MANAGEMENT SCIENCE
Vol. 59, No. 1, January 2013, pp. 84–101
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

http://dx.doi.org/10.1287/mnsc.1120.1564

Dynamic Pricing Competition with Strategic Customers Under Vertical Product Differentiation

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We consider dynamic pricing competition between two firms offering vertically differentiated products to strategic customers who are intertemporal utility maximizers. We show that price skimming arises as the unique pure-strategy Markov perfect equilibrium in the game under a simple condition. Our results highlight the asymmetric effect of strategic customer behavior on quality-differentiated firms. Even though the profit of either firm decreases as customers become more strategic, the low-quality firm suffers substantially more than the high-quality firm. Furthermore, we show that unilateral commitment to static pricing by either firm generally improves profits of both firms. Interestingly, both firms enjoy higher profit lifts when the high-quality firm commits rather than when the low-quality firm commits.

Key words: dynamic pricing; pricing competition; strategic customers; vertical differentiation
History: Received August 20, 2010; accepted March 13, 2012, by Yossi Aviv, operations management. Published online in *Articles in Advance* September 11, 2012.

1. Introduction

Product differentiation and pricing are among the most important marketing strategies for firms. Product differentiation is an effective way for firms to seek profitable niches in a competitive market and is prevalent for most consumer goods, including electronics, food products, and apparels. On the other hand, firms in many industries often systematically vary prices over time in order to better manage demand and increase profits. A classical explanation for this behavior is intertemporal price discrimination; high-valuation customers purchase at higher prices early, whereas low-valuation customers wait and purchase at lower prices later. The seminal work of Besanko and Winston (1990) establishes price skimming as a subgame perfect equilibrium in a market with a monopolistic seller and strategic customers who are intertemporal utility maximizers.

Product differentiation and pricing strategies are inextricably intertwined in a competitive market. Faced with differentiated products and dynamic pricing, strategic customers decide not only which products to purchase but also when to purchase them. Consequently, when making pricing decisions, a firm needs to take into consideration both the intratemporal demand competition and intertemporal demand substitution. It is therefore desirable to endogenize both customer choice behavior and purchase-timing behavior, capturing their joint effects on competing firms' pricing strategies. This is exactly the focal point of our work.

Particularly, this paper attempts to unravel the impact of strategic customer behavior on the pricing strategies of two competing firms offering vertically differentiated products. To this end, we adopt the classical vertical product differentiation model (Tirole 1988), which is appropriate when customers have unanimous assessment on the relative attractiveness of different products. Examples include national versus store brands, full service airlines versus low-cost carriers, executive suites versus regular hotel rooms, etc. Customers have private and heterogeneous valuations on product quality. The firms determine prices simultaneously in each period to maximize their respective profits over a finite selling season. Customers have rational expectations of firms' future prices and hence, in equilibrium, can correctly predict the prices charged by the firms. They weigh the expected payoffs of purchasing from different firms at different times and decide when and where to purchase so as to maximize their individual surpluses.

Two key factors that determine customer purchase behavior in our model are quality differentiation measured by the quality ratio of a low-quality product to a high-quality product, and customer rationality measured by customer discount factor. We focus on cases where quality ratio is greater than customer discount factor; in other words, customers attribute more value to purchasing the low-quality product immediately than waiting to purchase the high-quality product in the future when the two products are equally priced at the net present value.

Our model can be viewed as a competitive version of the model of Besanko and Winston (1990). Not surprisingly, we show that equilibrium profit of either firm decreases when customers become more strategic, echoing the results in Besanko and Winston (1990). More importantly, our competitive setup allows us to study the interaction between quality differentiation and strategic consumer behavior.

