to dominate static pricing, compared with not offering a price guarantee.

Proposition 5(ii) claims that the firm offers the sale price less frequently under price guarantees, which can be attributed to the firm's desire to issue fewer refunds. Compared with the situation without price guarantees, offering price guarantees can induce high-valuation type I customers to purchase immediately—even if the current price is high. Because they keep monitoring the price after purchase, offering flash sales less frequently would make the price guarantees more likely to expire unused. That is, offering flash sales less frequently can reduce the cannibalization effect of price guarantees on the high price.

Figure 6 illustrates the optimal market outcomes without price guarantees, using the same parameter values as in Figure 4. Note that Figures 6 and 4 have very similar structures. However, compared with Figures 4(b) and (c), the regions where the high/low pricing strategy is optimal shrinks in Figures 6(b) and (c), indicating that offering price guarantees makes the high/low pricing strategy more likely to be optimal, improving the firm's profit in the meantime.



(a)$\rho = 0$ (b)$\rho = -0.5$ (c)$\rho = -0.9$

Figure 6 The Firm's Optimal Pricing Strategy for a High Monitoring Cost.

How do the cost of price change m and monitoring cost c for type II customers affect the firm's pricing strategy? By Proposition 2, when $K \leq 2m\lambda$, the optimal pricing strategy is the same, with or without price guarantees. Therefore, price guarantees are adopted only when m is relatively small ($K > 2m\lambda$). Because $\beta V_L > K$, the cutoff on the pricing monitoring cost c in (5) is smaller than the cutoff in (8). Hence, high/low pricing with price guarantees is valid for a broader parameter range. A lower threshold value means that the condition on the monitoring cost c is less restrictive with price guarantees. This can be explained as follows. Recall that the high/low pricing strategy is optimal only if c is large enough such that type II customers do not wait for the sale price. Because the firm is less likely to offer the sale price in the presence of price guarantees, type II customers are less willing to wait than that without price guarantees. Therefore, the monitoring cost c does not have to be as large as that without price guarantees. To summarize, with price guarantees, the high/low pricing is more likely to be offered, not only because offering price guarantees boosts the firm profit, rendering it more likely to outperform the static pricing at either V_H or V_L, but also because the variation in customers' monitoring cost is less restrictive than that without price guarantees.

6.3 Customer Surplus and Social Welfare

In this section, we discuss the implication of offering price guarantees on the customer surplus and social welfare, compared with the situation without price guarantees. We focus on the case when $K > 2m\lambda$ and the monitoring cost is high for type II customers, because offering price guarantees has no effects on the customer surplus and firm profit otherwise.

Under static pricing at V_L and V_H, customer surplus is $\alpha(V_H - V_L)$ and 0, respectively, independent of whether the firm offers price guarantees. Under high/low pricing, the customer surplus is higher without price guarantees. Under the high/low pricing strategy without price guarantees,

Strategy	Without Price Guarantees	vs.	With Price Guarantees
Static pricing at V_H	0	$=$	0
Static pricing at V_L	$\alpha(V_H - V_L)$	$=$	$\alpha(V_H - V_L)$
High/low pricing	$\gamma\Big(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\Big)(V_H - V_L)$	$\geq$	$\gamma\Big(1-\sqrt{\frac{2m\lambda}{K}}\Big)(V_H - V_L)$

Table 2 Customer Surplus under Each Pricing Strategy with and without Price Guarantees.

high-valuation type I customers purchase with probability $1-\sqrt{\frac{2m\lambda}{\beta V_L}}$ and pay a price V_L, whereas the other purchasing customers pay a price equal to their valuation (see equation (9)). Hence, the aggregate customer surplus is $\gamma\Big(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\Big)(V_H - V_L)$. Under the high/low pricing strategy with price guarantees, high-valuation type I customers pay the expected price $\Big(1-\sqrt{\frac{2m\lambda}{K}}\Big)V_L+\sqrt{\frac{2m\lambda}{K}}V_H$, whereas the other purchasing customers pay a price equal to their valuation (see equation (6)). Hence, the aggregate customer surplus is $\gamma\Big(1-\sqrt{\frac{2m\lambda}{K}}\Big)(V_H - V_L)$. Because $K \leq \beta V_L$,

$$\gamma\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)(V_H - V_L) \geq \gamma\left(1-\sqrt{\frac{2m\lambda}{K}}\right)(V_H - V_L).$$

Therefore, offering price guarantees hurts the customer surplus when the firm uses the high/low pricing strategy because, even taking into account the price adjustment under the price guarantee, it is less likely for customers to pay the sale price V_L. We have the following corollary.

Corollary 2. *Customer surplus under each pricing strategy, with and without price guarantees, is summarized in Table 2.*

Proposition 5 shows that the firm is more likely to offer high/low pricing when a price guarantee is offered. Therefore, when a price guarantee is introduced, the firm may switch from static pricing to high/low pricing with price guarantees.[5] The firm may also switch from high/low pricing without price guarantees to high/low pricing with price guarantees.[6] Proposition 6 reports the changes to both customer surplus and social welfare before and after a price guarantee is introduced.

Proposition 6. *When a price guarantee is introduced, there are three possible switches in pricing strategies:*

(i) When the firm switches from static pricing at V_H (without price guarantees) to high/low pricing with price guarantees, both the customer surplus and social welfare increase;

(ii) When the firm switches from static pricing at V_L (without price guarantees) to high/low pricing with price guarantees, both the customer surplus and social welfare decrease;

[5] Suppose $K > 2m\lambda$ and condition (5) hold. If $\Phi^{B,*} > \alpha V_H$ (V_L), then the firm will switch from static pricing at V_H (V_L) to high/low pricing with price guarantees.

[6] Suppose $K > 2m\lambda$ and condition (8) hold. Then the firm will switch from high/low pricing without price guarantees to high/low pricing with price guarantees.

(iii) When the firm switches from high/low pricing without price guarantees to high/low pricing with price guarantees, the customer surplus decreases, and the social welfare either increases or decreases.

Proposition 6(i) shows that introducing a price guarantee can lead to a win-win outcome for the firm and customers when the firm switches from static pricing at V_H to high/low pricing with price guarantees. Unfortunately, this is the only price switch that leads to a win-win outcome. In the other two possible switches, the customer surplus decreases. This result is somewhat surprising because price guarantees are usually viewed positively by consumer groups, as they allow customers to take advantage of favorable price changes after purchase. However, when price guarantees are introduced, the firm is less likely to offer the sale price; hence, on average customers may pay a higher price even after taking into account the price adjustment made possible by price guarantees.

7. An Alternative Assumption on Customers' Monitoring Behavior

In the base model, we interpret customers' lifetime as their interest in the product. Once a customer makes a purchase, her lifetime does not matter anymore. Under this assumption, a purchased customer who decides to monitor for price refund keeps monitoring until the price guarantee applies or expires. Alternatively, one may interpret customers' lifetime as a patience parameter. Under this alternative interpretation, a customer who purchases at a high price may stop monitoring when her lifetime ends (her patience runs out) before the price guarantee expires. This section provides the results when we adopt such an alternative assumption, where a customer stops monitoring for price refund when the price guarantee expires or her lifetime ends, whichever occurs earlier. We find that our main results and insights still hold qualitatively and are therefore robust under the alternative assumption.

Following the same procedure as in the base model, we first analyze customer's optimal purchase decisions, based on which we derive the optimal Markovian pricing strategy. Due to the page limit, we relegate the detailed analysis to Section S.1 of the Online Supplement. Proposition 7 below, a counterpart of Proposition 2 in the base model, characterizes the optimal pricing strategy with price guarantees when type II customers' monitoring cost is high. Let

$$c_1(r_H, r_L, \mu_H, T) = \frac{(1-e^{-\lambda T})\mu_H + e^{-\lambda T}(\lambda+\mu_H)(1-e^{-\mu_H T})}{2-e^{-\lambda T}-e^{-\mu_H T}}(r_H - r_L).$$

PROPOSITION 7 **(The optimal Markovian pricing strategy for a high monitoring cost).** *Suppose the monitoring cost c is high (i.e., $c > c_1(r_H, r_L, \mu_H, T)$) such that type II customers do not monitor the price after they purchase at the price r_H. If $K \leq 2m\lambda$, then the firm's optimal* *pricing strategy reduces to that without price guarantees in the base model. If $K > 2m\lambda$, there are three possible outcomes for the firm's optimal pricing strategy: (i) static pricing at V_H; (ii) Static pricing at V_L;*

(iii) ***High/low pricing with price guarantees.*** *Only if*

$$c > \frac{\lambda}{2}\left(\sqrt{\frac{K}{2m\lambda}} - 1\right)(V_H - V_L), \tag{10}$$

the firm uses a Markovian pricing strategy with price guarantees where

$$r_H^{B,*} = V_H, \quad r_L^{B,*} = V_L, \quad T^* = \infty, \quad \mu_H^{B,*} = \lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right), \quad \mu_L^{B,*} = \infty.$$

The profit per unit time is

$$\begin{aligned}\Phi^{B,*} = {} & (\alpha-\gamma)V_H + (\beta-\gamma)\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L + \gamma\left[\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L + \sqrt{\frac{2m\lambda}{K}}V_H\right] \\ & - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right).\end{aligned} \tag{11}$$

Customer behavior is the same as that in Proposition 2(iii).

Observe that the firm's optimal Markovian pricing strategy and the corresponding customer behavior are almost the same as that in Proposition 2 in the base model. There are only two differences. One is the cutoff on the threshold of c. The threshold in condition (10) is lower than that in condition (5), implying that the high/low pricing strategy is valid for a broader parameter range under the alternative assumption. This can be explained as follows. Recall that the high/low pricing strategy is optimal only if c is large enough such that type II customers do not monitor for price refund. Under the alternative assumption, customers stop monitoring whenever their lifetime ends, making it less likely for customers to receive the price refund, compared to the scenario in the base model where customers keep monitoring until the price guarantee expires. In other words, type II customers are less willing to purchase and monitor under the alternative assumption. Therefore, the monitoring cost c does not have to be as large as that in the base model.

The other difference is the optimal expiration term of the price guarantee. Proposition 7 shows that the optimal guarantee duration is set to infinity under the alternative assumption. Since customers stop monitoring for price refund when their lifetime ends, $T^* = \infty$ means that the probability of getting the refund is equal to the probability that the price switches to r_L before the customer's lifetime ends ($\frac{\mu_H}{\lambda+\mu_H}$). Recall that in the base model, customers keep monitoring the price until the price guarantee expires, and the optimal guarantee duration is set to a finite value such that the probability of getting the refund ($1 - e^{\mu_H T^*}$, which is the probability that the price switches to r_L before the price guarantee expires) is also equal to the probability that the price switches to r_L before their lifetime ends ($\frac{\mu_H}{\lambda+\mu_H}$). That is, under either assumption, the expiration term is set to make the probability of getting the refund equal to the probability that the price switches to r_L before the customer's lifetime ends. Importantly, the pricing strategy (r_H, r_L, μ_H, μ_L), the firm's optimal revenue, and the corresponding customer behavior are the same under the alternative and original assumptions. This indicates that our main results and insights are robust against the assumption of whether customers keep or stop monitoring the price when their lifetime ends.

8. Summary and Concluding Remarks

This study examines the impact of price guarantees on a firm's profit and customer behavior under Markovian pricing strategies, where the firm sells to short-lived customers with an exponentially distributed lifetime duration. Customers differ in their valuations and price monitoring costs. With price guarantees, customers are refunded the price difference if the price drops within a given time window after purchase. We show that, compared with not offering price guarantees, offering price guarantees can improve the firm's profit. This is because high/low pricing with price guarantees not only differentiates customers with different price monitoring costs, but also price discriminates customers with different valuations and retains customers effectively by encouraging early purchases. Furthermore, we show that the firm offers the sale price less often under price guarantees, and offering price guarantees does not always improve customer welfare or social welfare.

Our work can be extended in several ways. First, although customers' lifetime can be used to capture certain competitive effects implicitly (i.e., customers within more competitive product categories are expected to have shorter lifetimes), it does not explicitly model competition. Therefore, one immediate direction is to investigate competitive Markovian pricing strategies, which is pursued in the recent work of Du et al. (2022). Second, several interesting behavioral considerations would enrich the model and analysis. One example is customer forgetfulness. Offering a price guarantee might entice customers to purchase sooner at higher prices; yet, some customers may forget to use the price guarantee in the event of a price drop. Such customer forgetfulness is likely to benefit the firm and make offering price guarantees even more appealing. Third, our model assumes that customers are refunded exactly the price difference when price guarantees are applied. However, the refund amount may exceed the price difference to compensate for customers' "hassle" or as a way of "penalizing" the firm for the price change (Cohen-Vernik and Pazgal 2017). Would a refund amount different from (especially above) the price difference ever be optimal? It is possible because allowing the seller to optimize over the price difference enlarges the seller's decision space. Finally, our model makes several empirically testable predictions. It would thus be interesting to relate market characteristics to the modeling elements to empirically test these results.

Acknowledgment

J. Wu was supported by National Natural Science Foundation of China [grant number 72271234] and Natural Science Foundation of Beijing Municipality [grant number 9212010]. Y. Liu was substantially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. PolyU15511123). Y. Liu was also supported by a grant from NSFC [72293564/72293560]. The authors thank the department editor Haresh Gurnani, a senior editor, and two reviewers for comments and suggestions that substantially improved the paper. All remaining errors are the authors' own. The authors contributed equally to this work.

References

Abe, Makoto. 2009. "Counting your customers" one by one: A hierarchical bayes extension to the pareto/nbd model. *Marketing Science* **28**(3) 541–553.

Aviv, Y., Y. Levin, M. Nediak. 2009. Counteracting strategic consumer behavior in dynamic pricing systems. S. Netessine, CS. Tang, eds., *Consumer-Driven Demand and Operations Management Models: A Systematic Study of IT-Enabled Sales Mechanisms*. Springer, New York, 323–352.

Bemmaor, Albert C., Nicolas Glady. 2012. Modeling purchasing behavior with sudden "death": A flexible customer lifetime model. *Management Science* **58**(5) 1012–1021.

Besanko, D., W. L. Winston. 1990. Optimal price skimming by a monopolist facing rational customers. *Management Science* **36**(5) 555–567.

Besbes, Omar, Ilan Lobel. 2015. Intertemporal price discrimination: Structure and computation of optimal policies. *Management Science* **61**(1) 92–110.

Board, Simon, Andrzej Skrzypacz. 2016. Revenue management with forward-looking buyers. *Journal of Political Economy* **124**(4) 1046–1087.

Caldentey, R., G. Vulcano. 2007. Online auction and list price revenue management. *Management Science* **53**(5) 795–813.

Çelik, Sabri, Alp Muharremoglu, Sergei Savin. 2009. Revenue management with costly price adjustments. *Operations research* **57**(5) 1206–1219.

Chen, Hongqiao, Ming Hu, Jiahua Wu. 2023. Intertemporal price discrimination via randomized promotions. *Manufacturing & Service Operations Management* **25**(3) 1176–1194.

Cohen-Vernik, Dinah, Amit Pazgal. 2017. Price adjustment policy with partial refunds. *Journal of Retailing* **93**(4) 507 – 526.

Conlisk, J., E. Gerstner, J. Sobel. 1984. Cyclic pricing by a durable goods monopolist. *The Quarterly Journal of Economics* **99** 489–505.

Corts, K. 1996. On the competitive effects of price-matching policies. *International Journal of Industrial Organization* **15**(3) 283–299.

Dilmé, F., F. Li. 2019. Revenue management without commitment dynamic pricing and periodic flash sales. *The Review of Economic Studies* **86**(5) 1999–2034.

Du, Haokun, Bin Hu, Elena Katok. 2022. Competitive markovian pricing. Working paper, Naveen Jindal School of Management, University of Texas, Dallas.

Elmaghraby, W., A. Gülcü, P. Keskinocak. 2008. Designing optimal pre-announced markdowns in the presence of rational customers with multi-unit demands. *Manufacturing and Service Operations Management* **10**(1) 126–148.

Elmaghraby, W., P. Keskinocak. 2003. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science* **49**(10) 1287–1309.

Fortin, David R. 2000. Clipping coupons in cyberspace: A proposed model of behavior for deal-prone consumers. *Psychology & Marketing* **17**(6) 515–534.

Hay, G. 1982. Oligopoly, shared monopoly, and antitrust law. *Cornell Law Review* **67**(3) 245–258.

Huang, Tingliang, Zhe Yin, Ying-Ju Chen. 2017. Managing posterior price matching: The role of customer boundedly rational expectations. *Manufacturing & Service Operations Management* **19**(3) 385–402.

Hviid, M. 2010. Summary of the literature on price-guarantees. Working paper, ESRC Center for competition Policy and UEA Law School.

Hviid, M., G. Shaffer. 1999. Hassle costs: the achilles heel of price-matching guarantees. *Journal of Economics and Management Strategy* **8**(4) 489–521.

Hviid, M., G. Shaffer. 2012. Optimal low-price guarantees with anchoring. *Quantitative Marketing and Economics* **10**(3) 393–417.

Jain, S., J. Srivastava. 2000. An experimental and theoretical analysis of price-matching refund policies. *Journal of Marketing Research* **37**(3) 351–362.

Janssen, M., A. Parakhonyak. 2013. Price matching guarantees and consumer search. *International Journal of Industrial Organization* **31**(1) 1–11.

Lai, G., L. G. Debo, K. Sycara. 2010. Buy now and match later: The impact of posterior price matching on profit with strategic consumers. *Manufacturing and Service Operations Management* **12**(1) 33–55.

Levin, Y., J. McGill, M. Nediak. 2007. Price guarantees in dynamic pricing and revenue management. *Operations Research* **55**(1) 75–97.

Levy, Daniel, Georg Müller, Haipeng Chen, Mark Bergen, Shantanu Dutta. 2010. Holiday price rigidity and cost of price adjustment. *Economica* **77**(305) 172–198.

Liu, Q., G. J. van Ryzin. 2008. Strategic capacity rationing to induce early purchases. *Management Science* **54**(6) 1115–1131.

Liu, Yan, William L Cooper. 2015. Optimal dynamic pricing with patient customers. *Operations Research* **63**(6) 1307–1319.

Lobel, Ilan. 2019. Dynamic pricing with heterogeneous patience levels. Forthcoming, *Operations Research*.

Mamadehussene, S. 2019. Price-matching guarantees as a direct signal of low prices. *Journal of Marketing Research* **56**(2) 245–258.

Moon, Ken, Kostas Bimpikis, Haim Mendelson. 2017. Randomized markdowns and online monitoring. *Management Science* **64**(3) 1271–1290.

Moorthy, S., R. Winter. 2006. Price-matching guarantees. *RAND Journal of Economics* **37**(2) 449–465.

Nalca, Arcan, Tamer Boyaci, Saibal Ray. 2013. Competitive price-matching guarantees: Equilibrium analysis of the availability verification clause under demand uncertainty. *Management Science* **59**(4) 971–986.

Netessine, S. 2006. Dynamic pricing of inventory/capacity with infrequent price chanegs. *European Journal of Operational Research* **174**(1) 553–580.

Netessine, S., C. S. Tang. 2009. *Consumer-Driven Demand and Operations Management Models*. Springer.

Png, I., D. Hirshleifer. 1987. Price discrimination through offers to match price. *Journal of Business* **60**(3) 365–383.

Puterman, M. L. 1994. *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley & Sons, New York.

Ross, S. M. 1996. *Stochastic Processes*. John Wiley & Sons, New York.

Salop, S. 1986. Practices that (credibly) facilitate oligopoly coordination. J. Stiglitz, F. Mathewson, eds., *New Developments in the Analysis of Market Structure*. MIT Press, 265–294.

Slade, M. E. 1998. Optimal pricing with costly adjustment: Evidence from retail-grocery prices. *Review of Economic Studies* **65** 87–107.

Su, X. 2007. Inter-temporal pricing with strategic customer behavior. *Management Science* **53**(5) 726–741.

Wang, Ruxian, Ozge Sahin. 2018. The impact of consumer search cost on assortment planning and pricing. *Management Science* **64**(8) 3649–3666.

Wu, Jianghua, Ling Li, Li Da Xu. 2014. A randomized pricing decision support system in electronic commerce. *Decision Support Systems* **58** 43–52.

Xu, Frances Zhiyun. 2011. Optimal best-price policy. *International Journal of Industrial Organization* **29**(5) 628 – 643.

Zhang, Huanan, Stefanus Jasin. 2022. Online learning and optimization of (some) cyclic pricing policies in the presence of patient customers. *Manufacturing & Service Operations Management* **24**(2) 1165–1182.

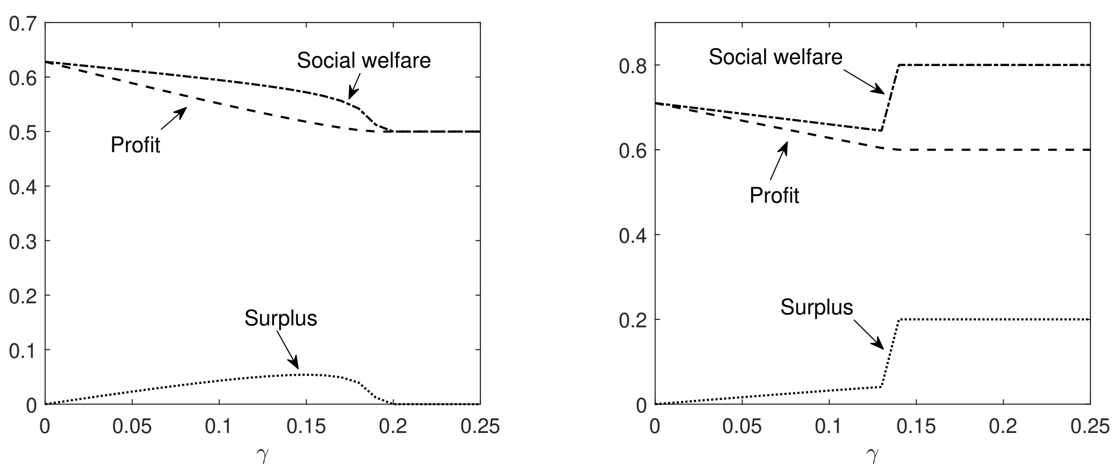
E-Companion
"Markovian Pricing with Price Guarantees"

Jianghua Wu, Dan Zhang, Yan Liu

This e-companion is divided into four sections. Section E.1 includes a sensitivity analysis to investigate how the optimal profit, customer surplus, and social welfare vary with respect to key model parameters. Section E.2 includes all auxiliary lemmas that will be used in Section E.3. Section E.3 provides the proofs of the lemmas and propositions for the base model. Section E.4 provides the proofs of auxiliary lemmas and Proposition 1.

E.1. Sensitivity Analysis

This section analyzes how the optimal profit, customer surplus, and social welfare vary with respect to key model parameters, such as the correlation between valuation and monitoring cost (measured by γ), and customers' lifetime duration (measured by λ). In our numerical experiments, we adopt the same parameter values as in Figure 4 with $(\alpha,\beta,V_H,c,m)=(0.5,0.5,1,0.1,0.1)$. Figures E.1 and E.2 present the results for different values of V_L. When $V_L=0.4$, static pricing at V_L is dominated by static pricing at V_H, as $\alpha V_H=0.5$. However, when $V_L=0.6$, static pricing at V_L dominates static pricing at V_H.



(a) $V_L=0.4$ (b) $V_L=0.6$

Figure E.1 Optimal profit, customer surplus, and social welfare v.s. γ

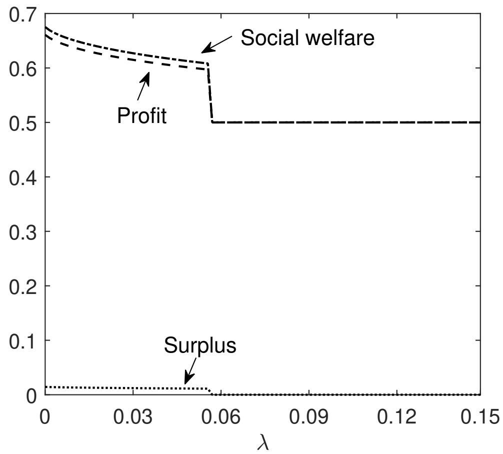
Figure E.1 depicts how the optimal profit, customer surplus, and social welfare change with respect to the fraction of high-valuation type I customers, denoted by γ. Note that the larger the fraction γ, the higher the correlation between the two dimensions (valuation and monitoring cost). When γ is relatively small (i.e., $\gamma\le 0.2$), high/low pricing with price guarantees performs better than static pricing. In this case, the optimal Markovian pricing strategy is the high/low pricing. Recall that Proposition 3 shows that a negative correlation is necessary for the profitability of the high/low pricing strategy. Therefore, the optimal profit decreases as γ increases. However, when γ is relatively large, static pricing outperforms the high/low pricing strategy. Consequently, the optimal profit is flat in this parameter region. This explains the observed pattern in Figures E.1(a) and (b), where the optimal profit first decreases and then remains flat as γ increases.

According to Table 2, the aggregate customer surplus under high/low pricing is $\gamma(1 - \sqrt{\frac{2m\lambda}{K}})(V_H - V_L)$, all of which is contributed by high-valuation type I customers. One might expect that customer surplus would increase as the fraction of high-valuation type I customers, γ, rises. However, it is important to note that μ_H^*, the rate of offering the low price V_L, decreases as γ increases. This is because the firm has an incentive to offer the sale price less frequently to avoid the loss from the price refund claimed by the high-valuation type I customers. In other words, a high-valuation type I customer is less likely to pay the low price V_L as γ increases. Therefore, the net effect on customer surplus depends on the relative magnitude of the two countervailing factors. Figure E.1(a) shows that when $V_L = 0.4$, the consumer surplus first increases and then decreases as γ increases under the high/low pricing. When γ is large such that static pricing at V_H dominates, customer surplus reduces to 0 as only high-valuation customers make a purchase. In contrast, Figure E.1(b) shows that when $V_L = 0.6$, the consumer surplus always increases in γ under the high/low pricing. When γ is large such that static pricing at V_L dominates, customer surplus increases to $\alpha(V_H - V_L)$ as all high-valuation customers pay the low price V_L.

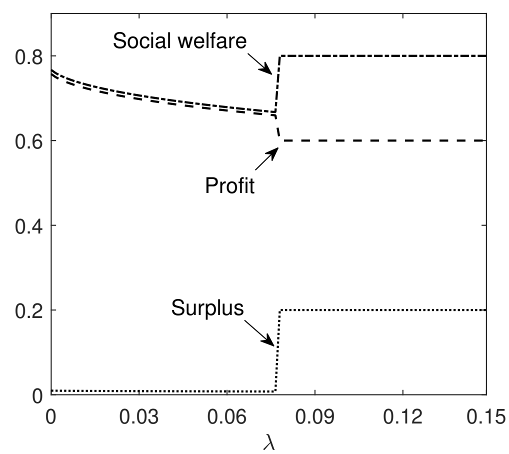
Figures E.1(a) and (b) also show that under the high/low pricing strategy, the social welfare decreases as the fraction of high-valuation type I customers, γ, increases. This is because the social welfare contributed by low-valuation type I customers decreases. Note that LI customers wait for the sale price when the price is high and pay a price equal to their valuation in their purchase. However, as the firm offers the sale price less frequently (i.e., μ_H^* decreases) with a higher γ, the purchasing probability of low-valuation type I customers decreases. This, in turn, leads to a lower overall social welfare.

This sensitivity analysis highlights the relationship between the fraction of high-valuation type I customers, the optimal pricing strategy, and its impact on customer surplus and social welfare. The different patterns observed for $V_L = 0.4$ and $V_L = 0.6$ further illustrate how the relative magnitudes of the high and low valuations can influence these dynamics.

Figure E.2 depicts how the optimal profit, customer surplus, and social welfare change with respect to customers' lifetime duration λ. When λ is relatively large (i.e., customers stay in the



(a) $V_L = 0.4$



(b) $V_L = 0.6$

Figure E.2 Optimal profit, customer surplus, and social welfare v.s. λ

market for a very short time on average), the high/low pricing strategy becomes less effective. Therefore, the high/low pricing is optimal only when λ is relatively small. Otherwise, static pricing is optimal. We make several observations. First, the profit under the high/low pricing decreases as λ increases. As customers' lifetime duration becomes shorter, although high-valuation type I customers are less likely to obtain the price refund, low-valuation type I customers who wait for the sale price are also less likely to make a purchase. Second, the aggregate customer surplus also decreases in λ under the high/low pricing. As discussed earlier, the aggregate customer surplus is contributed by high-valuation type I customers only. Since they are less likely to claim the price refund as λ increases, the effective price paid by this segment of customers become higher, leading to less customer surplus. Similar to the pattern observed in Figure E.1, the customer surplus reduces to 0 when static pricing at V_H dominates (i.e., $V_L = 0.4$) and increases to $\alpha(V_H - V_L)$ when static pricing at V_L dominates (i.e., $V_L = 0.6$). Third, not surprisingly, the social welfare decreases in λ under the high/low pricing, because both the optimal profit and customer surplus decrease. Finally, both Figures E.1 and E.2 show that when the firm switches from the high/low pricing with price guarantees to static pricing at V_H (V_L), both the customer surplus and social welfare decrease (increase), consistent with Proposition 6.

E.2. Auxiliary Lemmas

Lemmas E.1 and E.2 will be used in the proofs of Propositions 1, 2, and 4. Lemma E.1 demonstrates the revenue contribution of a type II customer when the monitoring cost c is relatively high, which corresponds to the optimal purchase strategy of a type II customer outlined in Lemma 2.

Lemma E.1. *Consider a type II customer with parameters* (v,c) *where* $c > \mu_H(r_H - r_L)$. *The revenue contribution of the customer is as follows:*

(a) If $v < r_L$*, then the customer never purchases and the revenue contribution is 0;*

(b) If $r_L \leq v < r_H$*, then the customer purchases upon arrival when the price is* r_L*, but leaves immediately without a purchase when the price is* r_H*. The revenue contribution is* $\frac{\mu_H}{\mu_H+\mu_L} r_L$*;*

(c) If $v \geq r_H$*, then the customer purchases immediately at both prices, and the revenue contribution is* $\frac{\mu_L}{\mu_H+\mu_L} r_H + \frac{\mu_H}{\mu_H+\mu_L} r_L$.

Lemma E.2 demonstrates the revenue contribution of a type II customer when the monitoring cost c is relatively low, which corresponds to the optimal purchase strategy of a type II customer outlined in Lemma 3.

Lemma E.2. *Consider a type II customer with parameters* (v,c) *where* $c \leq \mu_H(r_H - r_L)$. *The revenue contribution of the customer is as follows:*

(a) If $v < r_L$*, then the customer never purchases and the revenue contribution is 0;*

(b) If $r_L \leq v < r_L + \frac{c}{\mu_H}$*, then the customer purchases upon arrival when the price is* r_L*, but leaves immediately without a purchase when the price is* r_H*. The revenue contribution is* $\frac{\mu_H}{\mu_H+\mu_L} r_L$*;*

(c) If $r_L + \frac{c}{\mu_H} \leq v < \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$*, then the customer purchases immediately at price* r_L *and waits for the price* r_L *when the price is* r_H*. The revenue contribution is* $\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H} r_L + \frac{\mu_H}{\mu_H+\mu_L} r_L$*;*

(d) If $v \geq \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H}$*, then the customer purchases immediately at both prices. If the purchase is made at the price* r_H*, she would keep monitoring the price until the price guarantee is applied/expired. The revenue contribution is* $\frac{\mu_L}{\mu_H+\mu_L}[(1 - e^{-\mu_H T}) r_L + e^{-\mu_H T} r_H] + \frac{\mu_H}{\mu_H+\mu_L} r_L$.