Our paper highlights two main results. The first one is the asymmetric impact of strategic customer behavior on firms' profits. Although strategic customer behavior reduces both firms' profits compared to myopic customers, its impact is substantially different for quality-differentiated firms, with the low-quality firm usually suffering much more than the high-quality firm. This finding implies that a quality disadvantage in a differentiated market is further exacerbated by strategic customer behavior when firms engage in dynamic pricing competition. The intuition is as follows. In each period, myopic customers decide only which product to purchase as they ignore the option of future purchases. Thus, the low-quality firm competes with the high-quality firm in each period ignoring intertemporal demand substitution. However, as customers act strategically, they decide not only where to purchase, but also when to purchase because they may delay purchases in anticipation of future price discounts. Hence, the low-quality product competes directly with the high-quality product not only in the current period but also in the next period. As a result, the low-quality product is sandwiched between the high-quality product offerings. Furthermore, the more strategic the customers, the stronger the impact of intertemporal demand substitution across periods, resulting in much larger profit loss for the low-quality firm than for the high-quality firm.

The second result concerns firm commitment to static pricing. Commitment to static pricing is potentially an effective way to counteract negative effects of strategic customer behavior. The questions remain: How valuable is such price commitment when firms engage in price competition in a differentiated market? Furthermore, which firm is more likely to adopt static pricing when high-end and low-end businesses coexist in a market? We find that both firms in general benefit from unilateral price commitment of either firm. Furthermore, both firms enjoy higher profits when the high-quality firm commits rather than when the low-quality firm commits. This result is consistent with the business trend where firms are motivated to conduct fewer price promotions and sell more goods at face value.[1] For example, the Federated Department Stores are "being less promotional and more strategic" and claim that "regular-pricing selling is a focus" (Rozhon 2003). Another success story is Abercrombie and Fitch, which positioned the chain as a luxury teen retailer and stopped marking down most products (O'Donnell 2006).

The result that commitment to static pricing improves firms' profits is not surprising, in view of its effect even in the absence of competition, as static pricing eliminates customers' incentive to delay their purchases. However, the effect of price commitment is even stronger in a competitive environment as it can also alleviate price competition: firms only compete in prices in the first period because customers who purchase from the committing firm would do so as early as possible. To understand why the unilateral price commitment of the high-quality firm tends to be more desirable for both firms compared with that of the low-quality firm, we note that the residual market after the first period is more important for the low-quality firm. When the high-quality firm commits, it can grab a sizable portion of the customer base in the first period because of its quality advantage. At the same time, it alleviates the competitive pressure of the low-quality firm, which is effectively a monopoly from the second period onward. In contrast, when the low-quality firm unilaterally commits, it loses pricing flexibility and potential revenue from the residual market. Furthermore, because of its quality disadvantage, the low-quality firm has to slash its price to gain market share, imposing pressure for the high-quality firm to reduce its price as well.

2. Relation to the Literature

Our work is closely related to the growing literature on strategic customer behavior in operations management. This stream of literature investigates how strategic customer behavior affects firms' operational decisions in enriched contexts, including dynamic pricing (Aviv and Pazgal 2008); capacity rationing (Liu and van Ryzin 2008, Gallego et al. 2008, Ovchinnikov and Milner 2012); posterior price matching (Lai et al. 2010); quick response strategy (Cachon and Swinney 2009); and in-store display formats (Yin et al. 2009). Shen and Su (2007) and Netessine and Tang (2009) provide comprehensive reviews of the relevant literature.

Much of the existing literature focuses on a monopoly market in which strategic customers interact with a single seller for a single product, whereas

[1] There are alternative explanations on why firms conduct fewer price promotions; for example, frequent price promotions cause a decrease of reference price of customers thus jeopardize a firm's future profit.

we consider a duopoly market under vertical product differentiation. Several relevant papers consider either multiple products with a monopolistic seller (Parlaktürk 2012) or firms' competition under different selling environments, for example, opaque selling (Jerath et al. 2010) and advance selling (Cachon and Feldman 2010b).