Lemma E.3 will be used in the proof of Proposition 1.

Lemma E.3. *Suppose* $c \leq \mu_H(r_H - r_L)$*. In the presence of price guarantees,*

(a) If $r_L = V_L$ *and* $r_L \leq V_H < r_L + \frac{c}{\mu_H}$*, then the high/low pricing strategy is not optimal;*

(b) If $r_L = V_L - \frac{c}{\mu_H}$ *and* $V_L \geq r_L + \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L)}{\lambda}$*, then the high/low pricing strategy is not optimal.*

Lemmas E.4 and E.5 will be used in the proof of Proposition 4. Lemma E.4 characterizes a type II customer's optimal purchase decisions when the price monitoring cost is low ($c \leq \mu_H(r_H - r_L)$) without price guarantees. It follows immediately by taking $T = 0$ in Lemma 3. Lemma E.5 shows that if the monitoring cost c is low enough such that a type II customer behaves as stated in Lemma E.4, then a high/low pricing strategy is not optimal.

Lemma E.4. *Consider a type II customer with parameters (v,c) where $c \le \mu_H(r_H - r_L)$ and the firm does not offer a price guarantee. The optimal purchase strategy of the customer is as follows:*

(a) If $v < r_L$, then $G(r_H) = G(r_L) = 0$ and the customer never purchases;

(b) If $r_L \le v < r_L + \frac{c}{\mu_H}$, then $G(r_H) = 0$, $G(r_L) = v - r_L$, and the customer purchases upon arrival when the price is r_L, but leaves immediately without a purchase when the price is r_H;

(c) If $r_L + \frac{c}{\mu_H} \le v < r_H + \frac{\mu_H(r_H - r_L) - c}{\lambda}$, then

$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H}, \quad G(r_L) = v - r_L.$$

The customer purchases immediately upon arrival when the price is r_L. When the price is r_H, the customer would wait for the price r_L and leave without a purchase if r_L is not offered before she leaves the market;

(d) If $v \ge r_H + \frac{\mu_H(r_H - r_L) - c}{\lambda}$, then $G(r_H) = v - r_H$ and $G(r_L) = v - r_L$. The customer purchases immediately upon arrival.

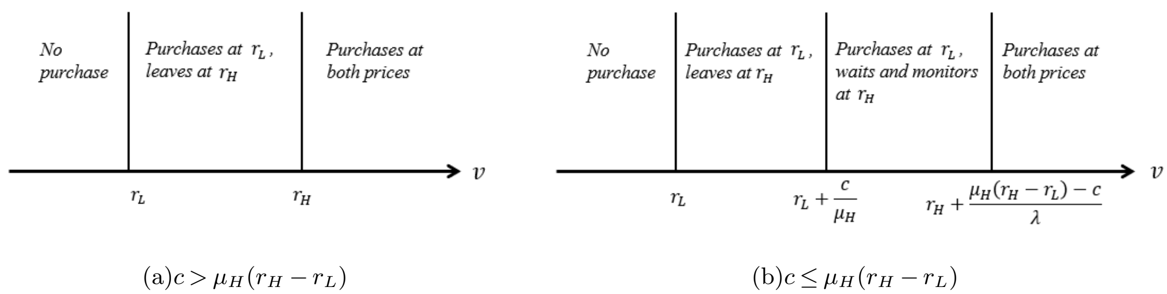


Figure E.3 A Type II Customer's Optimal Purchase Strategy without Price Guarantees.

Lemma E.5. *Without price guarantees, if the monitoring cost c is low enough (i.e., $c \le \mu_H(r_H - r_L)$) such that a type II customer behaves as stated in Lemma E.4, then a high/low pricing strategy is dominated by static pricing.*

E.3. Proofs of Lemmas and Propositions for the Base Model

Proof of Lemma 1

When the price is r_H, let X be the amount of time before the price is switched to r_L. Then X follows an exponential distribution with rate μ_H. A customer with valuation v purchasing at the price r_H earns an immediate surplus $v - r_H$. Suppose the customer monitors the price for t units of time (where $t \le T$), she may get a refund of $r_H - r_L$ in case the price drops before she stops monitoring ($X \le t$) but incur a price monitoring cost $c \cdot \min\{X, t\}$. Therefore, her total expected surplus is as follows:

$$v - r_H + (r_H - r_L) \cdot P(X \le t) - c \cdot E\Big[\min\{X, t\}\Big]$$

$$=v - r_H + (r_H - r_L)(1 - e^{-\mu_H t}) - c\Big[\int_0^t x dF(x) + t \cdot P(X > t)\Big]$$

$$=v - r_H + (r_H - r_L)(1 - e^{-\mu_H t}) - c\Big[x \cdot F(x)|_0^t - \int_0^t F(x)dx + t \cdot e^{-\mu_H t}\Big] \qquad \text{[by partial integration]}$$

$$=v - r_H + (r_H - r_L)(1 - e^{-\mu_H t}) - c\Big[t \cdot F(t) - \int_0^t (1 - e^{-\mu_H x})dx + t \cdot e^{-\mu_H t}\Big]$$

$$=v - r_H + (r_H - r_L)(1 - e^{-\mu_H t}) - c\Big[t \cdot (1 - e^{-\mu_H t}) - \Big[x + \frac{e^{-\mu_H x}}{\mu_H}\Big]_0^t + T \cdot e^{-\mu_H t}\Big]$$

$$=v - r_H + (r_H - r_L)(1 - e^{-\mu_H t}) - c\frac{1 - e^{-\mu_H t}}{\mu_H}$$

$$=v - r_H + (r_H - r_L - \frac{c}{\mu_H})(1 - e^{-\mu_H t}).$$

If the purchased customer chooses to monitor for price refund, it must be the case that $r_H - r_L - \frac{c}{\mu_H} \geq 0$, because otherwise, she will never choose to monitor. Given that $r_H - r_L - \frac{c}{\mu_H} \geq 0$, it follows immediately that the expected surplus above is maximized when $t = T$. Hence, if the customer who purchases at price r_H chooses to monitor the price, then it is optimal for her to keep monitoring until the price guarantee expires/applies. This completes the proof. ∎

Proof of Lemma 2

Part (a) is immediate.

Next, consider the situation when $r_L \leq v < r_H$. Note that $v - r_H < 0$, and thus the first term in equation (2) can be removed. Moreover, since $c > \mu_H(r_H - r_L)$, it follows immediately that $v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu}) < 0$, and thus the second term can also be removed. Therefore, equations (2) and (3) can be written as

$$G(r_H) = \max\Big\{\frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}, 0\Big\},$$
$$G(r_L) = v - r_L.$$

Recall that

$$\nu = \lambda + \mu_H + \mu_L.$$

We show $G(r_H) = 0$ by contradiction. Suppose for a contradiction that

$$G(r_H) = \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu} > 0.$$

Solving the equations gives

$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H} < \frac{\mu_H(v - r_L) - \mu_H(r_H - r_L)}{\lambda + \mu_H} = \frac{\mu_H(v - r_H)}{\lambda + \mu_H} \leq 0,$$

contradicting our supposition that $G(r_H) > 0$. Hence, it must be the case that $G(r_H) = 0$. This gives the solution in Part (b).

Now, suppose $v \geq r_H$. Note that $v - r_H \geq 0$, and thus the last term 0 in equation (2) can be removed. Because $c > \mu_H(r_H - r_L)$, it follows immediately that $v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu}) < v - r_H$, and thus the second term can also be removed. Therefore, equations (2) and (3) can be written as

$$\begin{aligned} G(r_H) &= \max\left\{v - r_H, \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}\right\}, \\ G(r_L) &= v - r_L. \end{aligned}$$

Following a similar approach as above, one can show that $G(r_H) = v - r_H$. This completes the proof. ■

Proof of Lemma 3

Part (a) is immediate.

Suppose $v \geq r_L$. We have

$$v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}) \geq v - r_H$$

when $c \leq \mu_H(r_H - r_L)$. Hence, equations (2)–(3) can be written as

$$\begin{aligned} G(r_H) &= \max\left\{v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}), \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu}, 0\right\}, \\ G(r_L) &= v - r_L. \end{aligned}$$

It remains to solve for $G(r_H)$, which we break into three cases. Recall that

$$\nu = \lambda + \mu_H + \mu_L.$$

<u>Case 1:</u> Suppose

$$v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}) \geq \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu}, \tag{E.1}$$

$$v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}) \geq 0. \tag{E.2}$$

It follows that

$$G(r_H) = v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}).$$

Using the expressions of $G(r_H)$ in (E.1) and simplifying (E.2), we obtain

$$v \geq r_L + \frac{c}{\mu_H} + \frac{(\lambda + \mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda},$$

$$v \geq r_H - (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}).$$

One can verify that

$$r_L + \frac{c}{\mu_H} + \frac{(\lambda + \mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} \geq r_H - (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H})$$

when $c \leq \mu_H(r_H - r_L)$. This gives the solution in Part (d).

Case 2: Suppose

$$\frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} \geq v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}), \tag{E.3}$$

$$\frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} \geq 0. \tag{E.4}$$

Then

$$G(r_H) = \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu}.$$

It follows that

$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H}.$$

Using the expression of $G(r_H)$ in (E.3) and (E.4), we obtain

$$v \leq r_L + \frac{c}{\mu_H} + \frac{(\lambda + \mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda},$$

$$v \geq r_L + \frac{c}{\mu_H}.$$

One can verify that

$$r_L + \frac{c}{\mu_H} < r_L + \frac{c}{\mu_H} + \frac{(\lambda + \mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda}$$

when $c \leq \mu_H(r_H - r_L)$. This provides the solution in Part (c).

Case 3: Suppose

$$v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}) < 0, \tag{E.5}$$

$$\frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} < 0. \tag{E.6}$$

It follows that $G(r_H) = 0$. Using the expression of $G(r_H)$ in (E.5) and (E.6), we obtain

$$v < r_H - (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}),$$

$$v < r_L + \frac{c}{\mu_H}.$$

One can verify that

$$r_H - (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}) > r_L + \frac{c}{\mu_H}$$

when $c \leq \mu_H(r_H - r_L)$. This leads to the solution in Part (b).

Combining the above cases completes the proof. ∎

A Sketch of Proof of Proposition 1

The proof of Proposition 1 is extensive, so we provide only a sketch of proof here. The complete proof is relegated to Section E.4.

According to Lemma 3, it is never optimal to charge a low price r_L that differs from V_L and $V_L - \frac{c}{\mu_H}$. Therefore, we consider two cases: $r_L = V_L$ and $r_L = V_L - \frac{c}{\mu_H}$.

When $r_L = V_L$, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, low-valuation type II customers also purchase at r_L, but they leave without making a purchase at r_H. Then, we only need to focus on the strategies of high-valuation type I and high-valuation type II customers. Since the monitoring cost c is low, these two segments of customers might choose either (a) to buy at the low price r_L while waiting at the high price r_H; or (b) to buy at both prices and monitor for price guarantees if a purchase is made at the high price r_H.

We analyze the four subcases (mixing the two options) one by one and find that each leads to a lower profit compared to static pricing. The proof for the case where $r_L = V_L - \frac{c}{\mu_H}$ follows a similar procedure. ■

Proof of Proposition 2

First, type I customers' behavior can be obtained by taking $c = 0$ in Lemma 3, while type II customers' behavior is the same as in Lemma 2. Taking into account the behavior of both types of customers, it is never optimal to charge the low price r_L different from V_L. The only remaining parameters are μ_H, μ_L, r_H, and T.

When $r_L = V_L$, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H, while low-valuation type II customers purchase at the price r_L but leave without a purchase when the price is r_H.

High-valuation type I customers purchase at both prices immediately if

$$V_H \geq r_L + \frac{(\lambda + \mu_H)e^{-\mu_H T}(r_H - r_L)}{\lambda}. \tag{E.7}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (E.7) is not satisfied, they would purchase at the price r_L immediately and wait and monitor when the price is r_H. Inequality (E.7) can be rewritten as

$$r_H \leq r_L + \frac{\lambda(V_H - r_L)}{(\lambda + \mu_H)e^{-\mu_H T}}.$$

For high-valuation type II customers, there are two possibilities. When $V_H < r_H$, they purchase at the price r_L but leave without a purchase when the price is r_H. At the same time, if high-valuation type I customers purchase at both prices, then customers' decision is the same as in Figure E.8; therefore, following the analysis in Lemma E.3(a), the high/low pricing strategy is never optimal. If high-valuation type I customers purchase at the price r_L but wait when the price is r_H, then no customers of the four segments purchase at the high price r_H; therefore, the high/low pricing strategy is also not optimal. To summarize, when $V_H < r_H$, the high/low pricing strategy is not optimal. Hereafter, we restrict our attention to the possibility with $V_H \geq r_H$, in which case high-valuation type II customers purchase at both prices but do not monitor for price guarantees.

We can analyze the firm's pricing problem based on the range of r_H. We consider two cases, labeled Cases I and II.

Case I: $r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}} < r_H \leq V_H$**.**

In this case, the purchase decisions of customers can be summarized in Figure E.4.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, do not monitor for price guarantee
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.4 Customer Purchase Decisions in Case I.

The analysis and result are the same as in the case without price guarantees (Case I in the proof of Proposition 4). Hence,

$$\Phi^{I,*} = \begin{cases} (\alpha-\gamma)V_H + \beta\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L - 2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}}-\lambda\right), & \text{if } \beta V_L - 2m\lambda > 0, \\ (\alpha-\gamma)V_H, & \text{if } \beta V_L - 2m\lambda \leq 0. \end{cases}$$

Case II: $r_H \leq r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$ **and** $r_H \leq V_H$**.**

In this case, the purchase decisions of customers can be summarized in Figure E.5.

Comparing $r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$ with V_H yields that

$$r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}} \geq V_H$$

if and only if $\frac{\lambda}{\lambda+\mu_H} \geq e^{-\mu_H T}$. Therefore, we have two subcases with respect to the range of T.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor for price guarantee	Buy at both prices, do not monitor for price guarantee
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.5 Customer Purchase Decisions in Case II.

Subcase II.A: $\frac{\lambda}{\lambda+\mu_H} \geq e^{-\mu_H T}$.

In this subcase, the optimal high price must be $r_H^* = V_H$. Let $\Phi^{II,A}(\mu_H,\mu_L,T)$ denote the firm's profit in this case. Then,

$$
\begin{aligned}
&\Phi^{II,A}(\mu_H,\mu_L,T)\\
=&\gamma\left[\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})V_L+e^{-\mu_H T}V_H]+\frac{\mu_H}{\mu_H+\mu_L}V_L\right]+(\alpha-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}V_H+\frac{\mu_H}{\mu_H+\mu_L}V_L\right]\\
&+(\beta-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}V_L+\frac{\mu_H}{\mu_H+\mu_L}V_L\right]+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}V_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\left\{\gamma[(1-e^{-\mu_H T})V_L+e^{-\mu_H T}V_H]+(\alpha-\gamma)V_H+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-V_L-2m\mu_H\right\}\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\left\{\gamma\big[V_L+e^{-\mu_H T}(V_H-V_L)\big]+(\alpha-\gamma)V_H+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-V_L-2m\mu_H\right\},
\end{aligned}
$$

where the revenue contribution of each customer segment follows Lemmas E.1 and E.2. To understand the cost of price changes, note that the prices go through cycles of high and low prices under Markovian pricing. The average cycle length is $\frac{1}{\mu_H}+\frac{1}{\mu_L}$, and there are two price changes in each cycle. Therefore, the number of price changes per unit time is $\frac{2\mu_H\mu_L}{\mu_H+\mu_L}$.

Note that this expression is decreasing in T, so $e^{-\mu_H T^*}=\frac{\lambda}{\lambda+\mu_H}$. The profit in this subcase is dominated by Subcase II.B analyzed below.

Subcase II.B: $\frac{\lambda}{\lambda+\mu_H} \leq e^{-\mu_H T}$.

In this subcase, the optimal high price must be $r_H^* = r_L+\frac{\lambda(V_H-r_L)}{(\lambda+\mu_H)e^{-\mu_H T}} = V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$. Let $\Phi^{II,B}(\mu_H,\mu_L,T)$ denote the firm's profit in this case. Then,

$$
\begin{aligned}
&\Phi^{II,B}(\mu_H,\mu_L,T)\\
=&\underbrace{\gamma\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\big[(1-e^{-\mu_H T})V_L+e^{-\mu_H T}r_H^*\big]+\frac{\mu_H}{\mu_H+\mu_L}\cdot V_L\right]}_{\text{revenue from HI customers}}
\end{aligned}
$$

$$
\begin{aligned}
&+\underbrace{(\alpha-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\left(V_L+\frac{\lambda(V_H-V_L)}{(\mu_H+\lambda)e^{-\mu_H T}}\right)+\frac{\mu_H}{\mu_H+\mu_L}\cdot V_L\right]}_{\text{revenue from HII customers}}\\
&+\underbrace{(\beta-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L+\frac{\mu_H}{\mu_H+\mu_L}V_L\right]}_{\text{revenue from LI customers}}\\
&+\underbrace{(1-\alpha-\beta+\gamma)\cdot\frac{\mu_H}{\mu_H+\mu_L}\cdot V_L}_{\text{revenue from LII customers}}-\underbrace{\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}}_{\text{cost of price changes}}\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\left[\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\alpha-\gamma)\left(V_L+\frac{\lambda(V_H-V_L)}{(\mu_H+\lambda)e^{-\mu_H T}}\right)\right.\\
&\left.+(\beta-\gamma)\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L-V_L-2m\mu_H\right],
\end{aligned}
$$

where the revenue contribution of each customer segment follows Lemmas E.1 and E.2. Note that the expression is increasing in T. At optimality, $e^{-\mu_H T^*}=\frac{\lambda}{\lambda+\mu_H}$. The profit expression simplifies to

$$
\begin{aligned}
&\Phi^{II,B}(\mu_H,\mu_L,T^*)\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\left[\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\alpha-\gamma)V_H+(\beta-\gamma)\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L-V_L-2m\mu_H\right].
\end{aligned}
$$

When the term in the square brackets is negative, the profit is less than V_L, which is the profit from static pricing at V_L. We proceed with the analysis assuming the term in the square brackets is positive. In our final analysis, we will compare the profit in this case with the optimal profit without price guarantees.

Since $\Phi^{II,B}(\mu_H,\mu_L,T^*)$ is increasing in μ_L, the optimal value of μ_L is ∞. We have

$$
\Phi^{II,B}(\mu_H,\infty,T^*)=\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\alpha-\gamma)V_H+(\beta-\gamma)\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L-2m\mu_H. \tag{E.8}
$$

It can be shown that when $K\le 0$, $\Phi^{II,B}(\mu_H,\infty,T^*)$ decreases in μ_H. Hence, the optimal $\mu_H=0$. The corresponding profit is

$$
\Phi^{II,B}(0,\infty,T^*)=\alpha V_H,
$$

which is the same as the revenue from static pricing at V_H.

When $K>0$, we can solve for μ_H using the first-order condition, which gives

$$
\mu_H^{II,B,*}=\begin{cases}\sqrt{\frac{\lambda K}{2m}}-\lambda, & \text{if } K-2m\lambda>0,\\ 0, & \text{if } K-2m\lambda\le 0.\end{cases}
$$

The corresponding profit is

$$\Phi^{II,B,*} = \begin{cases} \beta V_L + (\alpha-\gamma)V_H + (\gamma V_H - \beta V_L)\sqrt{\frac{2m\lambda}{K}} - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right), & \text{if } K - 2m\lambda > 0, \\ \alpha V_H, & \text{if } K - 2m\lambda \leq 0. \end{cases}$$

Moreover, when $K - 2m\lambda > 0$, by $e^{-\mu_H T^*} = \frac{\lambda}{\lambda+\mu_H}$, we obtain $T^* = \frac{\ln\sqrt{\frac{K}{2m\lambda}}}{\sqrt{\frac{\lambda K}{2m}}-\lambda}$, and thus $r_H^* = V_L + \frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)e^{-\mu_H T^*}} = V_H$. Putting $\mu_H^{II,B,*}$, r_H^*, and r_L^* back to $c > \mu_H(r_H - r_L)$ gives condition (5).

Summarizing the results for Cases I and II yields the following:

- If $K > 2m\lambda$, then

$$\Phi^{I,*} = (\alpha-\gamma)V_H + \beta\left(1 - \sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L - 2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}} - \lambda\right),$$
$$\Phi^{II,*} = \Phi^{II,B,*} = \beta V_L + (\alpha-\gamma)V_H + (\gamma V_H - \beta V_L)\sqrt{\frac{2m\lambda}{K}} - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right).$$

One can check that $\Phi^{II,*} \geq \Phi^{I,*}$. Hence,

$$\Phi^{B,*} = \beta V_L + (\alpha-\gamma)V_H + (\gamma V_H - \beta V_L)\sqrt{\frac{2m\lambda}{K}} - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right).$$

- If $K \leq 2m\lambda < \beta V_L$, then

$$\Phi^{I,*} = (\alpha-\gamma)V_H + \beta\left(1 - \sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L - 2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}} - \lambda\right),$$
$$\Phi^{II,*} = \Phi^{II,B,*} = \alpha V_H.$$

Hence,

$$\Phi^{B,*} = \max\left\{(\alpha-\gamma)V_H + \beta\left(1 - \sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L - 2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}} - \lambda\right), \alpha V_H\right\}.$$

- If $K \leq \beta V_L \leq 2m\lambda$, then $\Phi^{I,*} = (\alpha-\gamma)V_H < \alpha V_H = \Phi^{II,*}$. Hence, $\Phi^{B,*} = \Phi^{II,*} = \alpha V_H$.

Comparing V_L, αV_H, and $\Phi^{B,*}$ when $K \leq 2m\lambda$ shows that the pricing strategy is the same as that in Proposition 4. Comparing the profits when $K > 2m\lambda$ yields the results in Parts (i)–(iii). This completes the proof of Proposition 2. ■

Proof of Proposition 3

Note that $\Phi^{B,*} \geq \alpha V_H$ leads to the condition

$$\beta\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L - \gamma\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_H - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right) \geq 0. \tag{E.9}$$

And $\Phi^{B,*} \geq V_L$ leads to the condition

$$\alpha V_H - \gamma\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_H - \left(1-\beta+\beta\sqrt{\frac{2m\lambda}{K}}\right)V_L - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right) \geq 0. \tag{E.10}$$

It suffices to show that the conditions (E.9) and (E.10) for the optimality of the high/low pricing strategy do not hold when $\gamma \geq \alpha\beta$.

When $\gamma \geq \alpha\beta$, we have

$$\begin{aligned}
&\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_L - \gamma\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_H - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right)\\
\leq&\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_L - \alpha\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_H - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right)\\
=&\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)(V_L-\alpha V_H) - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right),
\end{aligned} \tag{E.11}$$

and

$$\begin{aligned}
&\alpha V_H - \gamma\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_H - \left(1-\beta+\beta\sqrt{\frac{2m\lambda}{K}}\right)V_L - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right)\\
\leq&\alpha V_H - \alpha\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_H - \left(1-\beta+\beta\sqrt{\frac{2m\lambda}{K}}\right)V_L - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right)\\
=&\alpha V_H\left[1-\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)\right] - \left[1-\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)\right]V_L - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right)\\
=&(\alpha V_H - V_L)\left[1-\beta\left(1-\sqrt{\frac{2m\lambda}{K}}\right)\right] - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right).
\end{aligned} \tag{E.12}$$

If $V_L \geq \alpha V_H$, then (E.12) is negative; otherwise, (E.11) is negative. That is, (E.9) and (E.10) cannot hold simultaneously. This completes the proof. ■

Proof of Proposition 4

We first consider static pricing. It is clear that the firm either uses the price V_L or the price V_H, with the corresponding profit rate of V_L or αV_H.

Lemma E.5 indicates that when c is small enough such that a type II customer behaves as in Lemma E.4, the high/low pricing strategy cannot improve the firm's profit. Now, suppose c is large enough such that a type II customer behaves as in Lemma 2.

A type I customer's behavior can be obtained by taking $c = 0$ in Lemma E.4. When $c = 0$, Lemma E.4(b) does not exist anymore. Taking into account the behavior of both types of customers, it is never optimal to charge a low price r_L different from V_L. Note that if $r_L = V_L - \frac{c}{\mu_H}$, then by $c > \mu_H(r_H - r_L)$, one can obtain $r_H < V_L$, and thus the firm's profit is no more than V_L. Hence, it is never optimal to set $r_L = V_L - \frac{c}{\mu_H}$. When $r_L = V_L$, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, low-valuation type II customers purchase at the price r_L but leave without a purchase when the price is r_H.

Next, we consider the high/low pricing strategy under which the firm needs to choose three parameters, r_H, μ_H, and μ_L. According to Lemma E.4, high-valuation type I customers purchase immediately if

$$V_H \geq r_H + \frac{\mu_H}{\lambda}(r_H - V_L). \tag{E.13}$$

If this condition is not satisfied, they purchase at the price V_L immediately and wait otherwise. Inequality (E.13) can be rewritten as

$$r_H \leq \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H}.$$

According to Lemma 2, high-valuation type II customers purchase immediately at both prices if $V_H \geq r_H$. If this condition is not satisfied, they purchase at the price r_L but leave without a purchase when the price is r_H. We ignore the possibility of $r_H > V_H$, because customers never purchase at $r_H > V_H$, in which case, the profit is no more than V_L.

From the condition on r_H, we can analyze the firm's pricing problem based on the range of r_H. We consider two cases, labeled Case I and Case II. We will show that the high/low pricing strategy in Case II can never be optimal. Therefore, finding the optimal solution to the firm's pricing problem entails comparing the solution to Case I with static pricing at either V_L or V_H, which leads to the results in Proposition 4.

Case I: $V_L < \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} < r_H \leq V_H$

In this case, the purchase decisions of customers can be summarized in Figure E.6.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.6 Customer Purchase Decisions in Case I.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^{I,*} = V_H$.

The only remaining parameters are μ_H and μ_L. Let $\Phi^I(\mu_H,\mu_L)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
\Phi^I(\mu_H,\mu_L) = & \underbrace{\beta\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\right]\cdot V_L}_{\text{revenue from HI and LI customers}} \\
& \underbrace{+(\alpha-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot V_H+\frac{\mu_H}{\mu_H+\mu_L}V_L\right]}_{\text{revenue from HII customers}} \\
& \underbrace{+(1-\alpha-\beta+\gamma)\cdot\frac{\mu_H}{\mu_H+\mu_L}\cdot V_L}_{\text{revenue from LII customers}} - \underbrace{\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}}_{\text{cost of price changes}} \\
= & V_L+\frac{\mu_L}{\mu_H+\mu_L}\left[(\alpha-\gamma)V_H-\frac{\lambda+(1-\beta)\mu_H}{\lambda+\mu_H}\cdot V_L-2m\mu_H\right]. \qquad \text{(E.14)}
\end{aligned}
$$

A few comments are in order. The revenue contribution of each customer segment follows Lemmas E.1 and E.2. To understand the cost of price changes, note that the prices go through cycles of high and low prices under Markovian pricing. The average cycle length is $\frac{1}{\mu_H}+\frac{1}{\mu_L}$, and there are two price changes in each cycle. Therefore, the number of price changes per unit time is $\frac{2\mu_H\mu_L}{\mu_H+\mu_L}$.

The profit is no more than that of static pricing at V_L if the term in the square brackets is negative. Hereafter, we assume that the term in the brackets is positive.

To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi^{I,*} = \max_{\mu_H\geq 0,\mu_L\geq 0}\Phi^I(\mu_H,\mu_L). \qquad \text{(E.15)}$$

Since the objective function is increasing in μ_L, $\mu_L^{I,*}=\infty$ at optimality. It follows that

$$\Phi^I(\mu_H,\infty) = \frac{\beta\mu_H}{\lambda+\mu_H}\cdot V_L+(\alpha-\gamma)V_H-2m\mu_H. \qquad \text{(E.16)}$$

It can be verified that $\Phi^I(\mu_H,\infty)$ is concave in μ_H. Therefore, the optimal μ_H can be obtained from the first-order condition, giving the solution

$$\mu_H^{I,*} = \begin{cases} \sqrt{\frac{\beta\lambda V_L}{2m}}-\lambda, & \text{if } \beta V_L-2m\lambda>0, \\ 0, & \text{if } \beta V_L-2m\lambda\leq 0. \end{cases}$$

The corresponding prices are

$$r_H^{I,*}=V_H, \quad r_L^{I,*}=V_L,$$

and the firm's profit is

$$\Phi^{I,*} = \begin{cases} (\alpha-\gamma)V_H+\beta\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L-2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}}-\lambda\right), & \text{if } \beta V_L-2m\lambda>0, \\ (\alpha-\gamma)V_H, & \text{if } \beta V_L-2m\lambda\leq 0. \end{cases}$$

We comment here that the expression for the firm's profit is obtained by plugging the optimal solution into the objective value. When $\beta V_L - 2m\lambda \le 0$, the profit is dominated by αV_H, which is the profit under static pricing at V_H. Therefore, the high/low pricing strategy when $\beta V_L - 2m\lambda \le 0$ is never optimal.

Putting μ_H^*, r_H^*, and r_L^* into the condition $c > \mu_H(r_H - r_L)$ yields the constraint

$$c > \lambda\Big(\sqrt{\frac{\beta V_L}{2m\lambda}} - 1\Big)(V_H - V_L).$$

Case II: $V_L < r_H \le \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} \le V_H$

In this case, the purchase decisions of customers can be summarized in Figure E.7.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.7 Customer Purchase Decisions in Case II.

Because the firm's profit is linear in the prices, the optimal high price must be $\frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H}$. Let $\Phi^{II}(\mu_H, \mu_L)$ denote the firm's profit per unit time in this case as a function of μ_H and μ_L. Then,

$$\begin{aligned}
\Phi^{II}(\mu_H,\mu_L) &= \underbrace{\alpha\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} + \frac{\mu_H}{\mu_H+\mu_L}\cdot V_L\right]}_{\text{revenue from HI and HII customers}} \\
&\quad + \underbrace{(\beta-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L + \frac{\mu_H}{\mu_H+\mu_L}V_L\right]}_{\text{revenue from LI customers}} \\
&\quad + \underbrace{(1-\alpha-\beta+\gamma)\cdot\frac{\mu_H}{\mu_H+\mu_L}\cdot V_L}_{\text{revenue from LII customers}} - \underbrace{\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}}_{\text{cost of price changes}} \\
&= V_L + \frac{\mu_L}{\mu_H+\mu_L}\left[\frac{\alpha(\lambda V_H+\mu_H V_L)+(\beta-\gamma)\mu_H V_L}{\lambda+\mu_H} - V_L - 2m\mu_H\right].
\end{aligned}$$

When the term in the square brackets is negative, the profit is less than V_L, which is the profit from charging the static price V_L. We proceed with our analysis by assuming that the term in the brackets is positive.

The profit maximization problem can be written as

$$\Phi^{II,*} = \max_{\mu_H \geq 0, \mu_L \geq 0} \Phi^{II}(\mu_H, \mu_L). \tag{E.17}$$

Since the objective function is increasing in μ_L, $\mu_L^{II,*} = \infty$ at optimality. It follows that

$$\Phi^{II}(\mu_H, \infty) = \frac{\alpha(\lambda V_H + \mu_H V_L) + (\beta - \gamma)\mu_H V_L}{\lambda + \mu_H} - 2m\mu_H. \tag{E.18}$$

It can be verified that $\Phi^{II}(\mu_H, \infty)$ is concave in μ_H. Therefore, the optimal μ_H can be obtained via the first-order condition, giving the solution

$$\mu_H^{II,*} = \begin{cases} \sqrt{\frac{\lambda[(\alpha+\beta-\gamma)V_L - \alpha V_H]}{2m}} - \lambda, & \text{if } \sqrt{\frac{\lambda[(\alpha+\beta-\gamma)V_L - \alpha V_H]}{2m}} - \lambda > 0, \\ 0 & \text{if } \sqrt{\frac{\lambda[(\alpha+\beta-\gamma)V_L - \alpha V_H]}{2m}} - \lambda \leq 0. \end{cases}$$

While we can write the solution to the optimization problem (E.17) explicitly, we instead show that the optimal profit is dominated by the profit under static pricing at either V_L or V_H. As a result, we can conclude that the high/low pricing strategy in this case is never optimal.