Our work is precedented by Levin et al. (2009), which formulates the dynamic pricing competition among multiple capacitated firms as a stochastic dynamic game. They consider stochastic demand and constrained capacity,[2] which we do not consider. Instead, our focus is on the interaction between strategic consumer behavior and quality differentiation in the absence of demand uncertainty and capacity constraints. Our study confirms several results of theirs. We both find that (1) strategic customer behavior reduces firms' profits, as compared to the case of myopic customers; and (2) a firm incurs profit loss when it wrongly assumes that customers are myopic, and the loss is higher for a low-quality firm. However, our work is different on a number of levels. First, by virtue of a relatively simple setup, we are able to fully characterize the unique pure-strategy Markov perfect equilibrium (MPE) in explicit recursive expressions. Second, we emphasize the asymmetric effect of product quality differentiation on firms' profits in the presence of strategic customers. Third, we also examine firm commitment to static pricing, and generate novel insights on the value of price commitment.

Price commitment has been featured in the existing literature as a promising approach to counteract negative effects of strategic customer behavior. Aviv and Pazgal (2008) and Liu and van Ryzin (2008) consider firm commitment to a price path, and the former shows that such commitment benefits the firm, compared to an inventory-contingent pricing policy. Su and Zhang (2008) demonstrate this point in a supply chain and examine how price commitment can be credibly made through various supply chain contracts. Nevertheless, price commitment may not always be beneficial for firms, which has been revealed by Dasu and Tong (2010), Cachon and Swinney (2009), and Parlaktürk and Kabul (2010). Another relevant paper by Cachon and Feldman (2010a) compares static pricing (committing to a single price) and dynamic pricing by postulating that static pricing imposes a rationing risk on customers whereas dynamic pricing imposes a price risk. Our work assumes no rationing risk and emphasizes the role of product quality differentiation in determining firms' pricing strategies. Interestingly, we show that price commitment of the high-quality firm tends to benefit both firms more than that of the low-quality firm, an insight not discussed in the existing literature.

Our focus of studying dynamic pricing is on intertemporal pricing discrimination in the presence of strategic customers.[3] In this sense, our paper is rooted in the economics literature on intertemporal demand substitution (Coase 1972, Stokey 1979, Bulow 1982, Lazear 1986, Besanko and Winston 1990). Among them, our model can be viewed as a generalization of Besanko and Winston (1990) to a competitive setup. They show that price skimming emerges as a subgame perfect equilibrium in a monopoly market with rational customers, whereas we extend the result to a duopoly market.

Another related stream of literature is competitive dynamic pricing under customer choice behavior. Recent papers in this stream include Xu and Hopp (2006), Lin and Sibdari (2009), Gallego and Hu (2006), Perakis and Sood (2006), and Martínez-de-Albéniz and Talluri (2011), among others. In all of those papers, customers decide where (which product) to buy, based on prices and inventory levels at the time of purchase; a customer goes away immediately if her demand is not fulfilled. An implicit result in this literature is that dynamic pricing improves firms' profits under competition. In contrast, our work assumes strategic customers who decide not only where to purchase, but also when to purchase. Because strategic customers take future price expectations into account in their purchase decisions, dynamic pricing can hurt firms' profits, compared with static pricing. Therefore, strategic customer behavior changes the equilibrium outcome *qualitatively*, emphasizing the importance of understanding such behavior in a competitive market. We point out that our focus is on intertemporal pricing in the context of strategic customers and vertical product differentiation, and we ignore stochastic demand and capacity constraints, which are nevertheless considered in several aforementioned papers.

3. The Model

Consider a market with two firms, firm H and firm L, each of which offers one product. The selling season for the products is divided into T consecutive periods. Time is counted forward, so the first period is period 1, and the last period is period T. The products can be sold at different prices in different time periods. The prices offered in period t by the two firms

[2] In fact, when random shock is assumed away from their linear random utility function, the customer choice model in Levin et al. (2009) can be reduced to our vertical differentiation with certain justifications.

[3] The adoption of dynamic pricing can be attributed to numerous other factors such as uncertain customer valuations and inventory considerations.

are denoted by $p_t = (p_{t,H}, p_{t,L})$, $t = 1, \ldots, T$. The per-period discount factor for each firm is α $(0 < \alpha \le 1)$. The firms determine prices simultaneously at the beginning of each period to maximize their respective total discounted profits over T periods.