We have

$$\begin{aligned} \Phi^{II,*} = \Phi^{II}(\mu_H^{II,*}, \infty) &= \frac{\alpha(\lambda V_H + \mu_H^{II,*} V_L) + (\beta - \gamma)\mu_H^{II,*} V_L}{\lambda + \mu_H^{II,*}} - 2m\mu_H^{II,*} \\ &= (\alpha + \beta - \gamma)V_L + \frac{\lambda(\alpha V_H - (\alpha + \beta - \gamma)V_L)}{\lambda + \mu_H^{II,*}} - 2m\mu_H^{II,*}. \end{aligned}$$

If $\alpha V_H - (\alpha + \beta - \gamma)V_L \leq 0$, then

$$\Phi^{II,*} \leq (\alpha + \beta - \gamma)V_L - 2m\mu_H^{II,*} < (\alpha + \beta - \gamma)V_L \leq V_L.$$

If $\alpha V_H - (\alpha + \beta - \gamma)V_L > 0$, then

$$\Phi^{II,*} < (\alpha + \beta - \gamma)V_L + \frac{\lambda(\alpha V_H - (\alpha + \beta - \gamma)V_L)}{\lambda} - 2m\mu_H^{II,*} = \alpha V_H - 2m\mu_H^{II,*} \leq \alpha V_H.$$

This completes the proof. ■

Proof of Proposition 5

The result follows by comparing the profit under price guarantees in Proposition 2 with that in Proposition 4. We first show that the profit under high/low pricing with price guarantees specified in Part (iii) is always higher than that under high/low pricing with flash sales specified in Part (iii) of Proposition 4. By (E.8), we have

$$\Phi^{B,*} \geq \gamma \frac{\lambda V_H + \mu_H^* V_L}{\lambda + \mu_H^*} + (\alpha - \gamma)V_H + (\beta - \gamma) \cdot \frac{\mu_H^*}{\lambda + \mu_H^*} \cdot V_L - 2m\mu_H^*$$

$$
\begin{aligned}
&= \frac{\beta \mu_H^* V_L + \gamma \lambda V_H}{\lambda + \mu_H^*} + (\alpha - \gamma) V_H - 2m\mu_H^* \\
&\geq \beta \frac{\mu_H^*}{\lambda + \mu_H^*} V_L + (\alpha - \gamma) V_H - 2m\mu_H^* \\
&= \Phi^*.
\end{aligned}
$$

In the above, the first inequality follows from the optimality of $\mu_H^{B,*}$. Because high/low pricing with price guarantees generates higher profits than high/low pricing without price guarantees, it is more likely for high/low pricing in Part (iii) to be optimal, compared with situations where no price guarantee is offered.

Because $K \leq \beta V_L$, it follows immediately that $\mu_H^{B,*} \leq \mu_H^*$. ■

Proof of Proposition 6

We use the customer surplus reported in Table 2 in our proof.

(i) Under static pricing at V_H without price guarantees, the customer surplus is zero. When the firm switches to high/low pricing with price guarantees (which implies higher firm profit), the customer surplus is positive. Therefore, both the customer surplus and social welfare (which is the sum of the customer surplus and firm profit) increase.

(ii) Under static pricing at V_L without price guarantees, the firm's profit is V_L and the customer surplus is $\alpha(V_H - V_L)$; see Table 2. Hence, the social welfare is $\alpha V_H + (1-\alpha)V_L$. When the firm switches to high/low pricing with price guarantees, the customer surplus is $\gamma\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)(V_H - V_L)$, which is lower than $\alpha(V_H - V_L)$. Hence, the customer surplus decreases. The social welfare is

$$
\begin{aligned}
&(\alpha-\gamma)V_H + (\beta-\gamma)\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L + \gamma\left\{\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L + \sqrt{\frac{2m\lambda}{K}}V_H\right\} - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right) \\
&\qquad + \gamma\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)(V_H - V_L) \\
&= \alpha V_H + (\beta-\gamma)\left(1 - \sqrt{\frac{2m\lambda}{K}}\right)V_L - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right) \\
&\leq \alpha V_H + (1-\alpha)V_L.
\end{aligned}
$$

Hence, the social welfare also decreases.

(iii) As shown in Table 2, the customer surplus is lower when the firm switches from high/low pricing without price guarantees to high/low pricing with price guarantees.

Next, we discuss the social welfare. Under high/low pricing without price guarantees, the social welfare is

$$
\alpha V_H - \gamma\sqrt{\frac{2m\lambda}{\beta V_L}}V_H + (\beta-\gamma)\left(1 - \sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L - 2m\lambda\left(\sqrt{\frac{\beta V_L}{2m\lambda}} - 1\right).
$$

Under high/low pricing with price guarantees, the social welfare is

$$\alpha V_H + (\beta - \gamma)\left(1 - \sqrt{\frac{2m\lambda}{K}}\right) V_L - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right).$$

The comparison reduces to checking whether

$$(\beta - \gamma)\left(\sqrt{\frac{2m\lambda}{\beta V_L}} - \sqrt{\frac{2m\lambda}{K}}\right) V_L + \gamma\sqrt{\frac{2m\lambda}{\beta V_L}} V_H + 2m\lambda\left(\sqrt{\frac{\beta V_L}{2m\lambda}} - \sqrt{\frac{K}{2m\lambda}}\right)$$

is positive. Note that $K < \beta V_L$, so the first term above is negative, while the second and third terms are positive. Hence, the comparison of the social welfare depends on the parameters. That is, social welfare can either increase or decrease when the firm introduces price guarantees. ■

E.4. Proofs of Auxiliary Lemmas and Proposition 1

Proof of Lemma E.1

Lemma E.1 demonstrates the revenue contribution of a type II customer when the monitoring cost c is relatively high, which corresponds to the optimal purchase strategy of a type II customer outlined in Lemma 2. In Lemma 2, if $v < r_L$, the customer never purchases, and thus the revenue contribution is zero. If $r_L \leq v < r_H$, the customer only purchases when the price is r_L, the probability of which is $\frac{\mu_H}{\mu_H+\mu_L}$, and leaves if the price is r_H; hence, the revenue contribution is $\frac{\mu_H}{\mu_H+\mu_L} \cdot r_L$. If $v \geq r_H$, the customer always purchases at the current price. Taking into account the stationary probabilities of the price, the total revenue is

$$\frac{\mu_L}{\mu_H + \mu_L} \cdot r_H + \frac{\mu_H}{\mu_H + \mu_L} \cdot r_L.$$

This completes the proof. ■

Proof of Lemma E.2

Lemma E.2 demonstrates the revenue contribution of a type II customer when the monitoring cost c is relatively low, which corresponds to the optimal purchase strategy of a type II customer outlined in Lemma 3. Here, we only focus on Lemma 3(c)-(d). In Lemma 3(c), the customer purchases only at price r_L and will wait and monitor if the price is r_H. The solution to $G(r_H)$ implies that when the current price is r_H, the customer eventually purchases with probability $\frac{\mu_H}{\lambda+\mu_H}$, where $\frac{\mu_H}{\lambda+\mu_H}$ is also the probability that the firm offers a low price before the customer's lifetime ends. Taking into account the stationary probabilities of the price, the total expected revenue is

$$\frac{\mu_L}{\mu_H + \mu_L} \cdot \frac{\mu_H}{\lambda + \mu_H} \cdot r_L + \frac{\mu_H}{\mu_H + \mu_L} \cdot r_L.$$

In Lemma 3(d), the customer always purchases at the current price. However, the customer will be refunded the price difference if the price guarantee is applied. According to the solution of $G(r_H)$ in Lemma 3(d), the price minus the expected refund claimed by a customer who purchases at the high price is $r_H - (1-e^{-\mu_H T})(r_H - r_L) = (1-e^{-\mu_H T})r_L + e^{-\mu_H T} r_H$, where $1-e^{-\mu_H T}$ is exactly the probability that the firm offers a sale before the price guarantee expires, in which case she receives the refund and her revenue contribution is r_L. Taking into account the stationary probabilities of the price, the total revenue is

$$\frac{\mu_L}{\mu_H+\mu_L} \cdot \left[(1-e^{-\mu_H T})r_L + e^{-\mu_H T} r_H\right] + \frac{\mu_H}{\mu_H+\mu_L} \cdot r_L.$$

This completes the proof. ■

Proof of Lemma E.3

Part (a): When $r_L = V_L$, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, low-valuation type II customers purchase at the price r_L but leave without a purchase when the price is r_H.

When $r_L \le V_H < r_L + \frac{c}{\mu_H}$, high-valuation type II customers purchase at the price r_L but leave without a purchase when the price is r_H, because $r_H \ge r_L + \frac{c}{\mu_H} > v_H$.

High-valuation type I customers purchase at both prices immediately (and monitor for price guarantee if the purchase is made at the price r_H) if

$$V_H \ge r_L + \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L)}{\lambda}. \tag{E.19}$$

Otherwise, they would purchase at the price r_L but wait and monitor when the price is r_H.

If high-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H, then none of the four customer segments purchase at the high price r_H. Therefore, it is impossible to obtain a profit higher than $r_L = V_L$.

Suppose they purchase at both prices immediately, that is, (E.19) holds. (E.19) can be rewritten as

$$r_H \le r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

The decision of each segment of customers can be summarized in Figure E.8.

Because the firm's profit is linear in prices, we must have the optimal high price $r_H^* = r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$. The only remaining parameters are μ_H and μ_L. Let $\Phi(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. We have

$$\Phi(\mu_H,\mu_L,T) = (1-\beta)\frac{\mu_H}{\mu_H+\mu_L} r_L + \gamma\Big(\frac{\mu_H}{\mu_H+\mu_L} r_L + \frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T} r_H]\Big)$$

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at low price, leave at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.8 Customer Purchase Decisions.

$$
\begin{aligned}
&+(\beta-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
&=\frac{\mu_H}{\mu_H+\mu_L}r_L+\gamma\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L+e^{-\mu_H T}r_H]+(\beta-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
&=r_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma[(1-e^{-\mu_H T})r_L+e^{-\mu_H T}r_H]+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L-r_L-2m\mu_H\Big\}\\
&=V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-V_L-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\begin{aligned}
\Phi(\mu_H,\infty,T)&=\gamma\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-2m\mu_H\\
&=\frac{\gamma\lambda V_H+\beta\mu_H V_L}{\lambda+\mu_H}-2m\mu_H\\
&=\beta V_L+\frac{\gamma\lambda V_H-\beta\lambda V_L}{\lambda+\mu_H}-2m\mu_H.
\end{aligned}
$$

If $\gamma\lambda V_H-\beta\lambda V_L<0$, then $\Phi(\mu_H,\infty,T)<V_L$. Otherwise,

$$\Phi(\mu_H,\infty,T)<\beta V_L+\frac{\gamma\lambda V_H-\beta\lambda V_L}{\lambda}-2m\mu_H=\gamma V_H-2m\mu_H<\alpha V_H.$$

This completes the proof of Part (a).

Part (b): When $r_L=V_L-\frac{c}{\mu_H}$, low-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H.

When $V_H\ge V_L\ge r_L+\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H-r_L)}{\lambda}$, both high- and low-valuation type I customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H.

High-valuation type II customers purchase immediately if

$$V_H\ge r_L+\frac{c}{\mu_H}+\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H-r_L-\frac{c}{\mu_H})}{\lambda}. \tag{E.20}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (E.20) is not satisfied, they would purchase at the price r_L but wait and monitor when the price is r_H.

Note that when customers purchase at the price r_H and then monitor for price guarantees, they pay an effective price $(1-e^{-\mu_H T})r_L + e^{-\mu_H T} r_H$ eventually. Putting $r_L = V_L - \frac{c}{\mu_H}$ into $V_L \geq r_L + \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L)}{\lambda}$ yields

$$r_H \leq V_L - \frac{c}{\mu_H} + \frac{\lambda}{(\lambda+\mu_H)e^{-\mu_H T}} \frac{c}{\mu_H}.$$

Therefore, the effective price

$$\begin{aligned}
&(1-e^{-\mu_H T})r_L + e^{-\mu_H T} r_H \\
\leq &(1-e^{-\mu_H T})(V_L - \frac{c}{\mu_H}) + e^{-\mu_H T}\Big[V_L - \frac{c}{\mu_H} + \frac{\lambda}{(\lambda+\mu_H)e^{-\mu_H T}} \frac{c}{\mu_H}\Big] \\
= &V_L - \frac{c}{\lambda+\mu_H}.
\end{aligned}$$

That is, none of the four customer segments pay an effective price higher than V_L. Therefore, it is impossible to obtain a profit higher than V_L by adopting such a pricing strategy. ■

Proof of Lemma E.4

It holds immediately by taking $T = 0$ in Lemma 3. ■

Proof of Lemma E.5

We first consider static pricing. It is clear that the firm either uses the price V_L or the price V_H, with the corresponding profit rate of V_L or αV_H.

Suppose c is small enough such that a type II customer behaves as in Lemma E.4. Now, let us consider a high/low pricing strategy under which the firm needs to decide four parameters, r_H, r_L, μ_H, and μ_L. According to Lemma E.4, it is never optimal to charge a low price r_L different from V_L and $V_L - \frac{c}{\mu_H}$. Therefore, we consider two cases.

Case 1: $r_L = V_L$.

According to Lemma E.4, low-valuation type I customers purchase at the price r_L, but wait and monitor for a price drop when the price is r_H. Meanwhile, low-valuation type II customers purchase at the price r_L, but leave immediately without a purchase when the price is r_H.

High-valuation type I customers purchase immediately if

$$V_H \geq r_H + \frac{\mu_H(r_H - r_L)}{\lambda}. \tag{E.21}$$

If condition (E.21) is not satisfied, they purchase at the price r_L immediately but wait and monitor when the price is r_H.[7] Inequality (E.21) can be rewritten as

$$r_H \le \frac{\lambda V_H + \mu_H r_L}{\lambda + \mu_H}.$$

High-valuation type II customers purchase immediately if

$$V_H \ge r_H + \frac{\mu_H(r_H - r_L) - c}{\lambda}. \tag{E.22}$$

If this condition does not hold, they purchase at the price r_L immediately but wait and monitor if the price is r_H (as shown in Lemma E.4(c)). Since $r_L + \frac{c}{\mu_H} \le r_L + \frac{\mu_H(r_H - r_L)}{\mu_H} = r_H \le V_H$, Lemma E.4(b) will not occur. Inequality (E.22) can be written as

$$r_H \le \frac{\lambda V_H + \mu_H r_L + c}{\lambda + \mu_H}.$$

From the conditions on r_H, we can analyze the firm's pricing problem based on the range of r_H. We consider three cases, labeled Cases 1.1-1.3. We will show that each of the three cases generates a profit lower than $\max\{V_L, \alpha V_H\}$. We restrict our attention to $V_H \ge r_H$, because otherwise, customers never purchase at the high price r_H, in which case the high/low pricing strategy cannot improve the firm's profit.

Case 1.1: $V_H \ge r_H \ge \frac{\lambda V_H + \mu_H r_L + c}{\lambda + \mu_H} \ge \frac{\lambda V_H + \mu_H r_L}{\lambda + \mu_H}$.

In this case, both high-valuation type I and high-valuation type II customers purchase at the price r_L, but wait and monitor for r_L if the price is r_H. The purchase decisions of customers can be summarized in Figure E.9.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.9 Customer Purchase Decisions in Case 1.1.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = V_H$.

[7] Note that when $c = 0$, Lemma E.4(b) does not exist anymore.

The only remaining parameters are μ_H and μ_L. Let $\Phi^{1.1}(\mu_H, \mu_L)$ denote the firm's profit per unit time. Then,

$$
\begin{aligned}
&\Phi^{1.1}(\mu_H, \mu_L)\\
=&(\alpha+\beta-\gamma)\Big(\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big)r_L+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}V_L+(\alpha+\beta-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}V_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
<&V_L.
\end{aligned}
$$

Note that the last term on the right-hand side of the first equality is the cost of price changes. Under high/low pricing, the prices go through cycles of high and low prices. The average cycle length is $\frac{1}{\mu_H}+\frac{1}{\mu_L}$, and there are two price changes in each cycle. Therefore, the number of price changes per unit time is $\frac{2\mu_H\mu_L}{\mu_H+\mu_L}$.

Case 1.2: $V_H \geq \frac{\lambda V_H+\mu_H r_L+c}{\lambda+\mu_H} \geq r_H \geq \frac{\lambda V_H+\mu_H r_L}{\lambda+\mu_H}$.

In this case, high-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, high-valuation type II customers purchase at both prices immediately. Customers' purchase decisions are summarized in Figure E.10.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.10 Customer Purchase Decisions in Case 1.2.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{\lambda V_H+\mu_H r_L+c}{\lambda+\mu_H}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{1.2}(\mu_H, \mu_L)$ denote the firm's profit per unit time. Then,

$$
\begin{aligned}
&\Phi^{1.2}(\mu_H, \mu_L)\\
=&\beta\Big(\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big)r_L+(\alpha-\gamma)\Big(\frac{\mu_L}{\mu_H+\mu_L}r_H+\frac{\mu_H}{\mu_H+\mu_L}r_L\Big)\\
&+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{\mu_H}{\mu_H+\mu_L}r_L+\beta\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L+(\alpha-\gamma)\frac{\mu_L}{\mu_H+\mu_L}r_H-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}V_L+\beta\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L+c}{\lambda+\mu_H}-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L+\frac{\mu_L}{\mu_L+\mu_H}\left[\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L+c}{\lambda+\mu_H}-V_L\right]-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}.
\end{aligned}
$$

A few comments are in order. First, as in Case 1.1, the last term on the right-hand side is the cost of price changes. Second, it is impossible to obtain a profit higher than charging a static price V_L if the value of the term in the brackets is negative. We ignore this condition for now, as we will compare the profit in this case with the profit $\max\{V_L, \alpha V_H\}$ from static pricing in our final analysis.

To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$
\Phi^{1.2}_* = \max_{\mu_H\geq 0, \mu_L\geq 0}\Phi^{1.2}(\mu_H,\mu_L).
$$

To solve this optimization problem, first note that the objective function is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\Phi^{1.2}(\mu_H,\infty)=\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L+c}{\lambda+\mu_H}-2m\mu_H. \tag{E.23}
$$

It can be verified that $\Phi^{1.2}(\mu_H,\infty)$ is concave in μ_H. Therefore, using the first-order condition, we obtain

$$
\mu_H^* = \begin{cases}\sqrt{\frac{\beta\lambda V_L+(\alpha-\gamma)(\lambda V_L-\lambda V_H-c)}{2m}}-\lambda, & \text{if } \beta V_L+(\alpha-\gamma)(V_L-V_H-c/\lambda)\geq 2m\lambda,\\ 0, & \text{otherwise.}\end{cases}
$$

Putting μ_H^* into (E.23) gives the optimal profit $\Phi^{1.2}_*$ in this case. Next, we show

$$
\Phi^{1.2}_*\leq\max\{V_L,\alpha V_H\}.
$$

If $\mu_H^*=0$, then

$$
\Phi^{1.2}_*=(\alpha-\gamma)V_H\leq\max\{V_L,\alpha V_H\}.
$$

Suppose

$$
\mu_H^*=\sqrt{\frac{\beta\lambda V_L+(\alpha-\gamma)(\lambda V_L-\lambda V_H-c)}{2m}}-\lambda.
$$

Then, by the constraint $\beta V_L+(\alpha-\gamma)(V_L-V_H-c/\lambda)\geq 2m\lambda$, we obtain

$$
c\leq\frac{\lambda\beta V_L}{\alpha-\gamma}+\lambda(V_L-V_H). \tag{E.24}
$$

Putting (E.24) into (E.23) yields

$$
\begin{aligned}
\Phi^{1.2}(\mu_H, \infty) &\leq \beta \frac{\mu_H}{\lambda + \mu_H} V_L + (\alpha - \gamma) \frac{\lambda V_H + \mu_H V_L + \frac{\lambda \beta V_L}{\alpha - \gamma} + \lambda (V_L - V_H)}{\lambda + \mu_H} - 2m\mu_H \\
&= (\alpha + \beta - \gamma) V_L - 2m\mu_H \\
&< V_L.
\end{aligned}
$$

Case 1.3: $V_H \geq \frac{\lambda V_H + \mu_H r_L + c}{\lambda + \mu_H} \geq \frac{\lambda V_H + \mu_H r_L}{\lambda + \mu_H} \geq r_H$.

In this case, both high-valuation type I and high-valuation type II customers purchase immediately at both prices. Customers' purchase decisions are summarized in Figure E.11.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.11 Customer Purchase Decisions in Case 1.3.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{\lambda V_H + \mu_H r_L}{\lambda + \mu_H}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{1.3}(\mu_H, \mu_L)$ denote the firm's profit per unit time. Then,

$$
\begin{aligned}
&\Phi^{1.3}(\mu_H, \mu_L) \\
=&\alpha \left(\frac{\mu_L}{\mu_H + \mu_L} r_H + \frac{\mu_H}{\mu_H + \mu_L} r_L \right) + (\beta - \gamma) \left(\frac{\mu_L}{\mu_H + \mu_L} \frac{\mu_H}{\lambda + \mu_H} + \frac{\mu_H}{\mu_H + \mu_L} \right) r_L \\
&+ (1 - \alpha - \beta + \gamma) \frac{\mu_H}{\mu_H + \mu_L} r_L - \frac{2m\mu_H \mu_L}{\mu_H + \mu_L} \\
=& \frac{\mu_H}{\mu_H + \mu_L} r_L + \alpha \frac{\mu_L}{\mu_H + \mu_L} r_H + (\beta - \gamma) \frac{\mu_L}{\mu_H + \mu_L} \frac{\mu_H}{\lambda + \mu_H} r_L - \frac{2m\mu_H \mu_L}{\mu_H + \mu_L} \\
=& \frac{\mu_H}{\mu_H + \mu_L} V_L + \alpha \frac{\mu_L}{\mu_H + \mu_L} \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + (\beta - \gamma) \frac{\mu_L}{\mu_H + \mu_L} \frac{\mu_H}{\lambda + \mu_H} V_L - \frac{2m\mu_H \mu_L}{\mu_H + \mu_L} \\
=& V_L + \frac{\mu_L}{\mu_L + \mu_H} \left[\alpha \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + (\beta - \gamma) \frac{\mu_H}{\lambda + \mu_H} V_L - V_L - 2m\mu_H \right].
\end{aligned}
$$

Again, if the term in the square brackets is negative, it is impossible to obtain a profit higher than V_L. Hereafter, we assume that the term in the brackets is positive.

To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi_*^{1.3} = \max_{\mu_H \geq 0, \mu_L \geq 0} \Phi^{1.3}(\mu_H, \mu_L).$$

First note that the objective function is increasing in μ_L, so $\mu_L^* = \infty$. It follows that

$$\begin{aligned}\Phi^{1.3}(\mu_H, \infty) &= \alpha \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + (\beta - \gamma)\frac{\mu_H}{\lambda + \mu_H} V_L - 2m\mu_H \\ &= \frac{\alpha(\lambda V_H + \mu_H V_L) + (\beta - \gamma)\mu_H V_L}{\lambda + \mu_H} - 2m\mu_H \\ &= (\alpha + \beta - \gamma)V_L + \frac{\lambda(\alpha V_H - (\alpha + \beta - \gamma)V_L)}{\lambda + \mu_H} - 2m\mu_H.\end{aligned}$$

Instead of writing the solution to the above optimization problem explicitly, we show that the optimal profit is dominated by the profit under static pricing at either V_L or V_H. As a result, we can conclude that the high/low pricing strategy in this case is not optimal.

If $\alpha V_H - (\alpha + \beta - \gamma)V_L \leq 0$, then

$$\Phi^{1.3}(\mu_H^*, \infty) \leq (\alpha + \beta - \gamma)V_L - 2m\mu_H^* < (\alpha + \beta - \gamma)V_L \leq V_L.$$

If $\alpha V_H - (\alpha + \beta - \gamma)V_L > 0$, then

$$\Phi^{1.3}(\mu_H^*, \infty) < (\alpha + \beta - \gamma)V_L + \frac{\lambda(\alpha V_H - (\alpha + \beta - \gamma)V_L)}{\lambda} - 2m\mu_H^* = \alpha V_H - 2m\mu_H^* \leq \alpha V_H.$$

This completes the proof of Case 1.

Case 2: $r_L = V_L - \frac{c}{\mu_H}$**.**

According to Lemma E.4, both low-valuation type I and low-valuation type II customers purchase at the price r_L but wait and monitor if the price is r_H. The behavior of high-valuation type I and high-valuation type II customers is the same as in Case 1. Similar to Case 1, we consider three cases, labeled Cases 2.1-2.3 based on the range of r_H. Again, we will show that each of the three cases generates a profit lower than $\max\{V_L, \alpha V_H\}$.

Case 2.1: $V_H \geq r_H \geq \frac{\lambda V_H + \mu_H r_L + c}{\lambda + \mu_H} \geq \frac{\lambda V_H + \mu_H r_L}{\lambda + \mu_H}$.

Customers' purchase decisions are summarized in Figure E.12.

Since customers never purchase at the high price r_H, it is impossible to obtain a profit higher than $r_L = V_L - \frac{c}{\mu_H} < V_L$.

Case 2.2: $V_H \geq \frac{\lambda V_H + \mu_H r_L + c}{\lambda + \mu_H} \geq r_H \geq \frac{\lambda V_H + \mu_H r_L}{\lambda + \mu_H}$.

Customers' purchase decisions are summarized in Figure E.13.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{\lambda V_H + \mu_H r_L + c}{\lambda + \mu_H}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{2.2}(\mu_H, \mu_L)$ denote the firm's profit per unit time. Then,

$$\Phi^{2.2}(\mu_H, \mu_L)$$

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure E.12 Customer Purchase Decisions in Case 2.1.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure E.13 Customer Purchase Decisions in Case 2.2.

$$
\begin{aligned}
=&(1-\alpha+\gamma)\Big(\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big)r_L+(\alpha-\gamma)\Big(\frac{\mu_L}{\mu_H+\mu_L}r_H+\frac{\mu_H}{\mu_H+\mu_L}r_L\Big)-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c}{\mu_H})+(1-\alpha+\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})\\
&+(\alpha-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H(V_L-\frac{c}{\mu_H})+c}{\lambda+\mu_H}-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L-\frac{c}{\mu_H}+\frac{\mu_L}{\mu_L+\mu_H}\Bigg[(1-\alpha+\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-(V_L-\frac{c}{\mu_H})-2m\mu_H\Bigg].
\end{aligned}
$$

It is impossible to obtain a profit higher than charging a static price V_L if the term in the square brackets is negative. Hereafter, we assume that the term in the brackets is positive.

To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi_*^{2.2}=\max_{\mu_H\geq 0,\mu_L\geq 0}\Phi^{2.2}(\mu_H,\mu_L).$$

First note that the objective function is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\begin{aligned}
\Phi^{2.2}(\mu_H,\infty)&=(1-\alpha+\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-2m\mu_H,\\
&=\frac{\mu_H V_L+(\alpha-\gamma)\lambda V_H-(1-\alpha+\gamma)c}{\lambda+\mu_H}-2m\mu_H\\
&=V_L+\frac{(\alpha-\gamma)\lambda V_H-(1-\alpha+\gamma)c-\lambda V_L}{\lambda+\mu_H}-2m\mu_H.
\end{aligned}
$$

If $(\alpha-\gamma)\lambda V_H-(1-\alpha+\gamma)c-\lambda V_L \le 0$, then $\Phi^{2.2}(\mu_H,\infty)\le V_L-2m\mu_H<V_L$. Otherwise,

$$\begin{aligned}\Phi^{2.2}(\mu_H,\infty) &< V_L+\frac{(\alpha-\gamma)\lambda V_H-(1-\alpha+\gamma)c-\lambda V_L}{\lambda}-2m\mu_H\\ &=(\alpha-\gamma)V_H-(1-\alpha+\gamma)\frac{c}{\lambda}-2m\mu_H\\ &<\alpha V_H.\end{aligned}$$

Case 2.3: $V_H \ge \frac{\lambda V_H+\mu_H r_L+c}{\lambda+\mu_H} \ge \frac{\lambda V_H+\mu_H r_L}{\lambda+\mu_H} \ge r_H$.

Customers' purchase decisions are summarized in Figure E.14.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure E.14 Customer Purchase Decisions in Case 2.3.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^*=\frac{\lambda V_H+\mu_H r_L}{\lambda+\mu_H}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{2.3}(\mu_H,\mu_L)$ denote the firm's profit per unit time. Then,

$$\begin{aligned}&\Phi^{2.3}(\mu_H,\mu_L)\\ =&\alpha\Big(\frac{\mu_L}{\mu_H+\mu_L}r_H+\frac{\mu_H}{\mu_H+\mu_L}r_L\Big)+(1-\alpha)\Big(\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big)r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\ =&\frac{\mu_H}{\mu_H+\mu_L}r_L+\alpha\frac{\mu_L}{\mu_H+\mu_L}r_H+(1-\alpha)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\ =&\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c}{\mu_H})+\alpha\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\alpha)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\ =&V_L-\frac{c}{\mu_H}+\frac{\mu_L}{\mu_L+\mu_H}\left[\alpha\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\alpha)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})-(V_L-\frac{c}{\mu_H})-2m\mu_H\right].\end{aligned}$$

Again, if the term in the square brackets is negative, it is impossible to obtain a profit higher than V_L. Hereafter, we assume that the term in the brackets is positive.

To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi_*^{2.3}=\max_{\mu_H\ge 0,\mu_L\ge 0}\Phi^{2.3}(\mu_H,\mu_L).$$

First note that the objective function is increasing in μ_L, so $\mu_L^* = \infty$. It follows that

$$\begin{aligned}\Phi^{2.3}(\mu_H, \infty) &= \alpha \frac{\lambda V_H + \mu_H V_L - c}{\lambda + \mu_H} + (1-\alpha)\frac{\mu_H}{\lambda + \mu_H}(V_L - \frac{c}{\mu_H}) - 2m\mu_H \\ &= \alpha V_H + \frac{(V_L - \alpha V_H)\mu_H - c}{\lambda + \mu_H} - 2m\mu_H.\end{aligned}$$

We again show that the optimal profit is dominated by the profit under static pricing at V_L or V_H. As a result, we can conclude that the high/low pricing strategy in this case is never optimal. To see this, note that if $(V_L - \alpha V_H)\mu_H - c \leq 0$, then $\Phi^{2.3}(\mu_H, \infty) \leq \alpha V_H - 2m\mu_H < \alpha V_H$. Otherwise,

$$\Phi^{2.3}(\mu_H, \infty) < \alpha V_H + \frac{(V_L - \alpha V_H)\mu_H - c}{\mu_H} - 2m\mu_H = V_L - \frac{c}{\mu_H} - 2m\mu_H \leq V_L.$$

This completes the proof of Case 2.

Combining the results for both cases establishes the lemma. ■

Proof of Proposition 1

Let's consider the high/low pricing strategy under which the firm needs to choose five parameters, r_H, r_L, μ_H, μ_L, and T. According to Lemma 3, it is never optimal to charge a low price r_L different than V_L and $V_L - \frac{c}{\mu_H}$. Therefore, we consider two cases.