The two firms offer quality-differentiated products. Specifically, firm H offers product H with a higher quality level q_H than firm L does; i.e., $q_H > q_L$. Without loss of generality, we normalize (q_H, q_L) to $(1, \beta)$ with $0 < \beta < 1$. We assume that quality levels are exogenously given, and our focus is on firms' price competition over time for any given quality ratio β. Customers have heterogeneous valuations (tastes) θ on product quality, following a Uniform distribution on $[0, 1]$. The valuation distribution is common knowledge for the firms and customers. If a customer with valuation θ purchases product i at price $p_{t,i}$ in period t, she earns a surplus of $\theta q_i - p_{t,i}$, $i = H, L$; she can also choose not to purchase and earn zero surplus.

We consider a linear cost structure where the per-unit cost of H and L is c and βc, respectively. To ensure nonnegative profits for the firms, we assume $c < 1$. Our cost assumption is a special case of those made in the literature on sequential quality-price competition for vertically differentiated products (Motta 1993). Because quality levels are exogenously given in our model, fixed cost can be reasonably assumed away when deriving firms' pricing strategies in equilibrium. Further, our linear cost structure is a special case of the often-assumed convex variable cost in the literature ever since Mussa and Rosen (1978). The linear cost facilitates us to derive analytical results in a multiperiod game, whereas a general convex cost function renders the game analytically intractable. For this reason, we leave the extension to a more general cost structure to future work.

To economize on notations, the total number of customers is normalized to one. All customers arrive at the beginning of the selling season prior to period 1, and each of them purchases at most one unit of the product. Customers are intertemporal utility maximizers and decide when and which product to purchase. Customers discount utilities over time via a per-period discount factor γ $(0 \le \gamma < 1)$. That is, when customers compare surpluses of purchasing now and waiting to purchase in the future, they discount future surpluses by γ. Hence, γ can be interpreted as the level of customer's strategicity/rationality; a higher γ implies that customers are more strategic. When $\gamma = 0$, customers purchase the product as long as they earn a positive surplus because the surplus of any future purchase is zero; in this case, customers are referred to as myopic customers. We assume that $\gamma < \beta$, which implies that customers earn higher surpluses by purchasing product L immediately than purchasing product H in the following period if the two products are equally priced at the net present value. We consider this assumption a limitation of our work and will discuss its implications in §4.

We formulate the model as a finite horizon dynamic game, where the state is the set of customers who remain in the market. The solution concept we adopt is MPE, which is a profile of Markov strategies that is subgame perfect for each player; see Fudenberg and Tirole (1991).

4. Equilibrium Analysis

We now analyze firms' equilibrium pricing strategies in a T-period dynamic game. A critical assumption we make is $\beta > \gamma$. The relationship between β and γ roughly captures the relative attractiveness of buying product L now versus buying product H in the next period. When $\beta > \gamma$, customers attribute more value to purchasing L now than purchasing H in the next period when the two products are equally priced at the net present value (i.e., $p_{t,L} = \gamma p_{t+1,H}$). Hence, firm L can price appropriately to capture customers in the second-highest valuation segment (who prefer not to wait to purchase H in the next period), whereas customers in the highest-valuation segment purchase H in the current period. In equilibrium, both firms incur positive sales in each period, and the remaining customers after each period have valuations in a continuous interval. Such a pattern repeats over periods. Therefore, the number of remaining customers in the market suffices to be the state variable for the game. The result is formally established in the following lemma.