Case 1: $r_L = V_L$.

According to Lemma 3, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, low-valuation type II customers purchase at the price r_L but leave without a purchase at the price r_H.

High-valuation type I customers purchase at both prices immediately if

$$V_H \geq r_L + \frac{(\lambda + \mu_H)e^{-\mu_H T}(r_H - r_L)}{\lambda}. \tag{E.25}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (E.25) is not satisfied, they would purchase at the price r_L immediately and wait and monitor when the price is r_H.[8] Inequality (E.25) can be rewritten as

$$r_H \leq r_L + \frac{\lambda(V_H - r_L)}{(\lambda + \mu_H)e^{-\mu_H T}}.$$

High-valuation type II customers purchase immediately if

$$V_H \geq r_L + \frac{c}{\mu_H} + \frac{(\lambda + \mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda}. \tag{E.26}$$

[8] Note that when $c = 0$, Lemma 3(b) does not exist.

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (E.26) is not satisfied, there are two possibilities. When $r_L + \frac{c}{\mu_H} \leq V_H < r_L + \frac{c}{\mu_H} + \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda}$, they would purchase at the price r_L but wait and monitor when the price is r_H. When $r_L \leq V_H < r_L + \frac{c}{\mu_H}$, they would purchase at the price r_L but leave without a purchase when the price is r_H. However, Lemma E.3(a) implies that the high/low pricing strategy is not optimal in this case. Therefore, we restrict our attention to the first possibility if (E.26) does not hold. Inequality (E.26) can be written as

$$r_H \leq r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

From the conditions on r_H, we can analyze the firm's pricing problem based on the range of r_H. We consider four cases, labeled Cases 1.1-1.4. We will show each of the four cases generates a profit lower than the profit from static pricing at either V_L or V_H.

Case 1.1: Suppose

$$r_H > r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}},$$
$$r_H > r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

Then both high-valuation type I and high-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure E.15.

	Type I 0 (β)	Type II c (1-β)
V_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
V_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.15 Customer Purchase Decisions in Case 1.1.

Since customers never purchase at the price r_H, it is impossible to obtain a profit higher than $r_L = V_L$.

Case 1.2: Suppose

$$r_H > r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}, \tag{E.27}$$

$$r_H \le r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}. \tag{E.28}$$

Then high-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure E.16.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.16 Customer Purchase Decisions in Case 1.2.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{1.2}(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
&\Phi^{1.2}(\mu_H,\mu_L,T)\\
=&\beta\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big) + (\alpha-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H]\Big)\\
&+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L + \beta\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L + (\alpha-\gamma)\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\beta\frac{\mu_H}{\lambda+\mu_H}r_L + (\alpha-\gamma)[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] - r_L - 2m\mu_H\Big\}\\
=&V_L + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\beta\frac{\mu_H}{\lambda+\mu_H}V_L + (\alpha-\gamma)\Big[\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} - (\frac{\lambda}{\lambda+\mu_H} - e^{-\mu_H T})\frac{c}{\mu_H}\Big] - V_L - 2m\mu_H\Big\}.
\end{aligned}$$

If the term in the brackets is negative, then $\Phi^{1.2}(\mu_H,\mu_L,T) < V_L$. Suppose the term is positive. Note that $\Phi^{1.2}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^* = \infty$. It follows that

$$\Phi^{1.2}(\mu_H,\infty,T) = \beta\frac{\mu_H}{\lambda+\mu_H}V_L + (\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} - (\alpha-\gamma)(\frac{\lambda}{\lambda+\mu_H} - e^{-\mu_H T})\frac{c}{\mu_H} - 2m\mu_H.$$

Note that (E.27) and (E.28) imply that $e^{-\mu_H T} > \frac{\lambda}{\lambda+\mu_H}$. Note also that $\Phi^{1.2}(\mu_H,\infty,T)$ is decreasing in T, so $T^*=0$, and thus $e^{-\mu_H T^*}=1$. It follows that

$$\Phi^{1.2}(\mu_H,\infty,0) = \beta\frac{\mu_H}{\lambda+\mu_H}V_L + (\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L+c}{\lambda+\mu_H} - 2m\mu_H.$$

The remaining analysis is the same as Case 1.2 in the proof of Lemma E.5. In the end, we obtain $\Phi^{1.2}(\mu_H,\infty,0) < \max\{V_L,\alpha V_H\}$.

Case 1.3: Suppose

$$r_H \le r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}},$$
$$r_H > r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

Then high-valuation type I customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. Meanwhile, high-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure E.17.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.17 Customer Purchase Decisions in Case 1.3.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{1.3}(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
&\Phi^{1.3}(\mu_H,\mu_L,T)\\
=&\gamma\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H]\Big) + (\alpha+\beta-2\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)\\
&+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L + \gamma\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] + (\alpha+\beta-2\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] + (\alpha+\beta-2\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L - r_L - 2m\mu_H\Big\}\\
=&V_L + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} + (\alpha+\beta-2\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L - V_L - 2m\mu_H\Big\}.
\end{aligned}$$

If the term in the brackets is negative, then $\Phi^{1.3}(\mu_H, \mu_L, T) < V_L$. Suppose the term is positive. Note that $\Phi^{1.3}(\mu_H, \mu_L, T)$ is increasing in μ_L, so $\mu_L^* = \infty$. It follows that

$$\begin{aligned}\Phi^{1.3}(\mu_H, \infty, T) &= \gamma \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + (\alpha + \beta - 2\gamma)\frac{\mu_H}{\lambda + \mu_H} V_L - 2m\mu_H \\ &= (\alpha + \beta - \gamma)V_L + \frac{\gamma\lambda V_H - (\alpha + \beta - \gamma)\lambda V_L}{\lambda + \mu_H} - 2m\mu_H.\end{aligned}$$

If $\gamma\lambda V_H - (\alpha + \beta - \gamma)\lambda V_L < 0$, then $\Phi^{1.3}(\mu_H, \infty, T) < (\alpha + \beta - \gamma)V_L < V_L$. Otherwise,

$$\Phi^{1.3}(\mu_H, \infty, T) < (\alpha + \beta - \gamma)V_L + \frac{\gamma\lambda V_H - (\alpha + \beta - \gamma)\lambda V_L}{\lambda} - 2m\mu_H = \gamma V_H - 2m\mu_H < \alpha V_H.$$

<u>Case 1.4:</u> Suppose

$$r_H \le r_L + \frac{\lambda(V_H - r_L)}{(\lambda + \mu_H)e^{-\mu_H T}}, \tag{E.29}$$

$$r_H \le r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda + \mu_H)e^{-\mu_H T}}. \tag{E.30}$$

Then both high-valuation type I and high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure E.18.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure E.18 Customer Purchase Decisions in Case 1.4.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \min\left\{r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}, r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}\right\}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{1.4}(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}&\Phi^{1.4}(\mu_H, \mu_L, T) \\ =&\alpha\Big(\frac{\mu_H}{\mu_H + \mu_L} r_L + \frac{\mu_L}{\mu_H + \mu_L}[(1 - e^{-\mu_H T})r_L + e^{-\mu_H T} r_H]\Big) + (\beta - \gamma)\Big(\frac{\mu_H}{\mu_H + \mu_L} r_L + \frac{\mu_L}{\mu_H + \mu_L}\frac{\mu_H}{\lambda + \mu_H} r_L\Big) \\ &+ (1 - \alpha - \beta + \gamma)\frac{\mu_H}{\mu_H + \mu_L} r_L - \frac{2m\mu_H\mu_L}{\mu_H + \mu_L}\end{aligned}$$

$$=\frac{\mu_H}{\mu_H+\mu_L}r_L+\alpha\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L+e^{-\mu_H T}r_H]+(\beta-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}$$

$$=r_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha[(1-e^{-\mu_H T})r_L+e^{-\mu_H T}r_H]+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L-r_L-2m\mu_H\Big\}$$

$$\le V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-V_L-2m\mu_H\Big\},$$

where the last inequality holds because $r_H\le V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$.

If the term in the brackets is negative, then $\Phi^{1.4}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{1.4}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$\Phi^{1.4}(\mu_H,\infty,T)=\alpha\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-2m\mu_H.$$

The remaining analysis is the same as Case 1.3 in the proof of Lemma E.5, establishing that $\Phi^{1.4}(\mu_H,\infty,T)<\max\{V_L,\alpha V_H\}$.

Case 2: $r_L=V_L-\frac{c}{\mu_H}$.

According to Lemma 3, low-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H.

For low-valuation type I customers, there are two possibilities. If $V_L>r_L+\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H-r_L)}{\lambda}$, they purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. However, Lemma E.3(b) shows that the high/low pricing strategy is not optimal in this case. If $V_L\le r_L+\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H-r_L)}{\lambda}$, they purchase at the price r_L but wait and monitor when the price is r_H. We will focus on the second possibility.

High-valuation type I customers purchase at both prices immediately if

$$V_H\ge r_L+\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H-r_L)}{\lambda}. \tag{E.31}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (E.31) is not satisfied, they would purchase at the price r_L immediately and wait and monitor when the price is r_H. Inequality (E.31) can be rewritten as

$$r_H\le r_L+\frac{\lambda(V_H-r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

High-valuation type II customers purchase immediately if

$$V_H\ge r_L+\frac{c}{\mu_H}+\frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H-r_L-\frac{c}{\mu_H})}{\lambda}. \tag{E.32}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (E.32) is not satisfied, they would purchase at the price r_L but wait and monitor when the price is r_H. Inequality (E.32) can be written as

$$r_H\le r_L+\frac{c}{\mu_H}+\frac{\lambda(V_H-r_L-\frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

From the conditions on r_H, we can analyze the firm's pricing problem based on the range of r_H. We consider four cases, labeled Cases 2.1-2.4. We will show each of the four cases generates a profit lower than $\max\{V_L, \alpha V_H\}$.

<u>Case 2.1:</u> Suppose

$$r_H > r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}},$$
$$r_H > r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

Then both high-valuation type I and high-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure E.19.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure E.19 Customer Purchase Decisions in Case 2.1.

Since customers never purchase at the price r_H, it is impossible to obtain a profit higher than $r_L < V_L$.

<u>Case 2.2:</u> Suppose

$$r_H > r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}},$$
$$r_H \le r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

Then high-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, high-valuation type II customers purchase at both prices and monitor if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure E.20.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}} = V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{2.2}(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$\Phi^{2.2}(\mu_H, \mu_L, T)$

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure E.20 Customer Purchase Decisions in Case 2.2.

$$
\begin{aligned}
=&(\alpha-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L+e^{-\mu_H T}r_H]\Big)\\
&\quad+(1-\alpha+\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L+(\alpha-\gamma)\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L+e^{-\mu_H T}r_H]+(1-\alpha+\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{(\alpha-\gamma)[(1-e^{-\mu_H T})r_L+e^{-\mu_H T}r_H]+(1-\alpha+\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L-r_L-2m\mu_H\Big\}\\
=&V_L-\frac{c}{\mu_H}\\
&+\frac{\mu_L}{\mu_H+\mu_L}\Big\{(\alpha-\gamma)\Big[\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}-(1-e^{-\mu_H T})\frac{c}{\mu_H}\Big]+(1-\alpha+\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}-(V_L-\frac{c}{\mu_H})-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^{2.2}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{2.2}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\begin{aligned}
\Phi^{2.2}(\mu_H,\infty,T)&=(\alpha-\gamma)\Big[\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}-(1-e^{-\mu_H T})\frac{c}{\mu_H}\Big]+(1-\alpha+\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}-2m\mu_H\\
&\le(\alpha-\gamma)\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(1-\alpha+\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}-2m\mu_H.
\end{aligned}
$$

The remaining analysis is the same as Case 2.2 in the proof of Lemma E.5. Therefore, we obtain $\Phi^{2.2}(\mu_H,\infty,T)<\max\{V_L,\alpha V_H\}$.

<u>Case 2.3:</u> Suppose

$$
\begin{aligned}
r_H&\le r_L+\frac{\lambda(V_H-r_L)}{(\lambda+\mu_H)e^{-\mu_H T}},\\
r_H&>r_L+\frac{c}{\mu_H}+\frac{\lambda(V_H-r_L-\frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.
\end{aligned}
$$

Then high-valuation type I customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. Meanwhile, high-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure E.21.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure E.21 Customer Purchase Decisions in Case 2.3.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}} = V_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - V_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{2.3}(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
&\Phi^{2.3}(\mu_H,\mu_L,T)\\
=&\gamma\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H]\Big)\\
&\quad + (1-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big) - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L + \gamma\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] + (1-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] + (1-\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L - r_L - 2m\mu_H\Big\}\\
=&V_L - \frac{c}{\mu_H} + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H + \mu_H V_L - c}{\lambda+\mu_H} + (1-\gamma)\frac{\mu_H V_L - c}{\lambda+\mu_H} - (V_L - \frac{c}{\mu_H}) - 2m\mu_H\Big\}.
\end{aligned}$$

If the term in the brackets is negative, then $\Phi^{2.3}(\mu_H,\mu_L,T) < V_L$. Suppose the term is positive. Note that $\Phi^{2.3}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^* = \infty$. It follows that

$$\begin{aligned}
\Phi^{2.3}(\mu_H,\infty,T) &= \gamma\frac{\lambda V_H + \mu_H V_L - c}{\lambda+\mu_H} + (1-\gamma)\frac{\mu_H V_L - c}{\lambda+\mu_H} - 2m\mu_H\\
&= V_L + \frac{\gamma\lambda V_H - \lambda V_L - c}{\lambda+\mu_H} - 2m\mu_H.
\end{aligned}$$

If $\gamma\lambda V_H - \lambda V_L - c < 0$, then $\Phi^{2.3}(\mu_H,\infty,T) < (\alpha+\beta-\gamma)V_L < V_L$. Otherwise,

$$\Phi^{2.3}(\mu_H,\infty,T) < V_L + \frac{\gamma\lambda V_H - \lambda V_L - c}{\lambda} - 2m\mu_H < \gamma V_H - 2m\mu_H < \alpha V_H.$$

<u>Case 2.4:</u> Suppose

$$\begin{aligned}
r_H &\le r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}},\\
r_H &\le r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}.
\end{aligned}$$

In this case, both high-valuation type I and high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure E.22.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure E.22 Customer Purchase Decisions in Case 2.4.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \min\left\{r_L + \frac{\lambda(V_H - r_L)}{(\lambda+\mu_H)e^{-\mu_H T}}, r_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - r_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}\right\}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{2.4}(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
&\Phi^{2.4}(\mu_H,\mu_L,T)\\
=&\alpha\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H]\Big) + (1-\alpha)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)\\
&-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L + \alpha\frac{\mu_L}{\mu_H+\mu_L}[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] + (1-\alpha)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha[(1-e^{-\mu_H T})r_L + e^{-\mu_H T}r_H] + (1-\alpha)\frac{\mu_H}{\lambda+\mu_H}r_L - r_L - 2m\mu_H\Big\}\\
\le&V_L - \frac{c}{\mu_H} + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha\frac{\mu_H V_L + \lambda V_H - c}{\lambda+\mu_H} + (1-\alpha)\frac{\mu_H V_L - c}{\lambda+\mu_H} - (V_L - \frac{c}{\mu_H}) - 2m\mu_H\Big\},
\end{aligned}
$$

where the last inequality holds because $r_H \le V_L - \frac{c}{\mu_H} + \frac{\lambda(V_H - V_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)e^{-\mu_H T}}$. The remaining analysis is the same as Case 2.3 in the proof of Lemma E.5. Therefore, we have $\Phi^{2.4}(\mu_H, \infty, T) < \max\{V_L, \alpha V_H\}$. This completes the proof of Proposition 1. ■

Online Supplement for "Markovian Pricing with Price Guarantees"

Jianghua Wu, Dan Zhang, Yan Liu

This online supplement provides supplemental materials for the paper and includes four sections:

- Section S.1 provides a detailed analysis under the assumption that a purchased customer stops monitoring for price refund when her lifetime ends.
- Section S.2 considers an extension where customers are heterogeneous in their lifetime.
- Section S.3 considers a non-zero monitoring cost for type I customers as a robustness check for our main results.
- Section S.4 provides proofs of the results in Sections S.1-S.3.

S.1. An Alternative Assumption on Customers' Monitoring Behavior

In the base model, we interpret customers' lifetime as their interest in the product. Once a customer makes a purchase, her lifetime does not matter anymore. Under this assumption, a purchased customer who decides to monitor for price refund keeps monitoring until the price guarantee applies or expires. Alternatively, one may interpret customers' lifetime as a patience parameter. Under this alternative interpretation, a customer who purchases at a high price may stop monitoring when her lifetime ends (her patience runs out) before the price guarantee expires. This section provides the results when we adopt such an alternative assumption, where a customer stops monitoring for price refund when the price guarantee expires or her lifetime ends, whichever occurs earlier. We find that our main results and insights still hold qualitatively and are therefore robust under the alternative assumption.

S.1.1 Customer's Decision Problem

Before writing down the decision problem for the customer as a dynamic program, we first derive the expected surplus if the customer chooses to purchase immediately at price r_H and then monitor the price until the price guarantee expires or her lifetime ends, as shown in the following lemma.

LEMMA S.1. *If the customer purchases at price r_H and then monitors the price until the price guarantee expires or her lifetime ends, then her expected surplus, denoted by M, is*

$$M = v - r_H + (1 - e^{-\mu_H T})\Big\{e^{-\lambda T}(r_H - r_L) - \frac{c}{\mu_H + \lambda}\Big\} + (1 - e^{-\lambda T})\frac{\mu_H (r_H - r_L) - c}{\mu_H + \lambda}.$$

When the price is r_H, let X be the amount of time before the price is switched to r_L and Y be the amount of the customer's lifetime in the market before she leaves. Then, X and Y follow an exponential distribution with rate μ_H and λ, respectively. A customer purchasing at the price r_H

earns an immediate surplus $v - r_H$. If the customer chooses to monitor the price after purchase, she may get a refund of $r_H - r_L$ in case the price drops before the guarantee expires and her lifetime ends ($X \leq \min\{T, Y\}$) but incur a price monitoring cost $c \cdot \min\{X, T, Y\}$. Therefore, her total expected surplus is

$$v - r_H + (r_H - r_L) \cdot P(X \leq \min\{T, Y\}) - c \cdot E[\min\{X, T, Y\}],$$

which leads to the expected surplus shown in Lemma S.1.

Let $G(\cdot)$ be the value function, which denotes the maximum surplus earned by the customer. Then, the dynamic program can be formulated as

$$G(r_H) = \max\left\{v - r_H, M, \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}, 0\right\}, \tag{S.1}$$

$$G(r_L) = \max\left\{v - r_L, \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}, 0\right\}. \tag{S.2}$$

Equations (S.1) and (S.2) are very similar to the dynamic program in the base model. The only difference is the expected surplus when the customer chooses to purchase immediately and then monitor for price refund in equation (S.1).

Lemmas S.2 and S.3 provide the optimal solution to the dynamic program in equations (S.1) and (S.2) and characterize the customer's optimal purchase strategy for a type II customer with parameters (v, c). The two lemmas consider cases with a high and a low price monitoring cost, respectively.[9]

To simplify the notations, we let

$$c_1(r_H, r_L, \mu_H, T) = \frac{(1 - e^{-\lambda T})\mu_H + e^{-\lambda T}(\lambda + \mu_H)(1 - e^{-\mu_H T})}{2 - e^{-\lambda T} - e^{-\mu_H T}}(r_H - r_L),$$

$$c_2(r_H, r_L, \mu_H, T) = \frac{\lambda\mu_H(1 - e^{-\lambda T}) + e^{-\mu_H T}\mu_H(\lambda + \mu_H)e^{-\lambda T}}{\lambda - \mu_H(1 - e^{-\lambda T} - e^{-\mu_H T})}(r_H - r_L),$$

$$\bar{v}(r_H, r_L, \mu_H, T, c) = r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right] + (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}.$$

Lemma S.2 **(Optimal purchase decisions when the price monitoring cost is high)**. *Consider a type II customer with parameters (v, c) where $c > c_1(r_H, r_L, \mu_H, T)$. The optimal solution to equations (S.1)–(S.2) and the optimal purchase strategy of the customer are as follows:*

(a) If $v < r_L$, then $G(r_H) = G(r_L) = 0$ and the customer never purchases;

(b) If $r_L \leq v < r_H$, then $G(r_H) = 0$, $G(r_L) = v - r_L$, and the customer purchases upon arrival when the price is r_L, but leaves immediately without a purchase when the price is r_H;

[9] There is a third case with an intermediate price monitoring cost. The result is summarized in Lemma S.5 and relegated to Section S.4.1.

(c) If $v \geq r_H$, then $G(r_H) = v - r_H$ and $G(r_L) = v - r_L$. The customer purchases immediately upon arrival.

LEMMA S.3 **(Optimal purchase decisions when the price monitoring cost is low)**. *Consider the purchase decisions for a type II customer with parameters (v, c) where either (i) $c \leq c_1(r_H, r_L, \mu_H, T)$ and $\mu_H \geq (\lambda + \mu_H)e^{-\lambda T}$, or (ii) $c \leq c_2(r_H, r_L, \mu_H, T)$ and $\mu_H < (\lambda + \mu_H)e^{-\lambda T}$. The optimal solution to equations (S.1)–(S.2) and the optimal purchase decision are given as follows:*

(a) If $v < r_L$, then $G(r_H) = G(r_L) = 0$; the customer never purchases and the price guarantee is never used;

(b) If $r_L \leq v < r_L + \frac{c}{\mu_H}$, then $G(r_H) = 0$, $G(r_L) = v - r_L$; the customer purchases immediately upon arrival at the price r_L, but leaves without a purchase if the price is r_H. The price guarantee is never used;

(c) If $r_L + \frac{c}{\mu_H} \leq v < \bar{v}(r_H, r_L, \mu_H, T, c)$, then

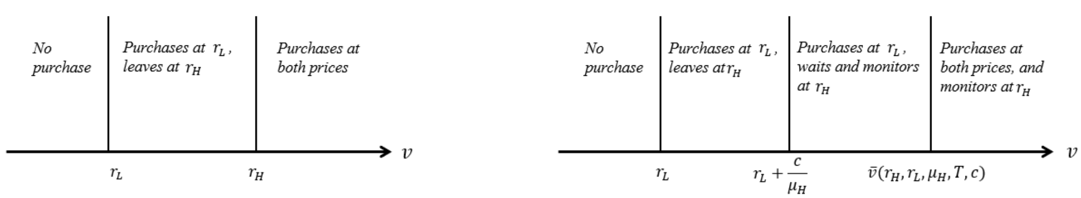
$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H}, \quad G(r_L) = v - r_L.$$

The customer purchases immediately upon arrival when the price is r_L. When the price is r_H, the customer would wait for the price r_L until she leaves the market. The price guarantee is never used;

(d) If $v \geq \bar{v}(r_H, r_L, \mu_H, T, c)$, then

$$G(r_H) = M, \quad G(r_L) = v - r_L.$$

The customer purchases immediately upon arrival. If the purchase is made at the price r_H, she would keep monitoring the price until the price guarantee is applied/expired or her lifetime ends.



(a)When the monitoring cost is high (b)When the monitoring cost is low

Figure S.1 A Type II Customer's Optimal Purchase Strategy with Price Guarantees.

Lemmas S.2 and S.3 are very similar to Lemmas 2 and 3 in the main text. Observe that the upper end of the range in Lemma S.3(c), $\bar{v}(r_H, r_L, \mu_H, T, c)$, may be above r_H. Consider two scenarios: one is when there are no price guarantees, i.e., $T = 0$, $\bar{v}(r_H, r_L, \mu_H, T, c) = r_H + \frac{\mu_H(r_H - r_L) - c}{\lambda} > r_H$ because $c \leq c_2(r_H, r_L, \mu_H, T) = \mu_H(r_H - r_L)$. The other scenario is when there is no expiration term for price guarantees, i.e, $T=\infty$, in which case $\bar{v}(r_H,r_L,\mu_H,T,c)=r_H+\frac{c}{\lambda}>r_H$. This means that, similar to Lemma 3, a type II customer with a valuation between r_H and $\bar{v}(r_H,r_L,\mu_H,T,c)$ may eventually leave without a purchase even though the price r_H is acceptable to her.

What is the average revenue contribution of a type II customer? Here, we only focus on Lemma S.3(d) because the other cases are the same as those in the main text. In Lemma S.3(d), the customer always purchases at the current price. However, the customer will be refunded the price difference if the price guarantee is applied. According to the solution of $G(r_H)$ in Lemma S.3(d), the price minus the expected refund claimed by a customer who purchases at the high price, denoted by $E[p^G]$, is

$$\begin{aligned} E[p^G] =& r_H - \Big[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big](r_H - r_L) \\ =& \Big[1-(1-e^{-\mu_H T})e^{-\lambda T} - (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big] r_H + \Big[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big] r_L, \end{aligned}$$

where $(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}$ is the probability that the firm offers a sale before the price guarantee expires and her lifetime ends, in which case she receives the refund and her revenue contribution is r_L. Taking into account the stationary probabilities of the price, the total revenue is

$$\frac{\mu_L}{\mu_H+\mu_L}\cdot E[p^G] + \frac{\mu_H}{\mu_H+\mu_L}\cdot r_L.$$

Next, we analyze the firm's optimal Markovian pricing strategy with price guarantees. Note that when a price guarantee is not offered, the above model reduces to the one without price guarantees in the main text. For example, one can verify that by taking $T=0$ in Lemmas S.2 and S.3, customers' optimal purchase strategy is the same as that in Lemmas 2 and E.4. Therefore, the firm's optimal Markovian pricing strategy without price guarantees is also the same as that in Proposition 4.

S.1.2 The Optimal Markovian Pricing Strategy with Price Guarantees

Our analysis in Section S.1.1 indicates that type II customers monitor the price after they purchase at the high price if the monitoring cost is low, and do not monitor otherwise; type I customers always monitor the price because their monitoring cost is zero. We first analyze the firm's problem when the monitoring cost for type II customers is low. The result is summarized in Proposition S.8.

PROPOSITION S.8. *If the monitoring cost c is low (as stated in Lemma S.3) such that type II customers who purchase at the high price monitor the price after purchase, then the high/low pricing with price guarantees cannot improve firm profit over static pricing.*[10]

[10] If the monitoring cost for type II customers is intermediate (as stated in Lemma S.5) such that they monitor the price after purchasing at the high price, we also show that offering price guarantees cannot improve firm profit. The result is summarized in Proposition S.15 and relegated to Section S.4.1.

Similar to the result in the main text, if type II customers monitor the price after purchasing at a high price, then offering price guarantees cannot improve firm profit.

Next, we analyze the firm's pricing problem when the monitoring cost is high ($c > c_1(r_H, r_L, \mu_H, T)$) such that type II customers do not monitor the price after they purchase at the price r_H. That is, only type I customers take advantage of the price guarantee. The result is summarized in Proposition S.9 below.

Proposition S.9 **(The optimal Markovian pricing strategy for a high monitoring cost)**. *Suppose the monitoring cost c is high ($c > c_1(r_H, r_L, \mu_H, T)$) such that type II customers do not monitor the price after they purchase at the price r_H. If $K \le 2m\lambda$, then the firm's optimal pricing strategy reduces to that without price guarantees in the base model. If $K > 2m\lambda$, there are three possible outcomes for the firm's optimal pricing strategy:*

(i) ***Static pricing at V_H.*** *The firm prices at V_H. All high-valuation customers purchase immediately and all low-valuation customers leave without a purchase. The profit per unit time is αV_H;*

(ii) ***Static pricing at V_L.*** *The firm prices at V_L. All customers purchase immediately. The profit per unit time is V_L;*

(iii) ***High/low pricing with price guarantees.*** *Only if*

$$c > \frac{\lambda}{2}\left(\sqrt{\frac{K}{2m\lambda}} - 1\right)(V_H - V_L), \tag{S.3}$$

the firm uses a Markovian pricing strategy with price guarantees where

$$r_H^{B,*} = V_H, \quad r_L^{B,*} = V_L, \quad T^* = \infty, \quad \mu_H^{B,*} = \lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right), \quad \mu_L^{B,*} = \infty.$$

The profit per unit time is

$$\begin{aligned}\Phi^{B,*} = (\alpha-\gamma)V_H + (\beta-\gamma)\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_L + \gamma\left[\left(1-\sqrt{\frac{2m\lambda}{K}}\right)V_L + \sqrt{\frac{2m\lambda}{K}}V_H\right] \\ - 2m\lambda\left(\sqrt{\frac{K}{2m\lambda}} - 1\right).\end{aligned} \tag{S.4}$$

All high-valuation customers purchase immediately at both prices; in particular, high-valuation type I customers try to take advantage of price guarantees by monitoring the price after purchase, while high-valuation type II customers leave immediately after purchase; low-valuation type I customers either purchase immediately at price V_L or wait for the price V_L; and low-valuation type II customers purchases immediately at V_L and leave without a purchase at V_H.

Observe that the firm's optimal Markovian pricing strategy and the corresponding customer behavior are almost the same as that in Proposition 2 in the base model. There are only two differences. One is the cutoff on the threshold of c. The threshold in condition (S.3) is lower than that in condition (5), implying that the high/low pricing strategy is valid for a broader parameter range under the alternative assumption. This can be explained as follows. Recall that the high/low pricing strategy is optimal only if c is large enough such that type II customers do not monitor for price refund. Under the alternative assumption, customers stop monitoring whenever their lifetime ends, making it less likely for customers to receive the price refund, compared to the base model where customers keep monitoring until the price guarantee expires. In other words, type II customers are less willing to purchase and monitor under the alternative assumption. Therefore, the monitoring cost c does not have to be as large as that in the base model.

The other difference is the optimal expiration term of the price guarantee. Proposition S.9 shows that the optimal guarantee duration is set to infinity under the alternative assumption. This means that the probability of getting the refund is equal to the probability that the price switches to r_L before the customer's lifetime ends ($\frac{\mu_H}{\lambda+\mu_H}$). Note that this result is derived under the alternative assumption that customers stop monitoring for price refund when their lifetime ends. Recall that in the base model, customers keep monitoring the price until the price guarantee expires, and the optimal guarantee duration is set to a finite value such that the probability of getting the refund ($1-e^{\mu_H T^*}$, which is the probability that the price switches to r_L before the price guarantee expires) is also equal to the probability that the price switches to r_L before their lifetime ends ($\frac{\mu_H}{\lambda+\mu_H}$). That is, under either assumption, the expiration term is set to make the probability of getting the refund equal to the probability that the price switches to r_L before the customer's lifetime ends. Importantly, the pricing strategy (r_H, r_L, μ_H, μ_L), the firm's optimal revenue, and the corresponding customer behavior are the same under the alternative and original assumptions. This indicates that our main results and insights are robust against the assumption of whether customers keep or stop monitoring the price when their lifetime ends.

S.2. Heterogeneity in Lifetimes

This section considers an extension where customers are heterogeneous in their lifetime. We assume that type I and type II customers' lifetimes follow an exponential distribution with rate $\lambda_1 = 0$ and $\lambda_2 = \lambda > 0$, respectively. This means that type I customers have an infinite lifetime in the market, whereas type II customers are short-lived with a limited lifetime. For tractability, we remove customers' heterogeneity in price monitoring costs by assuming a homogeneous monitoring cost $c > 0$. Hence, customers differ in two dimensions: production valuation and lifetime duration.

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	HI (γ)	HII (α-γ)
v_L (1-α)	LI (β-γ)	LII (1-α-β+γ)

Figure S.2 The Four Customer Segments.