Lemma 1. *Suppose $\beta > \gamma$. If a customer with valuation v' purchases in period t, $1 \le t \le T$, all customers in the market with valuations higher than v' will also purchase in period t, for any sequence of expected future prices $\{p_\tau\}_{\tau=t+1}^{T}$.*

When $\beta > \gamma$ is not satisfied, customers prefer purchasing product H in the next period to purchasing product L in the current period when the two products are equally priced at the net present value. This implies that firm H can set a price in the next period such that, for any current-period price chosen by firm L, a portion of high-valuation customers wait to purchase H later rather than purchase L immediately. Thus, the remaining customers after the current period cannot be characterized by a continuous interval. Unfortunately, when this is the case, the game becomes extremely difficult to analyze; see a detailed discussion in the online appendix (http://www.danzhang.com/cp_appendix.pdf).[4]

[4] When $\beta \le \gamma$, the state (the set of remaining customers) may not be represented by a continuous interval. When the state is a union

Table 1 Profits of Firm H and Firm L in the Last Period

Range of $p_{T,H}$	$[c, p_{T,L}/\beta)$	$[(p_{T,L}/\beta), p_{T,L}+(1-\beta)\theta_T)$	$[p_{T,L}+(1-\beta)\theta_T, 1]$
$r_{T,H}(\theta_T, p_T)$	$(p_{T,H}-c)(\theta_T-p_{T,H})$	$(p_{T,H}-c)\left(\theta_T-\dfrac{p_{T,H}-p_{T,L}}{1-\beta}\right)$	0
$r_{T,L}(\theta_T, p_T)$	0	$(p_{T,L}-\beta c)\left(\dfrac{p_{T,H}-p_{T,L}}{1-\beta}-\dfrac{p_{T,L}}{\beta}\right)$	$(p_{T,L}-\beta c)\left(\theta_T-\dfrac{p_{T,L}}{\beta}\right)$

Our analysis focuses on the characterization of a pure-strategy equilibrium. Although a mixed-strategy equilibrium may also exist, it is well known in the game theory literature that a mixed-strategy equilibrium can be difficult to interpret and implement. It is straightforward to show that when all the remaining customers have valuations less than c, i.e., $\theta_t < c$, there will be no sales for either firm. To avoid such a trivial case, we assume that $\theta_t \geq c$. This assumption is without loss of generality because θ_t will never fall below c on an equilibrium path. The game is analyzed by backward induction. We start with the last period problem in §4.1 and then proceed to the multiperiod problem in §4.2.

To simplify notations, we use a few sets of constants $\{A_{t,H}\}_{t=1}^T$, $\{A_{t,L}\}_{t=1}^T$, $\{B_{t,H}\}_{t=1}^T$, and $\{B_{t,L}\}_{t=1}^T$ in our analysis with

$$A_{T,H}=\frac{2(1-\beta)}{4-\beta}, \quad A_{T,L}=\frac{\beta(1-\beta)}{4-\beta},$$
$$B_{T,H}=\frac{4(1-\beta)}{(4-\beta)^2}, \quad B_{T,L}=\frac{\beta(1-\beta)}{(4-\beta)^2}. \tag{1}$$

We also define, for $t=1,\ldots,T$,

$$X_t=\beta-\gamma(1-A_{t,H}),$$
$$\Delta_t=3X_t^2+4(1-\beta)X_t-4\alpha(1-\beta)B_{t,L},$$
$$\Phi_t=X_t^2+(1-\beta)X_t-\alpha(1-\beta)B_{t,L}.$$

These notations will be used throughout the paper.

4.1. Analysis for the Last Period

Suppose the remaining customers have valuations over $[0,\theta_T]$, where $\theta_T \in [c,1]$. Given state θ_T and a price pair $p_T=(p_{T,H}, p_{T,L})$, a customer with valuation $\theta \leq \theta_T$ purchases product H if doing so leads to a positive surplus higher than purchasing product L; i.e., $\theta - p_{T,H} \geq (\beta\theta - p_{T,L})^+$, where the notation $(x)^+$ denotes the nonnegative part of x. Similarly, a customer with valuation $\theta \leq \theta_T$ purchases product L if $\beta\theta - p_{T,L} > (\theta - p_{T,H})^+$. Table 1 shows firm i's profit, $r_{T,i}(\theta_T, p_T)$, as a function of the state θ_T and price pair p_T for $i=H,L$. Proposition 1 characterizes the unique Nash equilibrium in the last period.

of disjoint intervals, we can show that, even for a single-period game, the firm's payoff function may not be quasi-concave, and Nash equilibrium in pure strategies may not exist.

Proposition 1. *Suppose the remaining customers in the last period have valuations in the range $[0,\theta_T]$, where $\theta_T \geq c$. There exists a unique Nash equilibrium in the last period. The equilibrium prices and profits are given by*

$$p^*_{T,H}(\theta_T)=A_{T,H}(\theta_T-c)+c, \quad r^*_{T,H}(\theta_T)=B_{T,H}(\theta_T-c)^2,$$
$$p^*_{T,L}(\theta_T)=A_{T,L}(\theta_T-c)+\beta c, \quad r^*_{T,L}(\theta_T)=B_{T,L}(\theta_T-c)^2.$$

In the last period, the profit margin of firm H is higher than that of firm L, and firm H earns a higher profit as well, because of its quality advantage. In addition, strategic customer behavior (i.e., γ) does not play any role in the last period game because there are no future purchase opportunities.

4.2. Multiperiod Analysis

We now analyze the T-period game for $T \geq 2$ by backward induction, building on the result in Proposition 1. A multiperiod game is substantially more complex because of intertemporal customer choice between the two products over different periods. In a multiperiod game, each customer decides when and which product to purchase, or not to purchase at all.

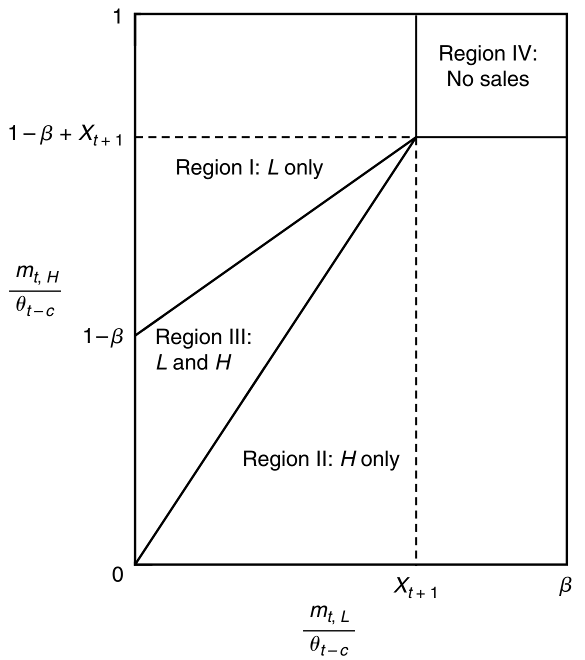
Assume that, in period $t+1$, the equilibrium margins and profits at state θ_{t+1} are given by

$$m^*_{t+1,H}(\theta_{t+1})=A_{t+1,H}(\theta_{t+1}-c),$$
$$m^*_{t+1,L}(\theta_{t+1})=A_{t+1,L}(\theta_{t+1}-c),$$
$$r^*_{t+1,H}(\theta_{t+1})=B_{t+1,H}(\theta_{t+1}-c)^2,$$
$$r^*_{t+1,L}(\theta_{t+1})=B_{t+1,L}(\theta_{t+1}-c)^2,$$

where $m^*_{t+1,H}(\theta_{t+1})=p^*_{t+1,H}(\theta_{t+1})-c$, $m^*_{t+1,L}(\theta_{t+1})=p^*_{t+1,L}(\theta_{t+1})-\beta c$ are the profit margins in period $t+1$ for the two firms, respectively.