Similar to the base model, we assume that a proportion γ of all customers are high-valuation type I customers. Consequently, $\alpha-\gamma$ ($\beta-\gamma$, $1-\alpha-\beta+\gamma$) proportion are high-valuation type II (low-valuation type I, low-valuation type II) customers. Figure S.2 illustrates the four customer segments. This setting allows us to analyze the implications of customer lifetime differences on the firm's optimal pricing strategies and the effectiveness of price guarantees.

For a type II customer with parameter (v, λ), the dynamic program is the same as (2) and (3) in the base model. We assume the price monitoring cost for both types of customers is low such that $c \le \mu_H(r_H - r_L)$. Otherwise, both types of customers would behave myopically as stated in Lemma 2, which makes the problem trivial. As a result, the optimal purchase decision for a type II customer is the same as in Lemma E.4 (without price guarantees) and Lemma 3 (with price guarantees). Thus, it suffices to focus on analyzing the optimal purchase decision of a type I customer. Let $J(\cdot)$ be the value function, which denotes the maximum surplus earned by a type I customer. Then, the dynamic program can be formulated as

$$J(r_H) = \max\{v - r_H, v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}), \frac{\mu_L}{\mu_H + \mu_L} J(r_H) + \frac{\mu_H}{\mu_H + \mu_L} J(r_L) - \frac{c}{\mu_H + \mu_L}, 0\}, \tag{S.5}$$

$$J(r_L) = \max\{v - r_L, 0\}. \tag{S.6}$$

The expressions for $J(r_H)$ and $J(r_L)$ are the same as (2) and (3) with $\lambda = 0$ in the base model.

Lemma S.4. *Consider the purchase decisions for a type I customer with parameter (v,c) where $c \le \mu_H(r_H - r_L)$. The optimal solution to equations* (S.5) *and* (S.6) *and the optimal purchase decisions are given as follows:*

(a) If $v < r_L$, then $J(r_H) = J(r_L) = 0$; the customer never purchases and the price guarantee is never used;

(b) If $r_L \le v < r_L + \frac{c}{\mu_H}$, then $J(r_H) = 0$ and $J(r_L) = v - r_L$; the customer purchases immediately upon arrival at the price r_L, but leaves without a purchase if the price is r_H. The price guarantee is never used;

(c) If $v \geq r_L + \frac{c}{\mu_H}$, then $J(r_H) = v - r_L - \frac{c}{\mu_H}$ and $J(r_L) = v - r_L$. The customer purchases immediately upon arrival when the price is r_L. When the price is r_H, the customer will wait for the price r_L. The price guarantee is never used.

Lemma S.4 characterizes the optimal purchase strategy for a type I customer. Compared to a type II customer, a type I customer will never purchase at the high price r_H and then monitor for a price refund. This is because a type I customer, who has an infinite lifetime in the market, is guaranteed to eventually obtain the product at the low price. In contrast, if a purchase is made at the high price, there is a chance that the customer cannot receive the price refund due to the limited duration of the price guarantee. Hence, it is never optimal for a type I customer to utilize price guarantees. Given that price guarantees have no effect on type I customers' purchase decisions, Lemma S.4 fully characterizes the optimal purchase strategy for a type I customer, regardless of whether the firm offers price guarantees or not.

We first analyze the firm's optimal Markovian pricing strategy without price guarantees. Given customers' optimal purchase decisions, the firm optimizes its Markovian pricing strategy by choosing the parameters r_H, r_L, μ_H, and μ_L. The result is summarized in the following proposition.

PROPOSITION S.10 **(The optimal Markovian pricing strategy without price guarantees)**. *There are three possible outcomes for the firm's optimal Markovian pricing strategy:*

(i) ***Static pricing at V_H.*** *The firm prices at V_H. All high-valuation customers purchase immediately and all low-valuation customers leave without a purchase. The profit per unit time is αV_H;*

(ii) ***Static pricing at V_L.*** *The firm prices at V_L. All customers purchase immediately. The profit per unit time is V_L;*

(iii) ***High/low pricing with flash sales.*** *Only if*

$$(\alpha - \gamma)V_H > (1 - \beta)V_L, \tag{S.7}$$

the firm uses a Markovian pricing strategy where

$$r_H^* = \frac{\lambda V_H + \mu_H^* V_L}{\lambda + \mu_H^*}, \quad r_L^* = V_L - \frac{c}{\mu_H^*}, \quad \mu_L^* = \infty,$$

and μ_H^* is the optimal solution to the following problem

$$\Phi^* = \max_{\mu_H} \beta(V_L - \frac{c}{\mu_H}) + (\alpha - \gamma)\frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + (1 - \alpha - \beta + \gamma)\frac{\mu_H}{\lambda + \mu_H}(V_L - \frac{c}{\mu_H}) - 2m\mu_H. \tag{S.8}$$

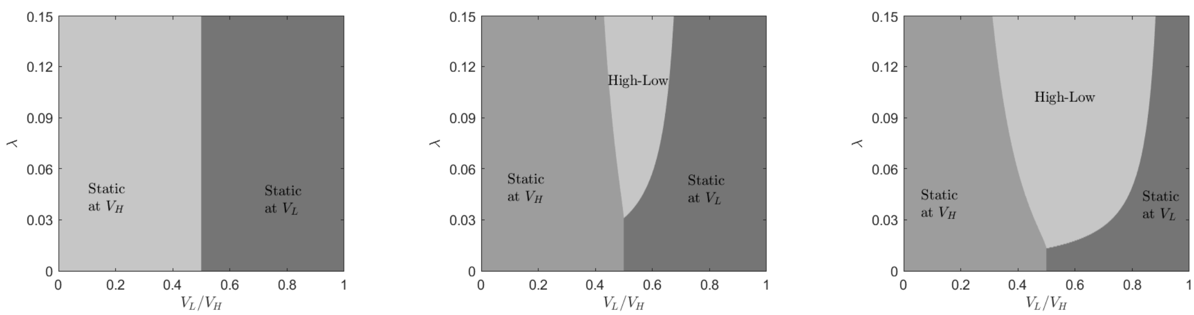
All type I and low-valuation type II customers either purchase immediately at the price r_L or wait for the price r_L; and high-valuation type II customers purchase immediately at both prices.

In general, the profit function (S.8) is not concave in μ, so it is not straightforward to obtain the expressions of μ^* and the corresponding profit Φ^*. However, the optimal pricing strategy is still either static pricing or high/low pricing with flash sales, with a structure very similar to that in the base model (see Proposition 4). The optimal pricing strategy can be obtained by comparing the profits in the three cases. In the high/low pricing strategy, the firm always charges a high price r_H^* (between V_H and V_L), except for occasional price drops to $V_L - \frac{c}{\mu_H^*}$. At optimality, both type I and low-valuation type II customers either purchase immediately at price $V_L - \frac{c}{\mu_H^*}$ or wait for the price drop when the current price is r_H. Although the two customer segments behave in the same way, their purchase probabilities are different. Equation (S.8) shows that type I customers purchase at the low price with a probability of 1, whereas low-valuation type II customers purchase at the low price with a probability of $\frac{\mu_H^*}{\mu_H^*+\lambda}$, which is less than 1. This is because type I customers have an infinite lifetime in the market and are guaranteed to encounter the price drop if they choose to wait, while low-valuation type II customers have a limited lifetime and may exit the market during the waiting period. High-valuation type II customers always purchase immediately at both prices.

According to the observed behavior of type I and type II customers, it is never optimal for the firm to charge a low price r_L that is different from V_L and $V_L - \frac{c}{\mu_H^*}$. We show that the firm's profit when $r_L = V_L$ is always lower than static pricing at either V_H or V_L. The key insight is that the low price is only offered occasionally, so the low-valuation customers have to wait for the price drop if they wish to make a purchase. However, all customers incur a monitoring cost c in this new setting. If the firm charges $r_L = V_L$, then no low-valuation customers would make a purchase, leading to a profit lower than static pricing at V_H. To induce low-valuation customers to purchase at the low price, the firm has to take into account the expected monitoring cost $\frac{c}{\mu_H^*}$ when deciding the low price. This explains why the optimal low price is set to $V_L - \frac{c}{\mu_H^*}$, instead of V_L.

High/low pricing can be beneficial to the firm because it allows the firm to discriminate against customers based on their lifetime duration. Type I customers only purchase at the low price, whereas high-valuation type II customers purchase at both the high and low prices. Since the low price is offered only occasionally, the effective price paid by high-valuation type II customers is predominantly the high price. The low-valuation type II customers also contribute to the firm's revenue, as they purchase at the low price r_L and wait for the price drop at the high price r_H. This is different from the base model, where low-valuation type II customers essentially do not contribute to the firm's revenue because the low price r_L is rarely offered and they only purchase at r_L and leave immediately at r_H. Therefore, under the high/low pricing strategy, all four customer segments contribute to the firm's revenue. Compared to static pricing at V_L, where all customers purchase at the same price V_L, the high/low pricing strategy results in type I and low-valuation type II customers paying a relatively lower price of $V_L - \frac{c}{\mu_H^*}$, while high-valuation type II customers pay a higher price than V_L. Hence, the profitability of high/low pricing over static pricing at V_L depends on the fraction of high-valuation type II customers. Only if this segment accounts for a large enough proportion would the high/low pricing lead to higher revenue than static pricing at V_L. This is similar to the insight from the base model, where the profitability of high/low pricing also depends on the correlation between valuation and lifetime.

Proposition S.11. *If $\gamma \geq \alpha\beta$, then the high/low pricing strategy with flash sales is no more profitable than static pricing.*



(a)$\rho = 0$ (b)$\rho = -0.5$ (c)$\rho = -0.9$

Figure S.3 The Firm's Optimal Pricing Strategy under Heterogeneous Lifetimes.

To help explain Proposition S.11, we conduct a set of numerical experiments to compare the three pricing strategies. We set $(\alpha, \beta, V_H, c, m) = (0.5, 0.5, 1, 0.005, 0.005)$ and vary V_L from 0 to 1 and λ from 0 to 0.15. The three subfigures correspond to the three levels of correlation between the valuation and lifetime duration: 0 (no correlation), and -0.5 (moderately negative correlation), -0.9 (highly positive correlation). The horizontal axis is V_L/V_H and the vertical axis is λ. The ratio V_L/V_H can be interpreted as a measure of valuation homogeneity, whereas λ can be interpreted as a measure of lifetime heterogeneity. Figure S.3(a) shows that when the correlation coefficient ρ is zero, the firm does not benefit from high/low pricing. Figures S.3(b) and (c) show that as the correlation becomes more negative, the firm is more likely to offer high/low pricing. One difference compared to Figure 6 in the base model is that in Figure S.3, the larger the value of λ, the more likely the firm will offer the high/low pricing strategy. This is because, in this new setting, the profitability of the high/low pricing strategy lies in its ability to allow the firm to discriminate against customers based on their lifetime duration. Recall that $\lambda = 0$ for type I customers, so the larger the value of λ for type II customers, the more heterogeneous the two customer types are in terms of their lifetime duration.

Next, we analyze the firm's optimal Markovian pricing strategy with price guarantees and compare it to the strategy without price guarantees. This allows us to investigate the effectiveness of price guarantees in this new setting. Surprisingly, we find that the optimal duration of the price guarantee is equal to 0. In other words, price guarantees cannot improve the firm's revenue in this new setting with heterogeneous customer lifetime duration. This result contrasts with the findings from the base model, where price guarantees are shown to be an effective tool for the firm to increase its revenue. The difference highlights how the presence of customer lifetime heterogeneity can fundamentally alter the effectiveness of price guarantees.

Proposition S.12. *When customers are heterogeneous in product valuation and lifetime duration, offering price guarantees cannot improve the firm's revenue, compared to that without price guarantees.*

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

(a) Without Price Guarantees

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

(b) With Price Guarantees

Figure S.4 Customer Behavior under the Optimal Markovian Pricing Strategy with and without Price Guarantees.

To understand Proposition S.12, we compare customers' purchase strategies under the optimal Markovian pricing strategy with and without price guarantees, as illustrated in Figure S.4. As discussed previously, price guarantees have no effect on type I customers' purchase strategy. Therefore, only the purchase behavior of high-valuation type II customers is affected by the presence of price guarantees. Without price guarantees, high-valuation type II customers purchase immediately at both the high and low prices. With price guarantees, they still purchase immediately at both prices, but would monitor for a potential price refund if their purchase was made at the high price. However, this change in customer behavior does not benefit the firm. Any price refunds issued by the firm would ultimately harm its revenue. From the firm's perspective, the optimal purchase strategy for high-valuation type II customers is to purchase immediately at both prices,

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	HI (γ)	HII (α-γ)
v_L (1-α)	LI (β-γ)	LII (1-α-β+γ)

Figure S.5 The Four Customer Segments.

without the need for price guarantees. Offering price guarantees would induce this segment of customers to monitor for potential refunds, which eventually hurts the firm's revenue. Therefore, in this new setting with heterogeneous customer lifetimes, providing price guarantees cannot improve the firm's revenue. Proposition S.12 implies that the driver of the effectiveness of price guarantees under the Markovian pricing is the heterogeneity in customers' monitoring costs, instead of the heterogeneity in lifetimes.

S.3. Non-zero Monitoring Cost for Type I customers

The base model assumes a zero monitoring cost for type I customers. To check the robustness of our main results, this section considers the case of a non-zero monitoring cost for type I customers. Assume the monitoring cost is c_1 for type I customers and c_2 for type II customers, where $0 < c_1 < c_2$. Figure S.5 illustrates the four customer segments.

We restrict our attention to the case where $c_1 \leq \mu_H(r_H - r_L) < c_2$. This is because otherwise, i.e., if $c_1 < c_2 \leq \mu_H(r_H - r_L)$ or $\mu_H(r_H - r_L) < c_1 < c_2$, the two types of customers would behave identically. In such cases, the high/low pricing strategy cannot effectively differentiate between the two customer types. The following two propositions characterize the optimal Markovian pricing strategy without and with price guarantees, respectively.

PROPOSITION S.13 **(The optimal Markovian pricing strategy without price guarantees)**. *There are three possible outcomes for the firm's optimal Markovian pricing strategy:*

(i) ***Static pricing at V_H.*** *The firm prices at V_H. All high-valuation customers purchase immediately and all low-valuation customers leave without a purchase. The profit per unit time is αV_H;*

(ii) ***Static pricing at V_L.*** *The firm prices at V_L. All customers purchase immediately. The profit per unit time is V_L;*

(iii) ***High/low pricing with flash sales.*** *Only if*

$$\beta(\lambda V_L + c_1) > 2m\lambda^2, \tag{S.9}$$

$$c_2 > \Big(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda\Big)(V_H - V_L) + c_1, \tag{S.10}$$

the firm uses a Markovian pricing strategy where

$$r_H^* = V_H, \quad r_L^* = V_L - \frac{c_1}{\mu_H^*}, \quad \mu_H^* = \sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda, \quad \mu_L^* = \infty.$$

The profit per unit time is

$$\Phi^* = (\alpha - \gamma)V_H + \beta\left(V_L - \sqrt{\frac{2m(\lambda V_L + c_1)}{\beta}}\right) - 2m\left(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda\right). \tag{S.11}$$

All type I customers either purchase immediately at the price r_L or wait for the price r_L; high-valuation type II customers purchase immediately at both prices; and low-valuation type II customers purchase at the price r_L but leave without a purchase at the price r_H.

Proposition S.14 **(The optimal Markovian pricing strategy with price guarantees)**. *If $\lambda K + \beta c_1 \le 2m\lambda^2$, then the firm's optimal pricing strategy reduces to that without price guarantees. If $\lambda K + \beta c_1 > 2m\lambda^2$, then there are three possible outcomes for the firm's optimal Markovian pricing strategy:*

(i) ***Static pricing at*** V_H***.*** *The firm prices at V_H. All high-valuation customers purchase immediately and all low-valuation customers leave without a purchase. The profit per unit time is αV_H;*

(ii) ***Static pricing at*** V_L***.*** *The firm prices at V_L. All customers purchase immediately. The profit per unit time is V_L;*

(iii) ***High/low pricing with price guarantees.*** *Only if*

$$c_2 > \left(\sqrt{\frac{\lambda K + \beta c_1}{2m}} - \lambda\right)(V_H - V_L) + c_1 \tag{S.12}$$

the firm uses a Markovian pricing strategy where

$$r_H^* = V_H, \quad r_L^* = V_L - \frac{c_1}{\mu_H^*}, \quad T^* = \frac{1}{\sqrt{\frac{\lambda K + \beta c_1}{2m}} - \lambda} \ln\sqrt{\frac{K + \beta c_1/\lambda}{2m\lambda}}, \quad \mu_H^* = \sqrt{\frac{\lambda K + \beta c_1}{2m}} - \lambda, \quad \mu_L^* = \infty.$$

The profit per unit time is

$$\begin{aligned}\Phi^* =& (\alpha - \gamma)V_H + (\beta - \gamma)\left(1 - \frac{\lambda}{\sqrt{\frac{\lambda K + \beta c_1}{2m}}}\right)\left(V_L - \frac{c_1}{\sqrt{\frac{\lambda K + \beta c_1}{2m}} - \lambda}\right) \\ &+ \gamma\left[\left(1 - \frac{\lambda}{\sqrt{\frac{\lambda K + \beta c_1}{2m}}}\right)\left(V_L - \frac{c_1}{\sqrt{\frac{\lambda K + \beta c_1}{2m}} - \lambda}\right) + \frac{\lambda}{\sqrt{\frac{\lambda K + \beta c_1}{2m}}} V_H\right] - 2m\left(\sqrt{\frac{\lambda K + \beta c_1}{2m}} - \lambda\right).\end{aligned} \tag{S.13}$$

All type I customers either purchase immediately at the price r_L or wait for the price r_L; high-valuation type II customers purchase immediately at both prices; and low-valuation type II customers purchase at the price r_L but leave without a purchase at the price r_H.

One can verify that if $c_1 = 0$ and $c_2 = c$, the optimal high/low pricing strategy without/with price guarantees reduces to that of the base model. Propositions S.13 and S.14 confirm the robustness of our main results against the inclusion of a non-zero monitoring cost for type I customers. This suggests that the key insights derived in the base model continue to hold even when the assumption of zero monitoring cost for type I customers is relaxed.

S.4. Proofs of Lemmas and Propositions in Sections S.1-S.3

S.4.1 Proofs of Lemmas and Propositions in Section S.1

Proof of Lemma S.1

According to the discussion immediately after Lemma S.1, we have

$$M = v - r_H + (r_H - r_L) \cdot P\Big(X \leq \min\big\{T, Y\big\}\Big) - c \cdot E\Big[\min\big\{X, T, Y\big\}\Big]. \tag{S.14}$$

Note that

$$\begin{aligned}
&P\Big(X \leq \min\big\{T, Y\big\}\Big) \\
=&P\Big(X \leq \min\big\{T, Y\big\} | Y = \min\big\{T, Y\big\}\Big) \cdot P\Big(Y = \min\big\{T, Y\big\}\Big) \\
&\quad + P\Big(X \leq \min\big\{T, Y\big\} | T = \min\big\{T, Y\big\}\Big) \cdot P\Big(T = \min\big\{T, Y\big\}\Big) \\
=&P(X \leq Y) \cdot P(Y \leq T) + P(X \leq T) \cdot P(Y > T) \\
=&\frac{\mu_H}{\mu_H + \lambda}(1 - e^{-\lambda T}) + (1 - e^{-\mu_H T})e^{-\lambda T},
\end{aligned} \tag{S.15}$$

where the last equality holds because X and Y follow exponential distributions with rate μ_H and λ, respectively, and T is a constant.

Note also that

$$\begin{aligned}
&E\Big[\min\big\{X, T, Y\big\}\Big] \\
=&E\Big[\min\big\{\min\{X, Y\}, T\big\}\Big] = E\Big[\min\big\{Z, T\big\}\Big] && \text{[by letting } Z = \min\{X, Y\}] \\
=&E\Big[\min\big\{Z, T\big\} | Z = X\Big] \cdot P(Z = X) + E\Big[\min\big\{Z, T\big\} | Z = Y\Big] \cdot P(Z = Y) \\
=&E\Big[\min\big\{X, T\big\}\Big] \cdot P(X \leq Y) + E\Big[\min\big\{Y, T\big\}\Big] \cdot P(X > Y).
\end{aligned} \tag{S.16}$$

One can check

$$E\Big[\min\big\{X, T\big\}\Big] = \int_0^T x dF(x) + T \cdot P(X > T)$$

$$
\begin{aligned}
&= x \cdot F(x)|_0^T - \int_0^T F(x)dx + T \cdot e^{-\mu_H T} && \text{[by partial integration]} \\
&= T \cdot F(T) - \int_0^T (1 - e^{-\mu_H x})dx + T \cdot e^{-\mu_H T} \\
&= T \cdot (1 - e^{-\mu_H T}) - \Big[x + \frac{e^{-\mu_H x}}{\mu_H}\Big]_0^T + T \cdot e^{-\mu_H T} \\
&= \frac{1 - e^{-\mu_H T}}{\mu_H}.
\end{aligned}
$$

Similarly,

$$E\Big[\min\big\{Y, T\big\}\Big] = \frac{1 - e^{-\lambda T}}{\lambda}.$$

Putting $E\Big[\min\big\{X, T\big\}\Big]$ and $E\Big[\min\big\{Y, T\big\}\Big]$ back to (S.16) yields

$$E\Big[\min\big\{X, T, Y\big\}\Big] = \frac{1 - e^{-\mu_H T}}{\mu_H} \cdot \frac{\mu_H}{\lambda + \mu_H} + \frac{1 - e^{-\lambda T}}{\lambda} \cdot \frac{\lambda}{\lambda + \mu_H} = \frac{1 - e^{-\mu_H T} + 1 - e^{-\lambda T}}{\lambda + \mu_H}. \tag{S.17}$$

Putting (S.15) and (S.17) back to (S.14) gives the expression of M. This completes the proof. ■

Proof of Lemma S.2

Part (a) is immediate.

Next, consider the situation when $r_L \le v < r_H$. Note that when $c > c_1(r_H, r_L, \mu_H, T)$,

$$
\begin{aligned}
M = {} & v - r_H + (1 - e^{-\mu_H T})\Big\{e^{-\lambda T}(r_H - r_L) - \frac{c}{\mu_H + \lambda}\Big\} + (1 - e^{-\lambda T})\Big\{\frac{\mu_H}{\mu_H + \lambda}(r_H - r_L) - \frac{c}{\mu_H + \lambda}\Big\} \\
& < v - r_H.
\end{aligned}
$$

Therefore, equations (S.1) and (S.2) can be written as

$$
\begin{aligned}
G(r_H) &= \max\Big\{\frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}, 0\Big\}, \\
G(r_L) &= v - r_L.
\end{aligned}
$$

Recall that

$$\nu = \lambda + \mu_H + \mu_L.$$

We show $G(r_H) = 0$ by contradiction. Suppose for a contradiction that

$$G(r_H) = \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu} > 0.$$

Solving the equations gives

$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H} < \frac{\mu_H(v - r_L) - \mu_H(r_H - r_L)}{\lambda + \mu_H} = \frac{\mu_H(v - r_H)}{\lambda + \mu_H} \le 0,$$

contradicting our supposition that $G(r_H) > 0$. Hence, it must be the case that $G(r_H) = 0$. This gives the solution in Part (b).

Now, suppose $v \geq r_H$. Equations (S.1) and (S.2) can be written as

$$G(r_H) = \max\left\{v - r_H, \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}G(r_L) - \frac{c}{\nu}\right\},$$
$$G(r_L) = v - r_L.$$

Following a similar approach as above, one can show that $G(r_H) = v - r_H$. This completes the proof. ■

Before proving Lemma S.3, we introduce another notation

$$\tilde{v}(r_H, r_L, \mu_H, T, c) = r_H - (r_H - r_L)\Big(e^{-\lambda T}\frac{\lambda}{\lambda + \mu_H}) - e^{-\lambda T}e^{-\mu_H T} + \frac{\mu_H}{\lambda + \mu_H}\Big) + (2 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda + \mu_H}.$$

Moreover, recall that

$$\bar{v}(r_H, r_L, \mu_H, T, c) = r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})] + (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}.$$

Proof of Lemma S.3

Part (a) is immediate.

Suppose $v \geq r_L$. We have

$$M \geq v - r_H$$

when $c \leq c_1(r_H, r_L, \mu_H, T)$. Hence, equations (S.1)–(S.2) can be written as

$$G(r_H) = \max\left\{M, \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu}, 0\right\},$$

$$G(r_L) = v - r_L.$$

It remains to solve for $G(r_H)$, which we break into three cases.

Case 1: Suppose

$$M \geq \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu}, \tag{S.18}$$
$$M \geq 0. \tag{S.19}$$

It follows that

$$G(r_H) = M = v - r_H + (1 - e^{-\mu_H T})\left\{e^{-\lambda T}(r_H - r_L) - \frac{c}{\mu_H + \lambda}\right\} + (1 - e^{-\lambda T})\left\{\frac{\mu_H}{\mu_H + \lambda}(r_H - r_L) - \frac{c}{\mu_H + \lambda}\right\}.$$

Using the expressions of $G(r_H)$ in (S.18) and simplifying (S.19), we obtain

$$v \geq \bar{v}(r_H, r_L, \mu_H, T, c),$$

$$v \geq \tilde{v}(r_H, r_L, \mu_H, T, c).$$

One can verify that if $\mu_H \geq (\lambda + \mu_H)e^{-\lambda T}$, then

$$c_1(r_H, r_L, \mu_H, T) \leq c_2(r_H, r_L, \mu_H, T),$$

and vice versa. One can also check that if $c \leq c_2(r_H, r_L, \mu_H, T)$, then

$$\bar{v}(r_H, r_L, \mu_H, T, c) \geq \tilde{v}(r_H, r_L, \mu_H, T, c),$$

and vice versa. Therefore, if either (i) $\mu_H \geq (\lambda + \mu_H)e^{-\lambda T}$ and $c \leq c_1(r_H, r_L, \mu_H, T)$, or (ii) $\mu_H < (\lambda + \mu_H)e^{-\lambda T}$ and $c \leq c_2(r_H, r_L, \mu_H, T)$, we have $\bar{v}(r_H, r_L, \mu_H, T, c) \geq \tilde{v}(r_H, r_L, \mu_H, T, c)$. This gives the solution in Part (d).

<u>Case 2:</u> Suppose

$$\frac{\mu_L}{\nu} G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} \geq M, \tag{S.20}$$

$$\frac{\mu_L}{\nu} G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} \geq 0. \tag{S.21}$$

Then

$$G(r_H) = \frac{\mu_L}{\nu} G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu}.$$

It follows that

$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H}.$$

Using the expression of $G(r_H)$ in (S.20) and (S.21), we obtain

$$v \leq \bar{v}(r_H, r_L, \mu_H, T, c),$$
$$v \geq r_L + \frac{c}{\mu_H}.$$

One can verify that if $c \leq c_2(r_H, r_L, \mu_H, T)$, then

$$r_L + \frac{c}{\mu_H} \leq \bar{v}(r_H, r_L, \mu_H, T, c).$$

Note that no matter whether condition (i) or condition (ii) holds, $c \leq c_2(r_H, r_L, \mu_H, T)$ always holds. This provides the solution in Part (c).

<u>Case 3:</u> Suppose

$$M < 0, \tag{S.22}$$

$$\frac{\mu_L}{\nu} G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} < 0. \tag{S.23}$$

It follows that $G(r_H)=0$. Using the expression of $G(r_H)$ in (S.22) and (S.23), we obtain

$$\begin{aligned} &v<\tilde{v}(r_H,r_L,\mu_H,T,c),\\ &v<r_L+\frac{c}{\mu_H}. \end{aligned}$$

One can verify that if $c\le c_2(r_H,r_L,\mu_H,T)$, then

$$r_L+\frac{c}{\mu_H}\le \tilde{v}(r_H,r_L,\mu_H,T,c).$$

Note that no matter whether condition (i) or condition (ii) holds, $c\le c_2(r_H,r_L,\mu_H,T)$ always holds. This leads to the solution in Part (b).

Combining the above cases completes the proof. ■

Lemma S.5 **(Optimal purchase decisions when the price monitoring cost is intermediate)**. *Consider a type II customer with parameters (v,c) where $c_2(r_H,r_L,\mu_H,T)<c\le c_1(r_H,r_L,\mu_H,T)$ and $\mu_H<(\lambda+\mu_H)e^{-\lambda T}$. The optimal solution to equations (S.1)–(S.2) and the optimal purchase strategy of the customer are as follows:*

(a) If $v<r_L$, then $J(r_H)=J(r_L)=0$ and the customer never purchases;

(b) If $r_L\le v<\tilde{v}(r_H,r_L,\mu_H,T,c)$, then $G(r_H)=0$, $G(r_L)=v-r_L$, and the customer purchases upon arrival when the price is r_L, but leaves immediately without a purchase when the price is r_H;

(c) If $v\ge \tilde{v}(r_H,r_L,\mu_H,T,c)$, then $G(r_H)=M$ and $G(r_L)=v-r_L$. The customer purchases immediately upon arrival. If the purchase is made at the price r_H, she would keep monitoring the price until the price guarantee is applied/expired or her lifetime ends.

Proof of Lemma S.5

Part (a) is immediate.

Suppose $v\ge r_L$. We have

$$M\ge v-r_H$$

when $c\le c_1(r_H,r_L,\mu_H,T)$. Hence, equations (S.1)–(S.2) can be written as

$$\begin{aligned} G(r_H)&=\max\left\{M,\frac{\mu_L}{\nu}G(r_H)+\frac{\mu_H}{\nu}(v-r_L)-\frac{c}{\nu},0\right\},\\ G(r_L)&=v-r_L. \end{aligned}$$

It remains to solve for $G(r_H)$, which we break into three cases.

Case 1: Suppose

$$M\ge\frac{\mu_L}{\nu}G(r_H)+\frac{\mu_H}{\nu}(v-r_L)-\frac{c}{\nu}, \tag{S.24}$$

$$M \geq 0. \tag{S.25}$$

It follows that

$$G(r_H) = M = v - r_H + (1 - e^{-\mu_H T})\Big\{e^{-\lambda T}(r_H - r_L) - \frac{c}{\mu_H + \lambda}\Big\} + (1 - e^{-\lambda T})\Big\{\frac{\mu_H}{\mu_H + \lambda}(r_H - r_L) - \frac{c}{\mu_H + \lambda}\Big\}.$$

Using the expressions of $G(r_H)$ in (S.24) and simplifying (S.25), we obtain

$$v \geq \bar{v}(r_H, r_L, \mu_H, T, c),$$
$$v \geq \tilde{v}(r_H, r_L, \mu_H, T, c).$$

One can verify that

$$c_2(r_H, r_L, \mu_H, T) \leq c_1(r_H, r_L, \mu_H, T)$$

if and only if $\mu_H < (\lambda + \mu_H)e^{-\lambda T}$. One can also check that if $c > c_2(r_H, r_L, \mu_H, T)$, then

$$\bar{v}(r_H, r_L, \mu_H, T, c) < \tilde{v}(r_H, r_L, \mu_H, T, c).$$

Therefore, we have $\bar{v}(r_H, r_L, \mu_H, T, c) < \tilde{v}(r_H, r_L, \mu_H, T, c)$. This gives the solution in Part (c).

Case 2: Suppose

$$\frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} \geq M, \tag{S.26}$$
$$\frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} \geq 0. \tag{S.27}$$

Then

$$G(r_H) = \frac{\mu_L}{\nu}G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu}.$$

It follows that

$$G(r_H) = \frac{\mu_H(v - r_L) - c}{\lambda + \mu_H}.$$

Using the expression of $G(r_H)$ in (S.26) and (S.27), we obtain

$$v \leq \bar{v}(r_H, r_L, \mu_H, T, c),$$
$$v \geq r_L + \frac{c}{\mu_H}.$$

One can verify that if $c > c_2(r_H, r_L, \mu_H, T)$, then

$$r_L + \frac{c}{\mu_H} > \bar{v}(r_H, r_L, \mu_H, T, c).$$

Hence, this case is infeasible.

Case 3: Suppose

$$M < 0, \tag{S.28}$$

$$\frac{\mu_L}{\nu} G(r_H) + \frac{\mu_H}{\nu}(v - r_L) - \frac{c}{\nu} < 0. \tag{S.29}$$

It follows that $G(r_H) = 0$. Using the expression of $G(r_H)$ in (S.28) and (S.29), we obtain

$$v < \tilde{v}(r_H, r_L, \mu_H, T, c),$$
$$v < r_L + \frac{c}{\mu_H}.$$

One can verify that if $c > c_2(r_H, r_L, \mu_H, T)$, then

$$r_L + \frac{c}{\mu_H} > \tilde{v}(r_H, r_L, \mu_H, T, c).$$

This leads to the solution in Part (b).

Combining the above cases completes the proof. ■

Before proving Proposition S.8, we first establish the following lemma.

Lemma S.6. *Consider the purchase decisions for a type II customer with parameters* (v, c) *where either (i)* $c \le c_1(r_H, r_L, \mu_H, T)$ *and* $\mu_H \ge (\lambda + \mu_H)e^{-\lambda T}$, *or (ii)* $c \le c_2(r_H, r_L, \mu_H, T)$ *and* $\mu_H < (\lambda + \mu_H)e^{-\lambda T}$. *In the presence of price guarantees,*

(a) if $r_L = V_L$ *and* $r_L \le V_H < r_L + \frac{c}{\mu_H}$, *then the high/low pricing strategy is not optimal;*

(b) if $r_L = V_L - \frac{c}{\mu_H}$ *and* $V_L \ge r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{\mu_H T})\right]$, *then the high/low pricing strategy is not optimal.*

Proof of Lemma S.6

Part (a): According to Lemma S.3(c), when $r_L = V_L$, low-valuation type I customers purchase at price r_L but wait and monitor when the price is r_H. Meanwhile, low-valuation type II customers purchase at price r_L but leave without a purchase when the price is r_H.

According to Lemma S.3(b), when $r_L \le V_H < r_L + \frac{c}{\mu_H}$, high-valuation type II customers purchase at price r_L but leave without a purchase when the price is r_H.

High-valuation type I customers purchase at both prices immediately (and monitor for price guarantee if the purchase is made at price r_H) if

$$V_H \ge r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{\mu_H T})\right]. \tag{S.30}$$

Otherwise, they would purchase at price r_L but wait and monitor when the price is r_H.

If high-valuation type I customers purchase at price r_L but wait and monitor when the price is r_H, then none of the four customer segments purchase at the high price r_H. Therefore, it is impossible to obtain a profit higher than $r_L = V_L$.

Suppose they purchase at both prices immediately, that is, (S.30) holds. (S.30) can be rewritten as

$$r_H \leq \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.$$

The decision of each segment of customers can be summarized in Figure S.6.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at low price, leave at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.6 Customer Purchase Decisions.

Because the firm's profit is linear in prices, we must have the optimal high price $r_H^* = r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{\mu_H T})\right]$. Moreover, the price minus the expected refund claimed by a HI customer who purchases at the high price is

$$\begin{aligned}
&E[p^G] \\
=&\left[1-(1-e^{-\mu_H T})e^{-\lambda T} - (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] r_H + \left[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] r_L \\
=&\left[1-(1-e^{-\mu_H T})e^{-\lambda T} - (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]} \\
&\qquad + \left[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] V_L \\
=&\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H}.
\end{aligned}$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. We have

$$\begin{aligned}
\Phi(\mu_H,\mu_L,T) = &(1-\beta)\frac{\mu_H}{\mu_H+\mu_L} r_L + \gamma\left(\frac{\mu_H}{\mu_H+\mu_L} r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H}\right) \\
&+ (\beta-\gamma)\left(\frac{\mu_H}{\mu_H+\mu_L} r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H} r_L\right) - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\mu_H}{\mu_H+\mu_L} r_L + \gamma \frac{\mu_L}{\mu_H+\mu_L} \frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H} + (\beta-\gamma) \frac{\mu_L}{\mu_H+\mu_L} \frac{\mu_H}{\lambda+\mu_H} r_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L} \\
&= r_L + \frac{\mu_L}{\mu_H+\mu_L} \left\{ \gamma \frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H} + (\beta-\gamma) \frac{\mu_H}{\lambda+\mu_H} r_L - r_L - 2m\mu_H \right\} \\
&= V_L + \frac{\mu_L}{\mu_H+\mu_L} \left\{ \gamma \frac{\mu_H V_L + \lambda V_H}{\lambda+\mu_H} + (\beta-\gamma) \frac{\mu_H}{\lambda+\mu_H} V_L - V_L - 2m\mu_H \right\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi(\mu_H,\mu_L,T) < V_L$. Suppose the term is positive. Note that $\Phi(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^* = \infty$. It follows that

$$
\begin{aligned}
\Phi(\mu_H,\infty,T) &= \gamma \frac{\mu_H V_L + \lambda V_H}{\lambda+\mu_H} + (\beta-\gamma) \frac{\mu_H}{\lambda+\mu_H} V_L - 2m\mu_H \\
&= \frac{\gamma\lambda V_H + \beta\mu_H V_L}{\lambda+\mu_H} - 2m\mu_H \\
&= \beta V_L + \frac{\gamma\lambda V_H - \beta\lambda V_L}{\lambda+\mu_H} - 2m\mu_H.
\end{aligned}
$$

If $\gamma\lambda V_H - \beta\lambda V_L < 0$, then $\Phi(\mu_H,\infty,T) < V_L$. Otherwise,

$$
\Phi(\mu_H,\infty,T) < \beta V_L + \frac{\gamma\lambda V_H - \beta\lambda V_L}{\lambda} - 2m\mu_H = \gamma V_H - 2m\mu_H < \alpha V_H.
$$

This completes the proof of Part (a).

Part (b): When $r_L = V_L - \frac{c}{\mu_H}$, low-valuation type II customers purchase at price r_L but wait and monitor when the price is r_H.

Putting $r_L = V_L - \frac{c}{\mu_H}$ into $V_L \geq r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{\mu_H T})\right]$ yields

$$
r_H \leq V_L - \frac{e^{-\lambda T}\left[\mu_H - (\lambda+\mu_H)(1-e^{\mu_H T})\right]}{\lambda + e^{-\lambda T}\left[\mu_H - (\lambda+\mu_H)(1-e^{\mu_H T})\right]} \cdot \frac{c}{\mu_H} < V_L.
$$

Therefore, none of the four customer segments pay a price higher than V_L. Therefore, it is impossible to obtain a profit higher than V_L by adopting such a pricing strategy. ■

Now, we are ready to prove Proposition S.8.

Proof of Proposition S.8

Let's consider the high/low pricing strategy under which the firm needs to decide five parameters, r_H, r_L, μ_H, μ_L, and T. According to Lemma S.3, it is never optimal to charge a low price r_L different than V_L and $V_L - \frac{c}{\mu_H}$. Therefore, we consider two cases.

Case 1: $r_L = V_L$.

According to Lemma S.3, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, low-valuation type II customers purchase at the price r_L but leave without a purchase at the price r_H.

High-valuation type I customers purchase at both prices immediately if

$$V_H \geq r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right]. \tag{S.31}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (S.31) is not satisfied, they would purchase at the price r_L immediately and wait and monitor when the price is r_H.[11] Inequality (S.31) can be rewritten as

$$r_H \leq \frac{V_H + V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right]}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right]}.$$

High-valuation type II customers purchase immediately if

$$V_H \geq r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right] + (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}. \tag{S.32}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (S.32) is not satisfied, there are two possibilities. When $r_L + \frac{c}{\mu_H} \leq V_H < r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right] + (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}$, they would purchase at the price r_L but wait and monitor when the price is r_H. When $r_L \leq V_H < r_L + \frac{c}{\mu_H}$, they would purchase at the price r_L but leave without a purchase when the price is r_H. However, Lemma S.6(a) implies that the high/low pricing strategy is not optimal in this case. Therefore, we restrict our attention to the first possibility if (S.32) does not hold. Inequality (S.32) can be written as

$$r_H \leq \frac{V_H + V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right] - (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right]}.$$

From the conditions on r_H, we can analyze the firm's pricing problem based on the range of r_H. We consider four cases, labeled Cases 1.1-1.4. We will show each of the four cases generates a profit lower than the profit from static pricing at either V_L or V_H.

<u>Case 1.1:</u> Suppose

$$r_H > \frac{V_H + V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right]}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right]},$$

$$r_H > \frac{V_H + V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right] - (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda + \mu_H)(1 - e^{-\mu_H T})\right]}.$$

Then both high-valuation type I and high-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure S.7.

[11] Note that when $c = 0$, Lemma S.3(b) does not exist.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.7 Customer Purchase Decisions in Case 1.1.

Since customers never purchase at the price r_H, it is impossible to obtain a profit higher than $r_L = V_L$.

Case 1.2: Suppose

$$r_H > \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}, \tag{S.33}$$

$$r_H \le \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right] - (1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}. \tag{S.34}$$

Then high-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure S.8.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.8 Customer Purchase Decisions in Case 1.2.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$. Moreover, the price minus the expected refund claimed by a HII customer who purchases at the high price is

$E[p^G]$

$$
\begin{aligned}
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_H+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_L\\
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}\\
&\quad+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]V_L\\
=&\frac{\lambda V_H+\mu_H V_L-(1-e^{-\lambda T}-e^{-\mu_H T})c}{\lambda+\mu_H}.
\end{aligned}
$$

The only remaining parameters are μ_H, μ_L and T. Let $\Phi^{1.2}(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
&\Phi^{1.2}(\mu_H,\mu_L,T)\\
=&\beta\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)+(\alpha-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-(1-e^{-\lambda T}-e^{-\mu_H T})c}{\lambda+\mu_H}\Big)\\
&\quad+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L+\beta\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L+(\alpha-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-(1-e^{-\lambda T}-e^{-\mu_H T})c}{\lambda+\mu_H}-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L-(1-e^{-\lambda T}-e^{-\mu_H T})c}{\lambda+\mu_H}-V_L-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^{1.2}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{1.2}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$\Phi^{1.2}(\mu_H,\infty,T)=\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-(\alpha-\gamma)\frac{1-e^{-\lambda T}-e^{-\mu_H T}}{\lambda+\mu_H}c-2m\mu_H.$$

Note that $\Phi^{1.2}(\mu_H,\infty)$ is decreasing in T, so $T^*=0$, and thus

$$\Phi^{1.2}(\mu_H,\infty,0)=\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L+c}{\lambda+\mu_H}-2m\mu_H.$$

The remaining analysis is the same as Case 1.2 in the proof of Lemma E.5 in E-Companion. In the end, we obtain $\Phi^{1.2}(\mu_H,\infty,0)<\max\{V_L,\alpha V_H\}$.

Case 1.3: Suppose

$$
\begin{aligned}
r_H&\le\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]},\\
r_H&>\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}.
\end{aligned}
$$

Then high-valuation type I customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. Meanwhile, high-valuation type II customers purchase at the

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.9 Customer Purchase Decisions in Case 1.3.

price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure S.9.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$. Moreover, the price minus the expected refund claimed by a HI customer who purchases at the high price is

$$
\begin{aligned}
&E[p^G]\\
=&\left[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_H+\left[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_L\\
=&\left[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}\\
&\quad+\left[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]V_L\\
=&\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}.
\end{aligned}
$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi^{1.3}(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
&\Phi^{1.3}(\mu_H,\mu_L,T)\\
=&\gamma\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}\Big)+(\alpha+\beta-2\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)\\
&\quad+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L+\gamma\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\alpha+\beta-2\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\alpha+\beta-2\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L-r_L-2m\mu_H\Big\}\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\alpha+\beta-2\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-V_L-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^{1.3}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{1.3}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$\begin{aligned}\Phi^{1.3}(\mu_H,\infty,T)&=\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\alpha+\beta-2\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-2m\mu_H\\&=(\alpha+\beta-\gamma)V_L+\frac{\gamma\lambda V_H-(\alpha+\beta-\gamma)\lambda V_L}{\lambda+\mu_H}-2m\mu_H.\end{aligned}$$

If $\gamma\lambda V_H-(\alpha+\beta-\gamma)\lambda V_L<0$, then $\Phi^{1.3}(\mu_H,\infty,T)<(\alpha+\beta-\gamma)V_L<V_L$. Otherwise,

$$\Phi^{1.3}(\mu_H,\infty,T)<(\alpha+\beta-\gamma)V_L+\frac{\gamma\lambda V_H-(\alpha+\beta-\gamma)\lambda V_L}{\lambda}-2m\mu_H=\gamma V_H-2m\mu_H<\alpha V_H.$$

Case 1.4: Suppose

$$r_H\le\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]},\tag{S.35}$$

$$r_H\le\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.\tag{S.36}$$

Then both high-valuation type I and high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure S.10.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.10 Customer Purchase Decisions in Case 1.4.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^*=\min\left\{\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]},\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}\right\}$. Moreover, the price minus the expected refund claimed by a high-valuation customer who purchases at the high price is

$$\begin{aligned}&E[p^G]\\&=\left[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_H+\left[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_L\end{aligned}$$

$$
\begin{aligned}
\leq&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}\\
&\qquad+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]V_L\\
=&\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}.
\end{aligned}
$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi^{1.4}(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
&\Phi^{1.4}(\mu_H,\mu_L,T)\\
\leq&\alpha\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}\Big)+(\beta-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)\\
&\quad+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L+\alpha\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-V_L-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^{1.4}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{1.4}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\Phi^{1.4}(\mu_H,\infty,T)=\alpha\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-2m\mu_H.
$$

The remaining analysis is the same as Case 1.3 in the proof of Lemma E.5 in E-Companion, establishing that $\Phi^{1.4}(\mu_H,\infty,T)<\max\{V_L,\alpha V_H\}$.

Case 2: $r_L=V_L-\frac{c}{\mu_H}$.

According to Lemma S.3, low-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H.

For low-valuation type I customers, there are two possibilities. If $V_L>r_H+(r_H-r_L)\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]$, they purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. However, Lemma S.6(b) shows that the high/low pricing strategy is not optimal in this case. If $V_L\leq r_H+(r_H-r_L)\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]$, they purchase at the price r_L but wait and monitor when the price is r_H. We will focus on the second possibility.

High-valuation type I customers purchase at both prices immediately if

$$
V_H\geq r_H+(r_H-r_L)\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]. \tag{S.37}
$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (S.37) is not satisfied, they would purchase at the price r_L immediately and wait and monitor when the price is r_H. Inequality (S.37) can be rewritten as

$$r_H \leq \frac{V_H + (V_L - \frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.$$

High-valuation type II customers purchase immediately if

$$V_H \geq r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right] + (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}. \tag{S.38}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (S.38) is not satisfied, they would purchase at the price r_L but wait and monitor when the price is r_H. Inequality (S.38) can be written as

$$r_H \leq \frac{V_H + (V_L - \frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right] - (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.$$

From the conditions on r_H, we can analyze the firm's pricing problem based on the range of r_H. We consider four cases, labeled Cases 2.1-2.4. We will show each of the four cases generates a profit lower than $\max\{V_L, \alpha V_H\}$.

Case 2.1: Suppose

$$r_H > \frac{V_H + (V_L - \frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]},$$

$$r_H > \frac{V_H + (V_L - \frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right] - (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.$$

Then both high-valuation type I and high-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure S.11.

Since customers never purchase at price r_H, it is impossible to obtain a profit higher than $r_L < V_L$.

Case 2.2: Suppose

$$r_H > \frac{V_H + (V_L - \frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]},$$

$$r_H \leq \frac{V_H + (V_L - \frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right] - (1 - e^{-\lambda T} - e^{-\mu_H T})\frac{c}{\lambda}}{1 + \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.$$

Then high-valuation type I customers purchase at price r_L but wait and monitor when the price is r_H. Meanwhile, high-valuation type II customers purchase at both prices and monitor if the purchase is made at price r_H. The purchase decisions of customers can be summarized in Figure S.12.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.11 Customer Purchase Decisions in Case 2.1.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.12 Customer Purchase Decisions in Case 2.2.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$. Moreover, the price minus the expected refund claimed by a HII customer who purchases at the high price is

$$
\begin{aligned}
&E[p^G] \\
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_H+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_L \\
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]\frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]-(1-e^{-\lambda T}-e^{-\mu_H T})}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]} \\
&\quad+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big](V_L-\frac{c}{\mu_H}) \\
=&\frac{\lambda V_H+\mu_H V_L-(2-e^{-\mu_H T}-e^{-\lambda T})c}{\lambda+\mu_H}.
\end{aligned}
$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi^{2.2}(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
&\Phi^{2.2}(\mu_H,\mu_L,T) \\
=&(\alpha-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-(2-e^{-\mu_H T}-e^{-\lambda T})c}{\lambda+\mu_H}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+(1-\alpha+\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L+(\alpha-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-(2-e^{-\mu_H T}-e^{-\lambda T})c}{\lambda+\mu_H}+(1-\alpha+\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L-(2-e^{-\mu_H T}-e^{-\lambda T})c}{\lambda+\mu_H}+(1-\alpha+\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L-r_L-2m\mu_H\Big\}\\
=&V_L-\frac{c}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L-(2-e^{-\mu_H T}-e^{-\lambda T})c}{\lambda+\mu_H}+(1-\alpha+\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}\\
&\qquad\qquad-(V_L-\frac{c}{\mu_H})-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^{2.2}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{2.2}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\begin{aligned}
\Phi^{2.2}(\mu_H,\infty,T)&=(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L-(2-e^{-\mu_H T}-e^{-\lambda T})c}{\lambda+\mu_H}+(1-\alpha+\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}-2m\mu_H\\
&\le(\alpha-\gamma)\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(1-\alpha+\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}-2m\mu_H.
\end{aligned}
$$

The remaining analysis is the same as Case 2.2 in the proof of Lemma E.5 in E-Companion. Therefore, we obtain $\Phi^{2.2}(\mu_H,\infty,T)<\max\{V_L,\alpha V_H\}$.

<u>Case 2.3:</u> Suppose

$$
\begin{aligned}
r_H&\le\frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]},\\
r_H&>\frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}.
\end{aligned}
$$

Then high-valuation type I customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. Meanwhile, high-valuation type II customers purchase at the price r_L but wait and monitor when the price is r_H. The purchase decisions of customers can be summarized in Figure S.13.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.13 Customer Purchase Decisions in Case 2.3.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$. Moreover, the price minus the expected refund claimed by a HI customer who purchases at the high price is

$$
\begin{aligned}
&E[p^G]\\
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_H+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_L\\
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]\frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}\\
&\quad+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big](V_L-\frac{c}{\mu_H})\\
=&\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}.
\end{aligned}
$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi^{2.3}(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
&\Phi^{2.3}(\mu_H,\mu_L,T)\\
=&\gamma\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}\Big)\\
&\quad+(1-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&\frac{\mu_H}{\mu_H+\mu_L}r_L+\gamma\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&r_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\gamma)\frac{\mu_H}{\lambda+\mu_H}r_L-r_L-2m\mu_H\Big\}\\
=&V_L-\frac{c}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}-(V_L-\frac{c}{\mu_H})-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^{2.3}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{2.3}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\begin{aligned}
\Phi^{2.3}(\mu_H,\infty,T)=&\gamma\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\gamma)\frac{\mu_H V_L-c}{\lambda+\mu_H}-2m\mu_H\\
=&V_L+\frac{\gamma\lambda V_H-\lambda V_L-c}{\lambda+\mu_H}-2m\mu_H.
\end{aligned}
$$

If $\gamma\lambda V_H-\lambda V_L-c<0$, then $\Phi^{2.3}(\mu_H,\infty,T)<(\alpha+\beta-\gamma)V_L<V_L$. Otherwise,

$$
\Phi^{2.3}(\mu_H,\infty,T)<V_L+\frac{\gamma\lambda V_H-\lambda V_L-c}{\lambda}-2m\mu_H<\gamma V_H-2m\mu_H<\alpha V_H.
$$

<u>Case 2.4:</u> Suppose

$$
r_H\le\frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]},
$$

$$r_H \leq \frac{V_H + (V_L - \frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right] - (1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.$$

In this case, both high-valuation type I and high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure S.14.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.14 Customer Purchase Decisions in Case 2.4.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \min\left\{\frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}, \frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]-(1-e^{-\lambda T}-e^{-\mu_H T})\frac{c}{\lambda}}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}\right\}$. Moreover, the price minus the expected refund claimed by a high-valuation customer who purchases at the high price is

$$\begin{aligned}
&E[p^G]\\
=&\left[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_H + \left[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_L\\
\leq&\left[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]\frac{V_H+(V_L-\frac{c}{\mu_H})\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}\\
&\quad+\left[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right](V_L-\frac{c}{\mu_H})\\
=&\frac{\lambda V_H+\mu_H V_L - c}{\lambda+\mu_H}.
\end{aligned}$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi^{2.4}(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
&\Phi^{2.4}(\mu_H,\mu_L,T)\\
=&\alpha\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}\Big)+(1-\alpha)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)\\
&-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}
\end{aligned}$$

$$
\begin{aligned}
&=\frac{\mu_H}{\mu_H+\mu_L}r_L+\alpha\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\alpha)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
&=r_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha\frac{\lambda V_H+\mu_H V_L-c}{\lambda+\mu_H}+(1-\alpha)\frac{\mu_H}{\lambda+\mu_H}r_L-r_L-2m\mu_H\Big\}\\
&=V_L-\frac{c}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha\frac{\mu_H V_L+\lambda V_H-c}{\lambda+\mu_H}+(1-\alpha)\frac{\mu_H V_L-c}{\lambda+\mu_H}-(V_L-\frac{c}{\mu_H})-2m\mu_H\Big\}.
\end{aligned}
$$

The remaining analysis is the same as Case 2.3 in the proof of Lemma E.5 in E-Companion. Therefore, we have $\Phi^{2.4}(\mu_H,\infty,T)<\max\{V_L,\alpha V_H\}$. This completes the proof of Proposition S.8.

■

Proof of Proposition S.9

First, type I customers' behavior can be obtained by taking $c=0$ in Lemma S.3, while type II customers' behavior is the same as in Lemma S.2. Taking into account the behavior of both types of customers, it is never optimal to charge the low price r_L different from V_L. The only remaining parameters are μ_H, μ_L, r_H, and T.

When $r_L=V_L$, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H, while low-valuation type II customers purchase at the price r_L but leave without a purchase when the price is r_H.

High-valuation type I customers purchase at both prices immediately if

$$
V_H\geq r_H+(r_H-r_L)\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]. \tag{S.39}
$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied or expires. If (S.39) is not satisfied, they would purchase at the price r_L immediately and wait and monitor when the price is r_H. Inequality (S.39) can be rewritten as

$$
r_H\leq\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}.
$$

For high-valuation type II customers, there are two possibilities. When $V_H<r_H$, they purchase at the price r_L but leave without a purchase when the price is r_H. At the same time, if high-valuation type I customers purchase at both prices, then customers' decision is the same as in Figure S.6; therefore, following the analysis in Lemma S.6(a), the high/low pricing strategy is never optimal. If high-valuation type I customers purchase at the price r_L but wait when the price is r_H, then no customers of the four segments purchase at the high price r_H; therefore, the high/low pricing strategy is also not optimal. To summarize, when $V_H<r_H$, the high/low pricing strategy is not optimal. Hereafter, we restrict our attention to the possibility with $V_H\geq r_H$, in which case high-valuation type II customers purchase at both prices but do not monitor for price guarantees.

We can analyze the firm's pricing problem based on the range of r_H. We consider two cases, labeled Cases I and II.

Case I: $\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]} < r_H \le V_H$**.**

In this case, the purchase decisions of customers can be summarized in Figure S.15.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, do not monitor for price guarantee
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.15 Customer Purchase Decisions in Case I.

The analysis and result are the same as in the case without price guarantees (Case I in the proof of Proposition 4 in E-Companion). Hence,

$$\Phi^{I,*} = \begin{cases} (\alpha-\gamma)V_H + \beta\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L - 2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}}-\lambda\right), & \text{if } \beta V_L - 2m\lambda > 0, \\ (\alpha-\gamma)V_H, & \text{if } \beta V_L - 2m\lambda \le 0. \end{cases}$$

Case II: $r_H \le \frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$ **and** $r_H \le V_H$**.**

In this case, the purchase decisions of customers can be summarized in Figure S.16.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor for price guarantee	Buy at both prices, do not monitor for price guarantee
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.16 Customer Purchase Decisions in Case II.

Comparing $\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$ with V_H yields that

$$\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]} \ge V_H$$

if and only if $\frac{\lambda}{\lambda+\mu_H} \geq e^{-\mu_H T}$. Therefore, we have two subcases with respect to the range of T.

Subcase II.A: $\frac{\lambda}{\lambda+\mu_H} \leq e^{-\mu_H T}$.

In this subcase, the optimal high price must be $r_H^* = \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$. Moreover, the price minus the expected refund claimed by a HI customer who purchases at the high price is

$$
\begin{aligned}
E[p^G] &= \left[1-(1-e^{-\mu_H T})e^{-\lambda T} - (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] r_H + \left[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] r_L \\
&= \left[1-(1-e^{-\mu_H T})e^{-\lambda T} - (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]} \\
&\quad + \left[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right] V_L \\
&= \frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H}.
\end{aligned}
$$

Let $\Phi^{II,A}(\mu_H,\mu_L,T)$ denote the firm's profit in this case. Then,

$$
\begin{aligned}
&\Phi^{II,A}(\mu_H,\mu_L,T) \\
=& \underbrace{\gamma\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} + \frac{\mu_H}{\mu_H+\mu_L}\cdot V_L\right]}_{\text{revenue from HI customers}} \\
&+ \underbrace{(\alpha-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]} + \frac{\mu_H}{\mu_H+\mu_L}\cdot V_L\right]}_{\text{revenue from HII customers}} \\
&+ \underbrace{(\beta-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L + \frac{\mu_H}{\mu_H+\mu_L}V_L\right]}_{\text{revenue from LI customers}} \\
&+ \underbrace{(1-\alpha-\beta+\gamma)\cdot\frac{\mu_H}{\mu_H+\mu_L}\cdot V_L}_{\text{revenue from LII customers}} - \underbrace{\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}}_{\text{cost of price changes}} \\
=& V_L + \frac{\mu_L}{\mu_H+\mu_L}\left[\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} + (\alpha-\gamma)\frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}\right. \\
&\left. + (\beta-\gamma)\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L - V_L - 2m\mu_H\right] \\
\leq& V_L + \frac{\mu_L}{\mu_H+\mu_L}\left[\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} + (\alpha-\gamma)V_H + (\beta-\gamma)\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L - V_L - 2m\mu_H\right],
\end{aligned}
$$

where the last inequality holds because $\frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]} \leq V_H$. Therefore, the profit in this subcase is dominated by Subcase II.B analyzed below.

<u>Subcase II.B:</u> $\frac{\lambda}{\lambda+\mu_H} \geq e^{-\mu_H T}$.

In this subcase, the optimal high price must be $r_H^* = V_H$. Moreover, the price minus the expected refund claimed by a HI customer who purchased at the high price is

$$E[p^G] = \Big[1-(1-e^{-\mu_H T})e^{-\lambda T} - (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]V_H + \Big[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]V_L$$
$$= V_H - \Big[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big](V_H - V_L).$$

Let $\Phi^{II,B}(\mu_H,\mu_L,T)$ denote the firm's profit in this case. Then,

$$\begin{aligned}
&\Phi^{II,B}(\mu_H,\mu_L,T)\\
=&\gamma\left[\frac{\mu_L}{\mu_H+\mu_L}\Big\{V_H - \Big[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big](V_H-V_L)\Big\} + \frac{\mu_H}{\mu_H+\mu_L}V_L\right]\\
&+(\alpha-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}V_H + \frac{\mu_H}{\mu_H+\mu_L}V_L\right] + (\beta-\gamma)\left[\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}V_L + \frac{\mu_H}{\mu_H+\mu_L}V_L\right]\\
&+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}V_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L + \frac{\mu_L}{\mu_H+\mu_L}\left\{\gamma\Big\{V_H - \Big[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big](V_H-V_L)\Big\} + (\alpha-\gamma)V_H\right.\\
&\left.\qquad + (\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L - V_L - 2m\mu_H\right\}.
\end{aligned}$$

The optimal T^* should minimize $e^{-\lambda T}(1-e^{-\mu_H T}) + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}$. Note that $1-e^{-\mu_H T} \geq \frac{\mu_H}{\lambda+\mu_H}$, so $e^{-\lambda T}$ should be as small as possible. Therefore, $T^* = \infty$ and $(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H} = \frac{\mu_H}{\lambda+\mu_H}$. Putting $T^* = \infty$ back to the profit function yields that

$$\begin{aligned}
&\Phi^{II,B}(\mu_H,\mu_L,\infty)\\
=&V_L + \frac{\mu_L}{\mu_H+\mu_L}\left[\gamma\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H} + (\alpha-\gamma)V_H + (\beta-\gamma)\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L - V_L - 2m\mu_H\right].
\end{aligned}$$

When the term in the square brackets is negative, the profit is less than V_L, which is the profit from static pricing at V_L. We proceed with the analysis assuming the term in the square brackets is positive. In our final analysis, we will compare the profit in this case with the optimal profit without price guarantees.

Since $\Phi^{II,B}(\mu_H,\mu_L,\infty)$ is increasing in μ_L, the optimal value of μ_L is ∞. We have

$$\Phi^{II,B}(\mu_H,\infty,\infty) = \gamma\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H} + (\alpha-\gamma)V_H + (\beta-\gamma)\cdot\frac{\mu_H}{\lambda+\mu_H}\cdot V_L - 2m\mu_H.$$

It can be shown that when $K \leq 0$, $\Phi^{II,B}(\mu_H,\infty,\infty)$ decreases in μ_H. Hence, the optimal $\mu_H = 0$. The corresponding profit is

$$\Phi^{II,B}(0,\infty,\infty) = \alpha V_H,$$

which is the same as the revenue from static pricing at V_H.

When $K>0$, we can solve for μ_H using the first-order condition, which gives

$$\mu_H^{II,B,*}=\begin{cases}\sqrt{\frac{\lambda K}{2m}}-\lambda, & \text{if } K-2m\lambda>0,\\ 0, & \text{if } K-2m\lambda\le 0.\end{cases}$$

The corresponding profit is

$$\Phi^{II,B,*}=\begin{cases}\beta V_L+(\alpha-\gamma)V_H+(\gamma V_H-\beta V_L)\sqrt{\frac{2m\lambda}{K}}-2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right), & \text{if } K-2m\lambda>0,\\ \\ \alpha V_H, & \text{if } K-2m\lambda\le 0.\end{cases}$$

Moreover, when $K-2m\lambda>0$, we obtain $T^*=\infty$, and $r_H^*=V_H$. Putting μ_H^*, r_H^*, r_L^*, and T^* into the condition $c>c_1(r_H,r_L,\mu_H,T)$ yields the constraint

$$c>\frac{\lambda}{2}\Big(\sqrt{\frac{K}{2m\lambda}}-1\Big)(V_H-V_L).$$

Summarizing the results for Cases I and II yields the following:

- If $K>2m\lambda$, then

$$\Phi^{I,*}=(\alpha-\gamma)V_H+\beta\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L-2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}}-\lambda\right),$$
$$\Phi^{II,*}=\Phi^{II,B,*}=\beta V_L+(\alpha-\gamma)V_H+(\gamma V_H-\beta V_L)\sqrt{\frac{2m\lambda}{K}}-2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right).$$

One can check that $\Phi^{II,*}\ge\Phi^{I,*}$. Hence,

$$\Phi^{B,*}=\beta V_L+(\alpha-\gamma)V_H+(\gamma V_H-\beta V_L)\sqrt{\frac{2m\lambda}{K}}-2m\lambda\left(\sqrt{\frac{K}{2m\lambda}}-1\right).$$

- If $K\le 2m\lambda<\beta V_L$, then

$$\Phi^{I,*}=(\alpha-\gamma)V_H+\beta\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L-2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}}-\lambda\right),$$
$$\Phi^{II,*}=\Phi^{II,B,*}=\alpha V_H.$$

Hence,

$$\Phi^{B,*}=\max\Big\{(\alpha-\gamma)V_H+\beta\left(1-\sqrt{\frac{2m\lambda}{\beta V_L}}\right)V_L-2m\left(\sqrt{\frac{\beta\lambda V_L}{2m}}-\lambda\right),\alpha V_H\Big\}.$$

- If $K\le\beta V_L\le 2m\lambda$, then $\Phi^{I,*}=(\alpha-\gamma)V_H<\alpha V_H=\Phi^{II,*}$. Hence, $\Phi^{B,*}=\Phi^{II,*}=\alpha V_H$.

Comparing V_L, αV_H, and $\Phi^{B,*}$ when $K\le 2m\lambda$ shows that the pricing strategy is the same as that in Proposition 4. Comparing the profits when $K>2m\lambda$ yields the results in Parts (i)–(iii). This completes the proof of Proposition S.9. ∎

Proposition S.15. *If the monitoring cost c is intermediate (as stated in Lemma S.5) such that type II customers who purchase at the high price monitor the price after purchase, high/low pricing with price guarantees cannot improve the firm profit, compared to static pricing.*

Proof of Proposition S.15

Let's consider the high/low pricing strategy under which the firm needs to decide five parameters, r_H, r_L, μ_H, μ_L, and T. According to Lemma S.3 with $c=0$ and Lemma S.5, it is never optimal to charge a low price r_L different than V_L. Therefore, we focus on the case where $r_L = V_L$.

According to Lemma S.3, low-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, according to Lemma S.5, low-valuation type II customers purchase at the price r_L but leave without a purchase at the price r_H.

High-valuation type I customers purchase at both prices immediately if

$$V_H \geq r_H + (r_H - r_L)\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]. \tag{S.40}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied/expired or their lifetime ends. If (S.40) is not satisfied, they would purchase at the price r_L immediately and wait and monitor when the price is r_H. Inequality (S.40) can be rewritten as

$$r_H \leq \frac{V_H + V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}.$$

High-valuation type II customers purchase immediately if

$$V_H \geq r_H - (r_H - r_L)\Big(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\Big) + (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}. \tag{S.41}$$

Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied/expired or their lifetime ends. If (S.41) is not satisfied, they would purchase at the price r_L but leave without a purchase when the price is r_H. Inequality (S.41) can be written as

$$r_H \leq \frac{V_H - V_L\Big(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\Big) - (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T}) + e^{-\lambda T}e^{-\mu_H T}}.$$

From the conditions on r_H, we can analyze the firm's pricing problem based on the range of r_H. We consider four cases, labeled Cases 1-4. We will show each of the four cases generates a profit lower than the profit from static pricing at either V_L or V_H.

<u>Case 1:</u> Suppose

$$r_H > \frac{V_H + V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]},$$

$$r_H > \frac{V_H - V_L\Big(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\Big) - (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T}) + e^{-\lambda T}e^{-\mu_H T}}.$$

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, leave at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.17 Customer Purchase Decisions in Case 1.

Then high-valuation type I customers purchase at price r_L but wait and monitor when the price is r_H, while high-valuation type II customers purchase at price r_L but leave when the price is r_H. The purchase decisions of customers can be summarized in Figure S.17.

Since customers never purchase at the price r_H, it is impossible to obtain a profit higher than $r_L = V_L$.

Case 2: Suppose

$$r_H > \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}, \tag{S.42}$$

$$r_H \le \frac{V_H - V_L\left(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\right) - (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T}) + e^{-\lambda T}e^{-\mu_H T}}. \tag{S.43}$$

Then high-valuation type I customers purchase at the price r_L but wait and monitor when the price is r_H. Meanwhile, high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure S.18.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.18 Customer Purchase Decisions in Case 2.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* =$

$\frac{V_H - V_L\left(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\right) - (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T}) + e^{-\lambda T}e^{-\mu_H T}}$. Moreover, the price minus the expected refund claimed by a HII customer who purchases at the high price is

$$\begin{aligned}
&E[p^G]\\
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_H+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]r_L\\
=&\Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]\frac{V_H - V_L\left(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\right) - (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T}) + e^{-\lambda T}e^{-\mu_H T}}\\
&\quad+\Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]V_L\\
=&V_H-(2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}.
\end{aligned}$$

The only remaining parameters are μ_H, μ_L and T. Let $\Phi^2(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
&\Phi^2(\mu_H,\mu_L,T)\\
=&\beta\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big)+(\alpha-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\big[V_H-(2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}\big]\Big)\\
&\quad+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)\big[V_H-(2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}\big]-V_L-2m\mu_H\Big\}.
\end{aligned}$$

If the term in the brackets is negative, then $\Phi^{1.2}(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^{1.2}(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$\begin{aligned}
\Phi^2(\mu_H,\infty,T)&=\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)V_H-(\alpha-\gamma)(2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}-2m\mu_H\\
&\le\beta\frac{\mu_H}{\lambda+\mu_H}V_L+(\alpha-\gamma)V_H-2m\mu_H\\
&\le\Phi^*,
\end{aligned}$$

which is the optimal profit of high/low pricing when there are no price guarantees.

<u>Case 3:</u> Suppose

$$\begin{aligned}
r_H&\le\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]},\\
r_H&>\frac{V_H - V_L\left(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\right) - (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T}) + e^{-\lambda T}e^{-\mu_H T}}.
\end{aligned}$$

Then high-valuation type I customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. Meanwhile, high-valuation type II customers purchase at the price r_L but leave when the price is r_H. The purchase decisions of customers can be summarized in Figure S.19.

	Type I 0 (β)	Type II c (1-β)
v_H (α)	Buy at both prices, monitor at high price	Buy at low price, leave at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.19 Customer Purchase Decisions in Case 3.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}$. Moreover, the price minus the expected refund claimed by a HI customer who purchases at the high price is

$$
\begin{aligned}
&E[p^G] \\
=&\left[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_H + \left[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_L \\
=&\left[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]\frac{V_H + V_L \frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]} \\
&\quad + \left[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]V_L \\
=&\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H}.
\end{aligned}
$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi^3(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
&\Phi^3(\mu_H,\mu_L,T) \\
=&\gamma\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H}\Big) + (\beta-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L + \frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big) \\
&\quad + (1-\beta)\frac{\mu_H}{\mu_H+\mu_L}r_L - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L} \\
=&V_L + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H} + (\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L - V_L - 2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^3(\mu_H,\mu_L,T) < V_L$. Suppose the term is positive. Note that $\Phi^3(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^* = \infty$. It follows that

$$\begin{aligned}\Phi^3(\mu_H,\infty,T) &= \gamma\frac{\lambda V_H + \mu_H V_L}{\lambda+\mu_H} + (\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L - 2m\mu_H \\ &= \beta V_L + \frac{\lambda(\gamma V_H - \beta V_L)}{\lambda+\mu_H} - 2m\mu_H.\end{aligned}$$

If $\gamma V_H - \beta V_L < 0$, then $\Phi^3(\mu_H,\infty,T) < \beta V_L < V_L$. Otherwise,

$$\Phi^3(\mu_H,\infty,T) < \beta V_L + \frac{\lambda(\gamma V_H - \beta V_L)}{\lambda} - 2m\mu_H = \gamma V_H - 2m\mu_H < \alpha V_H.$$

Case 4: Suppose

$$r_H \le \frac{V_H + V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H - (\lambda+\mu_H)(1-e^{-\mu_H T})\right]}, \tag{S.44}$$

$$r_H \le \frac{V_H - V_L\left(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T} - e^{-\mu_H T}e^{-\lambda T} + \frac{\mu_H}{\lambda+\mu_H}\right) - (2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T}) + e^{-\lambda T}e^{-\mu_H T}}. \tag{S.45}$$

Then both high-valuation type I and high-valuation type II customers purchase at both prices and monitor for price guarantees if the purchase is made at the price r_H. The purchase decisions of customers can be summarized in Figure S.20.

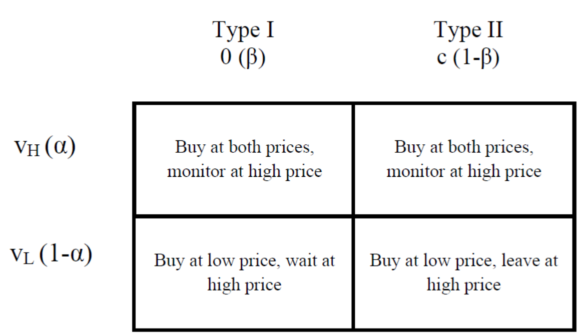


Figure S.20 Customer Purchase Decisions in Case 4.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^* = \min\left\{\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}{1+\frac{e^{-\lambda T}}{\lambda}\left[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\right]}, \frac{V_H-V_L\left(\frac{\lambda}{\lambda+\mu_H}e^{-\lambda T}-e^{-\mu_H T}e^{-\lambda T}+\frac{\mu_H}{\lambda+\mu_H}\right)-(2-e^{-\mu_H T}-e^{-\lambda T})\frac{c}{\lambda+\mu_H}}{\frac{\lambda}{\lambda+\mu_H}(1-e^{-\lambda T})+e^{-\lambda T}e^{-\mu_H T}}\right\}$. Moreover, the price minus the expected refund claimed by a high-valuation customer who purchases at the high price is

$$\begin{aligned}&E[p^G] \\ =&\left[1-(1-e^{-\mu_H T})e^{-\lambda T} - (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_H + \left[(1-e^{-\mu_H T})e^{-\lambda T} + (1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\right]r_L\end{aligned}$$

$$
\begin{aligned}
\leq & \Big[1-(1-e^{-\mu_H T})e^{-\lambda T}-(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]\frac{V_H+V_L\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]}{1+\frac{e^{-\lambda T}}{\lambda}\big[\mu_H-(\lambda+\mu_H)(1-e^{-\mu_H T})\big]} \\
& \quad + \Big[(1-e^{-\mu_H T})e^{-\lambda T}+(1-e^{-\lambda T})\frac{\mu_H}{\lambda+\mu_H}\Big]V_L \\
= & \frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}.
\end{aligned}
$$

The only remaining parameters are μ_H, μ_L, and T. Let $\Phi^4(\mu_H,\mu_L,T)$ denote the firm's profit per unit time in this case. Then,

$$
\begin{aligned}
& \Phi^4(\mu_H,\mu_L,T) \\
\leq & \alpha\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}\Big)+(\beta-\gamma)\Big(\frac{\mu_H}{\mu_H+\mu_L}r_L+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L\Big) \\
& \quad +(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L} \\
= & \frac{\mu_H}{\mu_H+\mu_L}r_L+\alpha\frac{\mu_L}{\mu_H+\mu_L}\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}r_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L} \\
= & V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-V_L-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi^4(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi^4(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\Phi^4(\mu_H,\infty,T)=\alpha\frac{\mu_H V_L+\lambda V_H}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}V_L-2m\mu_H.
$$

The remaining analysis is the same as Case 1.3 in the proof of Lemma E.5 in E-Companion, establishing that $\Phi^4(\mu_H,\infty,T)<\max\{V_L,\alpha V_H\}$. ■

S.4.2 Proof of Lemmas and Propositions in Section S.2

Proof of Lemma S.4

We first show that for any v, it is impossible for $J(r_H)$ to take $v-r_H+(1-e^{-\mu_H T})(r_H-r_L-\frac{c}{\mu_H})$. Suppose for a contradiction that

$$
J(r_H)=v-r_H+(1-e^{-\mu_H T})(r_H-r_L-\frac{c}{\mu_H}). \tag{S.46}
$$

Then,

$$
v-r_H+(1-e^{-\mu_H T})(r_H-r_L-\frac{c}{\mu_H})\geq\frac{\mu_L}{\mu_H+\mu_L}J(r_H)+\frac{\mu_H}{\mu_H+\mu_L}J(r_L)-\frac{c}{\mu_H+\mu_L}.
$$

Putting (S.46) into the right-hand side yields

$$
\frac{\mu_H}{\mu_H+\mu_L}\Big\{v-r_H+(1-e^{-\mu_H T})(r_H-r_L-\frac{c}{\mu_H})\Big\}\geq\frac{\mu_H}{\mu_H+\mu_L}(v-r_L-\frac{c}{\mu_H})
$$

$$\Rightarrow \quad v - r_H + (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}) \geq v - r_L - \frac{c}{\mu_H}$$
$$\Rightarrow \quad (1 - e^{-\mu_H T})(r_H - r_L - \frac{c}{\mu_H}) \geq r_H - r_L - \frac{c}{\mu_H},$$

which arrives at a contradiction given that $c \leq \mu_H(r_H - r_L)$. Therefore, equations (S.5) and (S.6) can be reduced to

$$J(r_H) = \max\Big\{v - r_H, \frac{\mu_L}{\mu_H + \mu_L} J(r_H) + \frac{\mu_H}{\mu_H + \mu_L} J(r_L) - \frac{c}{\mu_H + \mu_L}, 0\Big\}, \tag{S.47}$$
$$J(r_L) = \max\{v - r_L, 0\}. \tag{S.48}$$

Next, we solve equations (S.47) and (S.48). Part (a) is immediate.

Suppose $r_L \leq v < r_L + \frac{c}{\mu_H}$. Then, we can remove $v - r_H$ in (S.47). We show $J(r_H) = 0$ by contradiction. Suppose

$$J(r_H) = \frac{\mu_L}{\mu_H + \mu_L} J(r_H) + \frac{\mu_H}{\mu_H + \mu_L}(v - r_L) - \frac{c}{\mu_H + \mu_L} \tag{S.49}$$
$$\geq 0. \tag{S.50}$$

By (S.49), one can obtain $J(r_H) = v - r_L - \frac{c}{\mu_H} < 0$, contradicting (S.50). Therefore, $J(r_H) = 0$. This gives the solution in Part (b).

Suppose $r_L + \frac{c}{\mu_H} \leq v < r_H$. Again, we can remove $v - r_H$ in (S.47). Suppose $J(r_H) = 0$. Then,

$$\frac{\mu_L}{\mu_H + \mu_L} J(r_H) + \frac{\mu_H}{\mu_H + \mu_L}(v - r_L) - \frac{c}{\mu_H + \mu_L} < 0,$$

which leads to $\frac{\mu_H}{\mu_H + \mu_L}\big(v - r_L - \frac{c}{\mu_H}\big) < 0$, contradicting our supposition $v \geq r_L + \frac{c}{\mu_H}$. Therefore,

$$J(r_H) = \frac{\mu_L}{\mu_H + \mu_L} J(r_H) + \frac{\mu_H}{\mu_H + \mu_L}(v - r_L) - \frac{c}{\mu_H + \mu_L},$$

yielding $J(r_H) = v - r_L - \frac{c}{\mu_H}$.

Suppose $v \geq r_H$. Then, we can remove 0 in (S.47). Suppose $J(r_H) = v - r_H$, which means

$$\begin{aligned} v - r_H &> \frac{\mu_L}{\mu_H + \mu_L} J(r_H) + \frac{\mu_H}{\mu_H + \mu_L}(v - r_L) - \frac{c}{\mu_H + \mu_L} \\ &= \frac{\mu_L}{\mu_H + \mu_L}(v - r_H) + \frac{\mu_H}{\mu_H + \mu_L}(v - r_L) - \frac{c}{\mu_H + \mu_L}. \end{aligned}$$

This gives rise to

$$\frac{\mu_H}{\mu_H + \mu_L}(v - r_H) > \frac{\mu_H}{\mu_H + \mu_L}(v - r_L - \frac{c}{\mu_H}),$$

contradicting our supposition $c \leq \mu_H(r_H - r_L)$. Therefore,

$$J(r_H) = \frac{\mu_L}{\mu_H + \mu_L} J(r_H) + \frac{\mu_H}{\mu_H + \mu_L}(v - r_L) - \frac{c}{\mu_H + \mu_L},$$

yielding $J(r_H) = v - r_L - \frac{c}{\mu_H}$. This completes the proof. ∎

Proof of Proposition S.10

According to customers behavior in Lemmas S.4 and E.4, it is never optimal to charge a low price different from V_L and $V_L - \frac{c}{\mu_H}$. Therefore, we have two cases regarding the value of r_L.

Case I: $r_L = V_L - \frac{c}{\mu_H}$

By Lemma S.4, a type I customer (both high- and low-valuation) purchases immediately at r_L but waits at r_H. By Lemma E.4, a low-valuation type II customer also chooses to purchase immediately at r_L but waits at r_H.

For a high-valuation type II customer, according to Lemma E.4(d), if

$$V_H \geq r_H + \frac{\mu_H(r_H - r_L) - c}{\lambda}, \tag{S.51}$$

then she purchases immediately at both prices. Otherwise, she purchases at r_L but waits at r_H. Inequality (S.51) can be rewritten as

$$r_H \leq \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H}. \tag{S.52}$$

Case I.A: $r_H \leq \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} \leq V_H$

In this case, customers' purchase decisions can be summarized in Figure S.21.

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.21 Customer Purchase Decisions in Case I.A.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^{I,A,*} = \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{I,A}(\mu_H, \mu_L)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
\Phi^{I,A}(\mu_H, \mu_L) =& \beta\Big[\frac{\mu_L}{\mu_H + \mu_L} + \frac{\mu_H}{\mu_H + \mu_L}\Big]\big(V_L - \frac{c}{\mu_H}\big) + (\alpha - \gamma)\Big[\frac{\mu_L}{\mu_H + \mu_L}\frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + \frac{\mu_H}{\mu_H + \mu_L}\big(V_L - \frac{c}{\mu_H}\big)\Big] \\
&+ (1 - \alpha - \beta + \gamma)\Big[\frac{\mu_L}{\mu_H + \mu_L}\frac{\mu_H}{\lambda + \mu_H} + \frac{\mu_H}{\mu_H + \mu_L}\Big]\big(V_L - \frac{c}{\mu_H}\big) - \frac{2m\mu_H\mu_L}{\mu_H + \mu_L} \\
=& V_L - \frac{c}{\mu_H} + \frac{\mu_L}{\mu_H + \mu_L}\Big\{\beta\big(V_L - \frac{c}{\mu_H}\big) + (\alpha - \gamma)\frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + (1 - \alpha - \beta + \gamma)\frac{\mu_H}{\lambda + \mu_H}\big(V_L - \frac{c}{\mu_H}\big)
\end{aligned}$$

$$- \left(V_L - \frac{c}{\mu_H}\right) - 2m\mu_H \Big\}.$$

The profit is no more than that of static pricing at V_L if the term in the square brackets is negative. Hereafter, we assume that the term in the brackets is positive.

To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi^{I,A,*} = \max_{\mu_H \geq 0, \mu_L \geq 0} \Phi^{I,A}(\mu_H, \mu_L).$$

Since the objective function is increasing in μ_L, $\mu_L^{I,A,*} = \infty$ at optimality. It follows that

$$\Phi^{I,A}(\mu_H, \infty) = \beta\left(V_L - \frac{c}{\mu_H}\right) + (\alpha - \gamma)\frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} + (1 - \alpha - \beta + \gamma)\frac{\mu_H}{\lambda + \mu_H}\left(V_L - \frac{c}{\mu_H}\right) - 2m\mu_H.$$

It can be verified that if $(\alpha - \gamma)V_H \leq (1 - \beta)V_L$, then $\Phi^{I,A}(\mu_H, \infty) < V_L$.

Case I.B: $\frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} \leq r_H \leq V_H$

In this case, customers' purchase decisions can be summarized in Figure S.22.

	Type I 0 (β)	Type II λ (1-β)
V_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
V_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.22 Customer Purchase Decisions in Case I.B.

Figure S.22 shows that customers never purchase at r_H, so the profit in this case cannot exceed $V_L - \frac{c}{\mu_H}$, which is smaller than static pricing at V_L.

Case II: $r_L = V_L$

By Lemma S.4(b)-(c), a low-valuation type I customer purchases at r_L but leaves immediately at r_H, whereas a high-valuation type I customer purchases at r_L and wait at r_H. By Lemma E.4, a low-valuation type II customer purchases at r_L but leaves immediately at r_H.

For a high-valuation type II customer, according to Lemma E.4(d), if

$$V_H \geq r_H + \frac{\mu_H(r_H - r_L) - c}{\lambda}, \tag{S.53}$$

then she purchases immediately at both prices. Otherwise, she purchases at r_L but waits at r_H. Inequality (S.53) can be rewritten as

$$r_H \leq \frac{\lambda V_H + \mu_H V_L + c}{\lambda + \mu_H}. \tag{S.54}$$

Case II.A: $r_H \leq \frac{\lambda V_H + \mu_H V_L + c}{\lambda + \mu_H} \leq V_H$

In this case, customers' purchase decisions can be summarized in Figure S.23.

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, leave at high price	Buy at low price, leave at high price

Figure S.23 Customer Purchase Decisions in Case II.A.

Because the firm's profit is linear in the prices, we must have the optimal high price $r_H^{II,A,*} = \frac{\lambda V_H + \mu_H V_L + c}{\lambda + \mu_H}$. The only remaining parameters are μ_H and μ_L. Let $\Phi^{II,A}(\mu_H, \mu_L)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
\Phi^{II,A}(\mu_H, \mu_L) =& \gamma\left[\frac{\mu_L}{\mu_H + \mu_L} + \frac{\mu_H}{\mu_H + \mu_L}\right]V_L + (\alpha - \gamma)\left[\frac{\mu_L}{\mu_H + \mu_L}\frac{\lambda V_H + \mu_H V_L + c}{\lambda + \mu_H} + \frac{\mu_H}{\mu_H + \mu_L}V_L\right] \\
&+ (1-\alpha)\frac{\mu_H}{\mu_H + \mu_L}V_L - \frac{2m\mu_H\mu_L}{\mu_H + \mu_L} \\
=& V_L + \frac{\mu_L}{\mu_H + \mu_L}\left\{\gamma V_L + (\alpha - \gamma)\frac{\lambda V_H + \mu_H V_L + c}{\lambda + \mu_H} - V_L - 2m\mu_H\right\}.
\end{aligned}$$

The profit is no more than that of static pricing at V_L if the term in the square brackets is negative. Hereafter, we assume that the term in the brackets is positive.

To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi^{II,A,*} = \max_{\mu_H \geq 0, \mu_L \geq 0} \Phi^{II,A}(\mu_H, \mu_L).$$

Since the objective function is increasing in μ_L, $\mu_L^{II,A,*} = \infty$ at optimality. It follows that

$$\Phi^{II,A}(\mu_H, \infty) = \gamma V_L + (\alpha - \gamma)\frac{\lambda V_H + \mu_H V_L + c}{\lambda + \mu_H} - V_L - 2m\mu_H < \alpha V_H,$$

where the above inequality holds because $c \leq \mu_H(r_H - r_L)$, $r_H \leq V_H$, and $r_L = V_L$.

	Type I 0 (β)	Type II λ ($1-\beta$)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L ($1-\alpha$)	Buy at low price, leave at high price	Buy at low price, leave at high price

Figure S.24 Customer Purchase Decisions in Case II.B.

Case II.B: $\frac{\lambda V_H+\mu_H V_L+c}{\lambda+\mu_H} \le r_H \le V_H$

In this case, customers' purchase decisions can be summarized in Figure S.24.

Figure S.24 shows that customers never purchase at r_H, so the profit in this case cannot exceed V_L, which is the revenue collected by employing static pricing at V_L.

To summarize, only Case I.A is possible to be optimal. This gives the solution in Proposition S.10(iii). This completes the proof. ■

Proof of Proposition S.11

It suffices to show under condition (S.7), if $\gamma \ge \alpha\beta$, then the profit function Φ in equation (S.8) is smaller than V_L or αV_H.

One can check

$$
\begin{aligned}
\Phi &= \beta V_L + \frac{(\alpha-\gamma)\lambda V_H + (1-\beta)\mu_H V_L}{\lambda+\mu_H} - \beta\frac{c}{\mu_H} - (1-\alpha-\beta+\gamma)\frac{c}{\lambda+\mu_H} - 2m\mu_H \\
&< \beta\frac{\alpha-\gamma}{1-\beta}V_H + \frac{(\alpha-\gamma)\lambda V_H + (\alpha-\gamma)\mu_H V_H}{\lambda+\mu_H} - \beta\frac{c}{\mu_H} - (1-\alpha-\beta+\gamma)\frac{c}{\lambda+\mu_H} - 2m\mu_H \\
&= \alpha V_H + \frac{\alpha\beta-\gamma}{1-\beta}V_H - \beta\frac{c}{\mu_H} - (1-\alpha-\beta+\gamma)\frac{c}{\lambda+\mu_H} - 2m\mu_H \\
&< \alpha V_H,
\end{aligned}
$$

where the first inequality holds because of condition (S.7) and the last inequality holds because $\gamma \ge \alpha\beta$. This completes the proof. ■

Proof of Proposition S.12

According to customers behavior in Lemmas S.4 and 3, it is never optimal to charge a low price different from V_L and $V_L - \frac{c}{\mu_H}$. Therefore, we have two cases regarding the value of r_L.

Case I: $r_L = V_L - \frac{c}{\mu_H}$

By Lemma S.4, a type I customer (both high- and low-valuation) purchases immediately at r_L but waits at r_H. By Lemma 3, a low-valuation type II customer also chooses to purchase immediately at r_L but waits at r_H.

For a high-valuation type II customer, according to Lemma 3(d), if

$$V_H \geq \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H} = \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - V_L)}{\lambda} + V_L, \tag{S.55}$$

then she purchases immediately at both prices. Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied/expired. If (S.55) is not satisfied, she would purchase at r_L but wait at r_H. Inequality (S.55) can be rewritten as

$$r_H \leq V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)(e^{-\mu_H T})}. \tag{S.56}$$

<u>Case I.A: $r_H \leq V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)(e^{-\mu_H T})}$ and $r_H \leq V_H$</u>

In this case, customers' purchase decisions can be summarized in Figure S.25.

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.25 Customer Purchase Decisions in Case I.A.

Comparing $V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)(e^{-\mu_H T})}$ with V_H yields that

$$V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)(e^{-\mu_H T})} \geq V_H$$

if and only if $\frac{\lambda}{\lambda+\mu_H} \geq e^{-\mu_H T}$. Therefore, we have two subcases with respect to the range of T.

<u>Subcase I.A.I:</u> $\frac{\lambda}{\lambda+\mu_H} \geq e^{-\mu_H T}$.

In this subcase, the optimal high price must be $r_H^* = V_H$. Let $\Phi^{I,A,I}(\mu_H, \mu_L, T)$ denote the firm's profit in this case. Then,

$$\begin{aligned}&\Phi^{I,A,I}(\mu_H, \mu_L, T)\\ =&\beta\Big[\frac{\mu_L}{\mu_H+\mu_L} + \frac{\mu_H}{\mu_H+\mu_L}\Big](V_L - \frac{c}{\mu_H}) + (\alpha-\gamma)\Big\{\frac{\mu_L}{\mu_H+\mu_L}\Big[(1-e^{-\mu_H T})(V_L - \frac{c}{\mu_H}) + e^{-\mu_H T}V_H\Big]\end{aligned}$$

$$+\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c}{\mu_H})\Big\}+(1-\alpha-\beta+\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big](V_L-\frac{c}{\mu_H})-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}$$

$$=V_L-\frac{c}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\beta(V_L-\frac{c}{\mu_H})+(\alpha-\gamma)\Big[(1-e^{-\mu_H T})(V_L-\frac{c}{\mu_H})+e^{-\mu_H T}V_H\Big]$$
$$+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})-(V_L-\frac{c}{\mu_H})-2m\mu_H\Big\}.$$

Note that this expression is decreasing in T, so $e^{-\mu_H T^*}=\frac{\lambda}{\lambda+\mu_H}$. The profit in this subcase is dominated by Subcase I.A.II below.

Subcase I.A.II: $\frac{\lambda}{\lambda+\mu_H}\leq e^{-\mu_H T}$.

In this subcase, the optimal high price must be $r_H^*=V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)(e^{-\mu_H T})}$. Let $\Phi^{I,A,II}(\mu_H,\mu_L,T)$ denote the firm's profit in this case. Then,

$$\Phi^{I,A,II}(\mu_H,\mu_L,T)$$
$$=\beta\Big[\frac{\mu_L}{\mu_H+\mu_L}+\frac{\mu_H}{\mu_H+\mu_L}\Big](V_L-\frac{c}{\mu_H})+(\alpha-\gamma)\Big\{\frac{\mu_L}{\mu_H+\mu_L}\Big[(1-e^{-\mu_H T})(V_L-\frac{c}{\mu_H})+e^{-\mu_H T}\Big(V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)(e^{-\mu_H T})}\Big)\Big]$$
$$+\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c}{\mu_H})\Big\}+(1-\alpha-\beta+\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big](V_L-\frac{c}{\mu_H})-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}$$
$$=V_L-\frac{c}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\beta(V_L-\frac{c}{\mu_H})+(\alpha-\gamma)\Big[\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-(1-e^{-\mu_H T})\frac{c}{\mu_H}\Big]$$
$$+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})-(V_L-\frac{c}{\mu_H})-2m\mu_H\Big\}.$$

The profit is no more than that of static pricing at V_L if the term in the square brackets is negative. Hereafter, we assume that the term in the brackets is positive. To determine the optimal μ_H, μ_L and T, we solve the following optimization problem:

$$\Phi^{I,A,II,*}=\max_{\mu_H\geq 0,\mu_L\geq 0,T\geq 0}\Phi^{I,A,II}(\mu_H,\mu_L,T).$$

Since the objective function is increasing in μ_L, $\mu_L^{I,A,II,*}=\infty$ at optimality. It follows that

$$\Phi^{I,A,II}(\mu_H,\infty,T)=\beta(V_L-\frac{c}{\mu_H})+(\alpha-\gamma)\Big[\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-(1-e^{-\mu_H T})\frac{c}{\mu_H}\Big]$$
$$+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})-2m\mu_H.$$

Note that the expression is decreasing in T, so $T^*=0$, which means that no price guarantees should be offered. The profit expression is simplified to

$$\Phi^{I,A,II}(\mu_H,\infty,0)=\beta(V_L-\frac{c}{\mu_H})+(\alpha-\gamma)\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c}{\mu_H})-2m\mu_H,$$

which is exactly the same as that in Proposition S.10(iii).

Case I.B: $V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)(e^{-\mu_H T})}<r_H\leq V_H$

In this case, customers' purchase decisions can be summarized in Figure S.26.

Figure S.26 shows that customers never purchase at r_H, so the profit in this case cannot exceed $V_L-\frac{c}{\mu_H}$, which is smaller than the revenue collected by employing static pricing at V_L.

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, wait at high price

Figure S.26 Customer Purchase Decisions in Case I.B.

Case II: $r_L = V_L$

By Lemma S.4(b)-(c), a low-valuation type I customer purchases at r_L but leaves immediately at r_H, while a high-valuation type II customer purchases at r_L but wait at r_H. By Lemma 3(b), a low-valuation type II customer purchases at r_L but leaves immediately at r_H.

For a high-valuation type II customer, according to Lemma 3(d), if

$$V_H \geq \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c}{\mu_H})}{\lambda} + r_L + \frac{c}{\mu_H} = \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - V_L - \frac{c}{\mu_H})}{\lambda} + V_L + \frac{c}{\mu_H}, \tag{S.57}$$

then she purchases immediately at both prices. Moreover, if the purchase is made at the price r_H, they would keep monitoring the price until the price guarantee is applied/expired. If (S.57) is not satisfied, she would purchase at r_L but wait at r_H.

Inequality (S.57) can be rewritten as

$$r_H \leq V_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - V_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)(e^{-\mu_H T})}. \tag{S.58}$$

<u>Case II.A:</u> $r_H \leq V_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - V_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)(e^{-\mu_H T})}$ and $r_H \leq V_H$.

In this case, customers' purchase decisions can be summarized in Figure S.27.

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, monitor at high price
v_L (1-α)	Buy at low price, leave at high price	Buy at low price, leave at high price

Figure S.27 Customer Purchase Decisions in Case II.A.

Observe that all low-valuation customers purchase at r_L but leave immediately at r_H. Given that r_L is rarely offered in the Markovian pricing, low-valuation customers do not contribute any revenue. Similar to Case II.A in the proof of Proposition S.10, one can verify that the optimal profit cannot exceed αV_H.

<u>Case II.B:</u> $r_H \geq V_L + \frac{c}{\mu_H} + \frac{\lambda(V_H - V_L - \frac{c}{\mu_H})}{(\lambda+\mu_H)(e^{-\mu_H T})}$ and $r_H \leq V_H$.

	Type I 0 (β)	Type II λ (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at low price, wait at high price
v_L (1-α)	Buy at low price, leave at high price	Buy at low price, leave at high price

Figure S.28 Customer Purchase Decisions in Case II.B.

In this case, customers' purchase decisions can be summarized in Figure S.28.

Figure S.28 shows that customers never purchase at r_H, so the profit in this case cannot exceed V_L, which is the revenue collected by employing static pricing at V_L.

To summarize, only Case I.A.II is possible to be optimal. The profit function in Case I.A.II is the same as that in Proposition S.10(iii), implying that offering price guarantees cannot improve the firm's revenue. This completes the proof. ■

S.4.3 Proofs of Lemmas and Propositions in Section S.3

Proof of Proposition S.13

Due to the high monitoring cost, type II customers behave myopically as shown in Lemma 2, while type I customers follow the strategy in Lemma E.4. According to Lemmas 2 and E.4, it is never optimal to charge a low price r_L different from $V_L - \frac{c_1}{\mu_H}$ and V_L. Therefore, we have two cases regarding the value of r_L.

Case I: $r_L = V_L - \frac{c_1}{\mu_H}$

By Lemma E.4(c), a low-valuation type I customer purchases immediately at r_L and waits for a sale at r_H. By Lemma 2(b), a low-valuation type II customer purchases at r_L but leaves immediately at r_H. By Lemma E.4(d), a high-valuation type I customer purchases at both prices immediately if

$$V_H \geq r_H + \frac{\mu_H(r_H - r_L) - c_1}{\lambda} = r_H + \frac{\mu_H(r_H - V_L)}{\lambda}. \tag{S.59}$$

If this condition is not satisfied, then she purchases at r_L immediately and waits for a sale otherwise. Inequality (S.59) can be rewritten as

$$r_H \leq \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H}.$$

By Lemma 2(c), a high-valuation type II customer purchases immediately at both prices if $V_H \geq r_H$. Otherwise, she purchases at r_L but leaves immediately at r_H.

Subcase I.A: $\frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} < r_H \leq V_H$

In this case, the purchase decisions of customers can be summarized in Figure S.29.

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.29 Customer Purchase Decisions in Subcase I.A.

Because the firm's profit is linear in prices, we must have the optimal high price $r_H^{I,A,*} = V_H$. Let $\Phi^{I,A}(\mu_H, \mu_L)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}\Phi^{I,A}(\mu_H,\mu_L) =& \beta\Big[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\mu_H}{\lambda+\mu_H} + \frac{\mu_H}{\mu_H+\mu_L}\Big](V_L - \frac{c_1}{\mu_H}) + (\alpha-\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}V_H + \frac{\mu_H}{\mu_H+\mu_L}(V_L - \frac{c_1}{\mu_H})\Big] \\ &+ (1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}(V_L - \frac{c_1}{\mu_H}) - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L} \\ =& V_L - \frac{c_1}{\mu_H} + \frac{\mu_L}{\mu_H+\mu_L}\Big\{(\alpha-\gamma)V_H - \frac{\lambda+(1-\beta)\mu_H}{\lambda+\mu_H}(V_L - \frac{c_1}{\mu_H}) - 2m\mu_H\Big\}.\end{aligned}$$

The profit is no more than that of static pricing at V_L if the term in the brackets is negative. Hereafter, we assume that the term in the brackets is positive. To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi^{I,A,*} = \max_{\mu_H \geq 0, \mu_L \geq 0} \Phi^{I,A}(\mu_H, \mu_L).$$

Since the objective function is increasing in μ_L, $\mu_L^* = \infty$ at optimality. It follows that

$$\Phi^{I,A}(\mu_H, \infty) = \frac{\beta\mu_H}{\lambda + \mu_H}(V_L - \frac{c_1}{\mu_H}) + (\alpha - \gamma)V_H - 2m\mu_H.$$

It can be verified that $\Phi^{I,A}(\mu_H, \infty)$ is concave in μ_H. Therefore,

$$\mu_H^{I,A,*} = \begin{cases} \sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda, & \text{if } \beta(\lambda V_L + c_1) - 2m\lambda^2 > 0, \\ 0, & \text{otherwise.} \end{cases}$$

The corresponding prices are

$$r_H^{I,*} = V_H, \quad r_L^{I,*} = V_L - \frac{c}{\mu_H^{I,A,*}},$$

and the firm's profit is

$$\Phi^{I,A,*} = \begin{cases} (\alpha - \gamma)V_H + \beta\left(V_L - \sqrt{\frac{2m(\lambda V_L + c_1)}{\beta}}\right) - 2m\left(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda\right), & \text{if } \beta(\lambda V_L + c_1) - 2m\lambda^2 > 0, \\ (\alpha - \gamma)V_H, & \text{otherwise.} \end{cases}$$

Putting μ_H^*, r_H^*, and r_L^* into the condition $c_2 > \mu_H(r_H - r_L)$ and $c_1 \leq \mu_H(r_H - r_L)$ yields the constraint

$$c_2 > \left(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda\right)(V_H - V_L) + c_1,$$
$$c_1 \leq \left(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda\right)(V_H - V_L) + c_1,$$

where the second one holds automatically.

Subcase I.B: $r_H \leq \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H} \leq V_H$

In this case, the purchase decisions of customers can be summarized in Figure S.30.

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	Buy at both prices	Buy at both prices
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.30 Customer Purchase Decisions in Subcase I.B.

It is immediate to see $r_H^* = \frac{\lambda V_H + \mu_H V_L}{\lambda + \mu_H}$. Let $\Phi^{I,B}(\mu_H, \mu_L)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
\Phi^{I,B}(\mu_H,\mu_L) =& \alpha\Big[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H} + \frac{\mu_H}{\mu_H+\mu_L}(V_L - \frac{c_1}{\mu_H})\Big] + (\beta-\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H} + \frac{\mu_H}{\mu_H+\mu_L}\Big](V_L - \frac{c_1}{\mu_H}) \\
&+ (1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}(V_L - \frac{c_1}{\mu_H}) - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L} \\
=& V_L - \frac{c_1}{\mu_H} + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\frac{\alpha(\lambda V_H+\mu_H V_L)+(\beta-\gamma)(\mu_H V_L - c_1)}{\lambda+\mu_H} - (V_L - \frac{c_1}{\mu_H}) - 2m\mu_H\Big\}.
\end{aligned}$$

The profit is no more than that of static pricing at V_L if the term in the brackets is negative. Hereafter, we assume that the term in the brackets is positive. To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi^{I,B,*} = \max_{\mu_H\ge 0,\mu_L\ge 0} \Phi^{I,B}(\mu_H,\mu_L).$$

Since the objective function is increasing in μ_L, $\mu_L^* = \infty$ at optimality. It follows that

$$\begin{aligned}
\Phi^{I,B}(\mu_H,\infty) =& \frac{\alpha(\lambda V_H+\mu_H V_L)+(\beta-\gamma)(\mu_H V_L - c_1)}{\lambda+\mu_H} - 2m\mu_H \\
=& (\alpha+\beta-\gamma)V_L + \frac{\lambda(\alpha V_H - (\alpha+\beta-\gamma)V_L)}{\lambda+\mu_H} - \frac{(\beta-\gamma)c_1}{\lambda+\mu_H} - 2m\mu_H.
\end{aligned}$$

If $\alpha V_H - (\alpha+\beta-\gamma)V_L \le 0$, then $\Phi^{I,B}(\mu_H,\infty) < (\alpha+\beta-\gamma)V_L < V_L$. Otherwise,

$$\begin{aligned}
\Phi^{I,B}(\mu_H,\infty) <& (\alpha+\beta-\gamma)V_L + \frac{\lambda(\alpha V_H - (\alpha+\beta-\gamma)V_L)}{\lambda} - \frac{(\beta-\gamma)c_1}{\lambda+\mu_H} - 2m\mu_H \\
=& \alpha V_H - \frac{(\beta-\gamma)c_1}{\lambda+\mu_H} - 2m\mu_H \\
<& \alpha V_H.
\end{aligned}$$

Hence, the optimal revenue in this subcase is dominated by that under static pricing at either V_L or V_H.

Case II: $r_L = V_L$

By Lemmas 2(b) and E.4(b), a low-valuation customer (both type I and II) purchases immediately at r_L but leaves immediately at r_H. By Lemma E.4(d), a high-valuation type I customer purchases at both prices immediately if

$$V_H \ge r_H + \frac{\mu_H(r_H - r_L) - c_1}{\lambda} = r_H + \frac{\mu_H(r_H - V_L) - c_1}{\lambda}. \tag{S.60}$$

If this condition is not satisfied, then she purchases at r_L immediately and waits for a sale otherwise. Inequality (S.60) can be rewritten as

$$r_H \le \frac{\lambda V_H + \mu_H V_L - c_1}{\lambda + \mu_H}.$$

By Lemma 2(c), a high-valuation type II customer purchases immediately at both prices if $V_H \ge r_H$. Otherwise, she purchases at r_L but leaves immediately at r_H.

Subcase II.A: $\frac{\lambda V_H+\mu_H V_L-c_1}{\lambda+\mu_H} < r_H \leq V_H$

In this case, the purchase decisions of customers can be summarized in Figure S.31.

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices
v_L (1-α)	Buy at low price, leave at high price	Buy at low price, leave at high price

Figure S.31 Customer Purchase Decisions in Subcase II.A.

Because the firm's profit is linear in prices, we must have the optimal high price $r_H^{II,A,*} = V_H$. Let $\Phi^{II,A}(\mu_H,\mu_L)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}\Phi^{II,A}(\mu_H,\mu_L) =& \gamma\Big[\frac{\mu_L}{\mu_H+\mu_L}\cdot\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big]V_L+(\alpha-\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}V_H+\frac{\mu_H}{\mu_H+\mu_L}V_L\Big]\\ &+(1-\alpha)\frac{\mu_H}{\mu_H+\mu_L}V_L-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\ =& V_L+\frac{\mu_L}{\mu_H+\mu_L}\Big\{(\alpha-\gamma)V_H-\frac{\lambda+(1-\gamma)\mu_H}{\lambda+\mu_H}V_L-2m\mu_H\Big\}.\end{aligned}$$

The profit is no more than that of static pricing at V_L if the term in the brackets is negative. Hereafter, we assume that the term in the brackets is positive. To determine the optimal μ_H and μ_L, we solve the following optimization problem:

$$\Phi^{II,A,*} = \max_{\mu_H\geq 0,\mu_L\geq 0}\Phi^{II,A}(\mu_H,\mu_L).$$

Since the objective function is increasing in μ_L, $\mu_L^*=\infty$ at optimality. It follows that

$$\begin{aligned}\Phi^{II,A}(\mu_H,\infty) &= V_L+(\alpha-\gamma)V_H-\frac{\lambda+(1-\gamma)\mu_H}{\lambda+\mu_H}V_L-2m\mu_H\\ &=(\alpha-\gamma)V_H+\gamma\frac{\mu_H}{\lambda+\mu_H}V_L-2m\mu_H\\ &=\alpha V_H-\gamma(1-\frac{\mu_H}{\lambda+\mu_H})V_L-2m\mu_H\\ &\leq \alpha V_H.\end{aligned}$$

Hence, the optimal revenue in this subcase is dominated by that under static pricing at V_H.

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	Buy at both prices	Buy at both prices
v_L (1-α)	Buy at low price, leave at high price	Buy at low price, leave at high price

Figure S.32 Customer Purchase Decisions in Subcase II.B.

Subcase II.B: $r_H \leq \frac{\lambda V_H + \mu_H V_L - c_1}{\lambda + \mu_H} \leq V_H$

In this case, the purchase decisions of customers can be summarized in Figure S.32.

Observe that both types of low-valuation customers only purchase at r_L and leave immediately at r_H. Since r_L is offered occasionally, the low-valuation customers do not contribute to the firm's revenue. Therefore, the optimal revenue in this subcase is dominated by that under static pricing at V_H.

To summarize, only Subcase I.A is possible to be optimal given that all other cases are dominated by static pricing. The solution in Subcase I.A gives the high/low pricing in Proposition S.13(iii). This completes the proof. ■

Before proving Proposition S.14, we first establish the following lemma.

Lemma S.7. *Suppose $c_1 \leq \mu_H(r_H - r_L)$. In the presence of price guarantees, if $r_L = V_L - \frac{c_1}{\mu_H}$ and $r_H > V_H \geq \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - V_L)}{\lambda} + V_L$, then the high/low pricing strategy is not optimal.*

Proof of Lemma S.7

Since $r_L = V_L - \frac{c}{\mu_H}$, by Lemma 3(c), a low-valuation type I customer purchases immediately at r_L but wait for a sale at r_H. By Lemma 2(b), a low-valuation type II customer purchases at r_L but leaves immediately at r_H.

When $V_H \geq \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - V_L)}{\lambda} + V_L$, by Lemma 3(d), a high-valuation type I customer purchases immediately at both prices. Since $V_H \leq r_H$, a high-valuation type II customer purchases at price r_L but leaves without a purchase at price r_H. The decision of each segment of customers can be summarized in Figure S.33.

Note that $V_H \geq \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - V_L)}{\lambda} + V_L$ is equivalent to $r_H \leq V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$, so the optimal high price is $r_H^* = V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$. Let $\Phi(\mu_H, \mu_L, T)$ denote the firm's revenue per unit time in this case. We have

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	Buy at both prices, monitor for price guarantee	Buy at low price, leave at high price
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.33 Customer Purchase Decisions in this case.

$$
\begin{aligned}
\Phi(\mu_H,\mu_L,T) =& (1-\beta)\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c_1}{\mu_H})+\gamma\Big\{\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c_1}{\mu_H})+\frac{\mu_L}{\mu_H+\mu_L}\Big[(1-e^{-\mu_H T})(V_L-\frac{c_1}{\mu_H})+e^{-\mu_H T}r_H^*\Big]\Big\}\\
&+(\beta-\gamma)\Big[\frac{\mu_H}{\mu_H+\mu_L}+\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}\Big](V_L-\frac{c_1}{\mu_H})-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=& V_L-\frac{c_1}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-\gamma(1-e^{-\mu_H T})\frac{c_1}{\mu_H}\\
&+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c_1}{\mu_H})-(V_L-\frac{c_1}{\mu_H})-2m\mu_H\Big\}.
\end{aligned}
$$

If the term in the brackets is negative, then $\Phi(\mu_H,\mu_L,T)<V_L$. Suppose the term is positive. Note that $\Phi(\mu_H,\mu_L,T)$ is increasing in μ_L, so $\mu_L^*=\infty$. It follows that

$$
\begin{aligned}
\Phi(\mu_H,\infty,T) =& \gamma\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-\gamma(1-e^{-\mu_H T})\frac{c_1}{\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c_1}{\mu_H})-2m\mu_H\\
=& \beta V_L+\frac{\gamma\lambda V_H-\beta\lambda V_L}{\lambda+\mu_H}-(\beta-\gamma)\frac{c_1}{\lambda+\mu_H}-\gamma(1-e^{-\mu_H T})\frac{c_1}{\mu_H}-2m\mu_H.
\end{aligned}
$$

If $\gamma\lambda V_H-\beta\lambda V_L<0$, then $\Phi(\mu_H,\infty,T)<V_L$. Otherwise,

$$
\begin{aligned}
\Phi(\mu_H,\infty,T) <& \beta V_L+\frac{\gamma\lambda V_H-\beta\lambda V_L}{\lambda}-(\beta-\gamma)\frac{c_1}{\lambda+\mu_H}-\gamma(1-e^{-\mu_H T})\frac{c_1}{\mu_H}-2m\mu_H\\
=& \gamma V_H-(\beta-\gamma)\frac{c_1}{\lambda+\mu_H}-\gamma(1-e^{-\mu_H T})\frac{c_1}{\mu_H}-2m\mu_H\\
<& \alpha V_H.
\end{aligned}
$$

This completes the proof. ■

Proof of Proposition S.14

Due to the high monitoring cost, type II customers behave myopically as shown in Lemma 2, while type I customers follow the strategy in Lemma 3. According to Lemmas 2 and 3, it is never optimal to charge a low price r_L different from $V_L-\frac{c_1}{\mu_H}$ and V_L. Therefore, we have two cases regarding the value of r_L. However, one can see that if $r_L=V_L$, then both types of low-valuation customers would leave immediately without purchase when the price is high. Since the low price is offered occasionally, all low-valuation customers do not contribute to the firm's revenue. Therefore, the revenue in this case cannot exceed αV_H given that only high-valuation customers make a purchase. As a result, the case with $r_L = V_L$ can never be optimal. Hereafter, we restrict our attention to the case with $r_L = V_L - \frac{c_1}{\mu_H}$.

By Lemma 3(c), a low-valuation type I customer purchases immediately at r_L and waits for a sale at r_H. By Lemma 2(b), a low-valuation type II customer purchases at r_L but leaves immediately at r_H. By Lemma 3(d), a high-valuation type I customer purchases at both prices immediately if

$$V_H \geq \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - r_L - \frac{c_1}{\mu_H})}{\lambda} + r_L + \frac{c_1}{\mu_H} = \frac{(\lambda+\mu_H)e^{-\mu_H T}(r_H - V_L)}{\lambda} + V_L. \qquad \text{(S.61)}$$

Moreover, if the purchase is made at a high price r_H, then she would keep monitoring the price until the price guarantee is applied or expires. If this condition is not satisfied, then she purchases at r_L immediately and waits for a sale otherwise. Inequality (S.61) can be rewritten as

$$r_H \leq V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}.$$

For high-valuation type II customers, there are two possibilities: $V_H < r_H$ and $V_H \geq r_H$. If $V_H < r_H$, they purchase at a price r_L but leave without a purchase when the price is r_H. Meanwhile, if high-valuation type I customers purchase at both prices, then customers' decision is the same as in Figure S.33; therefore, by Lemma S.7, the high/low pricing is never optimal. If high-valuation type I customers purchase at r_L but wait for a sale at r_H, then no customers of the four segments purchase at the high price r_H; therefore, the high/low pricing strategy is also not optimal. To summarize, when $V_H < r_H$, the high/low pricing strategy is not optimal. Hereafter, we restrict our attention to the possibility with $V_H \geq r_H$, in which case high-valuation type II customers purchase at both prices but do not monitor for price refund.

Case I: $V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}} < r_H \leq V_H$

In this case, the purchase decisions of customers can be summarized in Figure S.34.

The analysis and result are the same as in the case without price guarantees (Subcase I.A in the proof of Proposition S.13). Hence,

$$\Phi^{I,*} = \begin{cases} (\alpha-\gamma)V_H + \beta\left(V_L - \sqrt{\frac{2m(\lambda V_L + c_1)}{\beta}}\right) - 2m\left(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda\right), & \text{if } \beta(\lambda V_L + c_1) - 2m\lambda^2 > 0, \\ (\alpha-\gamma)V_H, & \text{otherwise.} \end{cases}$$

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	Buy at low price, wait at high price	Buy at both prices, do not monitor for price guarantee
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.34 Customer Purchase Decisions in Case I.

	Type I c_1 (β)	Type II c_2 (1-β)
v_H (α)	Buy at both prices, monitor for price guarantee	Buy at both prices, do not monitor for price guarantee
v_L (1-α)	Buy at low price, wait at high price	Buy at low price, leave at high price

Figure S.35 Customer Purchase Decisions in Case II.

Case II: $r_H \le V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$ **and** $r_H \le V_H$

In this case, the purchase decisions of customers can be summarized in Figure S.35.

Comparing $V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$ with V_H yields that

$$V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}} \ge V_H \quad \Leftrightarrow \quad \frac{\lambda}{\lambda+\mu_H} \ge e^{-\mu_H T}.$$

Subcase II.A: $\frac{\lambda}{\lambda+\mu_H} \ge e^{-\mu_H T}$ In this case, $r_H^* = V_H$. Let $\Phi^{II,A}(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
\Phi^{II,A}(\mu_H, \mu_L, T) =& \gamma\Big\{\frac{\mu_L}{\mu_H+\mu_L}\Big[(1-e^{-\mu_H T})(V_L - \frac{c_1}{\mu_H}) + e^{-\mu_H T}V_H\Big] + \frac{\mu_H}{\mu_H+\mu_L}(V_L - \frac{c_1}{\mu_H})\Big\} \\
&+ (\alpha-\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}V_H + \frac{\mu_H}{\mu_H+\mu_L}(V_L - \frac{c_1}{\mu_H})\Big] \\
&+ (\beta-\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H} + \frac{\mu_H}{\mu_H+\mu_L}\Big](V_L - \frac{c_1}{\mu_H}) \\
&+ (1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}(V_L - \frac{c_1}{\mu_H}) - \frac{2m\mu_H\mu_L}{\mu_H+\mu_L} \\
=& V_L - \frac{c_1}{\mu_H} + \frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\Big[V_L - \frac{c_1}{\mu_H} + e^{-\mu_H T}(V_H - V_L + \frac{c_1}{\mu_H})\Big] \\
&\qquad + (\alpha-\gamma)V_H + (\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L - \frac{c_1}{\mu_H}) - (V_L - \frac{c_1}{\mu_H}) - 2m\mu_H\Big\}.
\end{aligned}$$

Note that this expression is decreasing in T, so $e^{-\mu_H T^*} = \frac{\lambda}{\lambda+\mu_H}$. The profit in this subcase is dominated by Subcase II.B below.

Subcase II.B: $\frac{\lambda}{\lambda+\mu_H} \le e^{-\mu_H T}$ In this case, $r_H^* = V_L + \frac{\lambda(V_H - V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}$. Let $\Phi^{II,B}(\mu_H, \mu_L, T)$ denote the firm's profit per unit time in this case. Then,

$$\begin{aligned}
\Phi^{II,B}(\mu_H,\mu_L,T) =&\gamma\Big\{\frac{\mu_L}{\mu_H+\mu_L}\Big[(1-e^{-\mu_H T})(V_L-\frac{c_1}{\mu_H})+e^{-\mu_H T}\Big(V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}\Big)\Big]+\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c_1}{\mu_H})\Big\}\\
&+(\alpha-\gamma)\Big\{\frac{\mu_L}{\mu_H+\mu_L}\Big[V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}\Big]+\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c_1}{\mu_H})\Big\}\\
&+(\beta-\gamma)\Big[\frac{\mu_L}{\mu_H+\mu_L}\frac{\mu_H}{\lambda+\mu_H}+\frac{\mu_H}{\mu_H+\mu_L}\Big](V_L-\frac{c_1}{\mu_H})\\
&+(1-\alpha-\beta+\gamma)\frac{\mu_H}{\mu_H+\mu_L}(V_L-\frac{c_1}{\mu_H})-\frac{2m\mu_H\mu_L}{\mu_H+\mu_L}\\
=&V_L-\frac{c_1}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\Big[\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}-(1-e^{-\mu_H T})\frac{c_1}{\mu_H}\Big]\\
&+(\alpha-\gamma)\Big[V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}\Big]+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c_1}{\mu_H})-(V_L-\frac{c_1}{\mu_H})-2m\mu_H\Big\}.
\end{aligned}$$

Taking derivatives with respect to T yields

$$\begin{aligned}
\frac{d\Phi^{II,B}(\mu_H,\mu_L,T)}{dT} &= -\gamma c_1 e^{-\mu_H T}+(\alpha-\gamma)\frac{\lambda(V_H-V-L)}{\lambda+\mu_H}e^{\mu_H T}\mu_H,\\
\frac{d^2\Phi^{II,B}(\mu_H,\mu_L,T)}{dT^2} &= \gamma c_1 e^{-\mu_H T}\mu_H+(\alpha-\gamma)\frac{\lambda(V_H-V-L)}{\lambda+\mu_H}e^{\mu_H T}\mu_H^2>0.
\end{aligned}$$

Observe that $\Phi^{II,B}(\mu_H, \mu_L, T)$ is convex in T, so it suffices to compare the profits when $e^{-\mu_H T} = 1$ and $e^{-\mu_H T} = \frac{\lambda}{\lambda+\mu_H}$, respectively. One can check that when $e^{-\mu_H T} = 1$,

$$\begin{aligned}
\Phi^{II,B}(\mu_H,\mu_L,T) =&V_L-\frac{c_1}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\alpha\frac{\lambda V_H+\mu_H V_L}{\lambda+\mu_H}+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c_1}{\mu_H})\\
&-(V_L-\frac{c_1}{\mu_H})-2m\mu_H\Big\},
\end{aligned}$$

which is the same as $\Phi^{I,B}(\mu_H, \mu_L)$ in Subcase I.B in the proof of Proposition S.13. Therefore, it is dominated by static pricing.

When $e^{-\mu_H T} = \frac{\lambda}{\lambda+\mu_H}$,

$$\begin{aligned}
\Phi^{II,B}(\mu_H,\mu_L,T) =&V_L-\frac{c_1}{\mu_H}+\frac{\mu_L}{\mu_H+\mu_L}\Big\{\gamma\frac{\lambda V_H+\mu_H V_L-c_1}{\lambda+\mu_H}+(\alpha-\gamma)V_H\\
&+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c_1}{\mu_H})-(V_L-\frac{c_1}{\mu_H})-2m\mu_H\Big\}.
\end{aligned}$$

Since the objective function is increasing in μ_L, $\mu_L^* = \infty$ at optimality. It follows that

$$\Phi^{II,B}(\mu_H,\infty,T^*) = \gamma\frac{\lambda V_H+\mu_H V_L-c_1}{\lambda+\mu_H}+(\alpha-\gamma)V_H+(\beta-\gamma)\frac{\mu_H}{\lambda+\mu_H}(V_L-\frac{c_1}{\mu_H})-2m\mu_H.$$

It can be shown that when $\lambda K+\beta c_1 \le 0$, $\Phi^{II,B}(\mu_H,\infty,T^*)$ decreases in μ_H. Hence, $\mu_H^*=0$. The corresponding profit is

$$\Phi^{II,B}(\mu_H,\infty,T^*)=\alpha V_H-\gamma\frac{c_1}{\lambda}<\alpha V_H.$$

When $\lambda K+\beta c_1>0$, we solve for μ_H using the first-order condition, which gives

$$\mu_H^{II,B,*}=\begin{cases}\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda, & \text{if } \lambda K+\beta c_1-2m\lambda^2>0,\\ 0, & \text{otherwise.}\end{cases}$$

Moreover, when $\lambda K+\beta c_1>2m\lambda^2$, by $e^{-\mu_H T}=\frac{\lambda}{\lambda+\mu_H}$, we obtain $T^*=\frac{1}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda}\ln\sqrt{\frac{K+\beta c_1/\lambda}{2m\lambda}}$, and thus

$$r_H^*=V_L+\frac{\lambda(V_H-V_L)}{(\lambda+\mu_H)e^{-\mu_H T}}=V_H.$$

Putting μ_H^*, r_H^*, and r_L^* into the condition $c_2>\mu_H(r_H-r_L)$ and $c_1\le\mu_H(r_H-r_L)$ yields the constraint

$$c_2>\Big(\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda\Big)(V_H-V_L)+c_1,$$
$$c_1\le\Big(\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda\Big)(V_H-V_L)+c_1,$$

respectively, where the second one holds automatically.

Summarizing the results for Case I and Case II yields the following:

- If $\lambda K+\beta c_1>2m\lambda^2$, then

$$\Phi^{I,*}=(\alpha-\gamma)V_H+\beta\left(V_L-\sqrt{\frac{2m(\lambda V_L+c_1)}{\beta}}\right)-2m\left(\sqrt{\frac{\beta(\lambda V_L+c_1)}{2m}}-\lambda\right),$$

$$\Phi^{II,*}=\Phi^{II,B,*}=(\alpha-\gamma)V_H+(\beta-\gamma)\left(1-\frac{\lambda}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}}\right)\left(V_L-\frac{c_1}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda}\right)$$
$$+\gamma\left[\left(1-\frac{\lambda}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}}\right)\left(V_L-\frac{c_1}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda}\right)+\frac{\lambda}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}}V_H\right]-2m\Big(\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda\Big).$$

One can check that $\Phi^{II,*}\ge\Phi^{I,*}$, so

$$\Phi^{*}=\Phi^{II,*}=(\alpha-\gamma)V_H+(\beta-\gamma)\left(1-\frac{\lambda}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}}\right)\left(V_L-\frac{c_1}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda}\right)$$
$$+\gamma\left[\left(1-\frac{\lambda}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}}\right)\left(V_L-\frac{c_1}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda}\right)+\frac{\lambda}{\sqrt{\frac{\lambda K+\beta c_1}{2m}}}V_H\right]-2m\Big(\sqrt{\frac{\lambda K+\beta c_1}{2m}}-\lambda\Big).$$

- If $\lambda K + \beta c_1 \leq 2m\lambda^2 < \lambda\beta V_L + \beta c_1$, then

$$\Phi^{I,*} = (\alpha - \gamma)V_H + \beta \left(V_L - \sqrt{\frac{2m(\lambda V_L + c_1)}{\beta}} \right) - 2m \left(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda \right),$$
$$\Phi^{II,*} = \Phi^{II,B,*} = \alpha V_H - \gamma \frac{c_1}{\lambda}.$$

Hence,

$$\Phi^* = \max \left\{ (\alpha - \gamma)V_H + \beta \left(V_L - \sqrt{\frac{2m(\lambda V_L + c_1)}{\beta}} \right) - 2m \left(\sqrt{\frac{\beta(\lambda V_L + c_1)}{2m}} - \lambda \right), \alpha V_H - \gamma \frac{c_1}{\lambda} \right\}.$$

In this case, the optimal high/low pricing policy reduces to that without price guarantees.

- If $\lambda\beta V_L + \beta c_1 \leq 2m\lambda^2$, then $\Phi^{I,*} = (\alpha - \gamma)V_H$, and $\Phi^{II,*} = \alpha V_H - \gamma \frac{c_1}{\lambda}$. Hence, $\Phi^* = \max\{(\alpha - \gamma)V_H, \alpha V_H - \gamma \frac{c_1}{\lambda}\}$. In this case, the optimal high/low pricing policy is dominated by static pricing.

Comparing V_L, αV_H, and Φ^* when $\lambda K + \beta c_1 \leq 2m\lambda^2$ shows that the pricing strategy is the same as that in Proposition S.13. Comparing the profits when $\lambda K + \beta c_1 > 2m\lambda^2$ yields the results in Parts (i)-(iii). This completes the proof. ■