We now consider the game in period t with state θ_t. To write down the payoff functions, we need to analyze customer purchase decisions. Each customer determines (i) whether to purchase in period t, or wait until the next period; and (ii) in case she purchases in period t, which product (H or L) to choose. Let $\hat{\theta}_{t+1}$ be the valuation of a marginal customer who is indifferent between purchasing in period t and period $t+1$. The consumer surpluses under different

Figure 1 **Regions of Strategy Space in Period t**



purchase options for a customer with valuation θ are given by

$$\theta - m_{t,H} - c \qquad \text{(purchase } H \text{ in period } t\text{)},$$

$$\beta\theta - m_{t,L} - \beta c \qquad \text{(purchase } L \text{ in period } t\text{)},$$

$$\gamma(\theta - m^*_{t+1,H}(\hat{\theta}_{t+1}) - c) \qquad \text{(wait to purchase } H \text{ in period } t+1\text{)}.$$

Depending on m_t and the values of β and γ, firm L may or may not incur demand in period t. When firm L incurs positive demand, the marginal valuation $\hat{\theta}_{t+1}$ is determined by comparing the surpluses of purchasing L in period t and purchasing H in period $t+1$, satisfying $\beta\hat{\theta}_{t+1} - m_{t,L} - \beta c = \gamma(1-A_{t+1,H})(\hat{\theta}_{t+1} - c)$.[5] Hence, $\hat{\theta}_{t+1} = m_{t,L}/X_{t+1} + c$. On the other hand, if firm L does not incur positive demand in period t, $\hat{\theta}_{t+1}$ is determined by comparing the surpluses of purchasing H in period t and in period $t+1$; that is, $\hat{\theta}_{t+1} - m_{t,H} - c = \gamma(1-A_{t+1,H})(\hat{\theta}_{t+1} - c)$. Hence, $\hat{\theta}_{t+1} = m_{t,H}/(1-\beta+X_{t+1}) + c$.

The strategy space for the two firms can be divided into four regions shown in Figure 1. Specifically, only firm L (H) incurs sales in region I (II); both firms incur sales in region III; neither firm has sales in region IV. Accordingly, Table 2 summarizes each firm's sales in period t and the resulting state in period $t+1$ in the four regions. Our analysis reveals that a Nash equilibrium can only be sustained in region III where both firms incur positive sales in the current period t. The resulting state (the remaining market after period t) is hence determined by the indifference point between purchasing product L in the current period t and purchasing product H in the next period $t+1$ (see Table 2). A key driver for this equilibrium result is the assumption $\beta > \gamma$. Recall that this assumption implies that customers earn a higher surplus when purchasing product L immediately than waiting to buy product H in the future if the two products are equally priced at the net present value. In this case, for any anticipated price of firm H in the immediate future, firm L can always choose an appropriate price such that the high-valuation residual customers prefer to purchase L in the current period than to wait. As a result, both firm H and firm L incur positive sales in equilibrium, with the highest-valuation segment purchasing H and the second highest-valuation purchasing L, in each period. Proposition 2 below provides a formal statement of the existence of a unique MPE in pure strategy.

Table 2 **Sales and State Transition in Period t**

Region	Sales of H	Sales of L	State in period $t+1$
I	0	$\theta_t - \frac{m_{t,L}}{X_{t+1}} - c$	$\frac{m_{t,L}}{X_{t+1}} + c$
II	$\theta_t - \frac{m_{t,H}}{1-\beta+X_{t+1}} - c$	0	$\frac{m_{t,H}}{1-\beta+X_{t+1}} + c$
III	$\theta_t - \frac{m_{t,H}-m_{t,L}}{1-\beta} - c$	$\frac{m_{t,H}-m_{t,L}}{1-\beta} - \frac{m_{t,L}}{X_{t+1}}$	$\frac{m_{t,L}}{X_{t+1}} + c$
IV	0	0	θ_t

Proposition 2. *There exists a unique pure-strategy MPE in the multiperiod dynamic game when $\beta > \gamma$. The unique pure-strategy equilibrium in period T is given in Proposition 1. Given state θ_t in period $t < T$, the equilibrium prices and profits can be characterized by*

$$p^*_{t,H}(\theta_t) = A_{t,H}(\theta_t - c) + c, \quad r^*_{t,H}(\theta_t) = B_{t,H}(\theta_t - c)^2,$$
$$p^*_{t,L}(\theta_t) = A_{t,L}(\theta_t - c) + \beta c, \quad r^*_{t,L}(\theta_t) = B_{t,L}(\theta_t - c)^2,$$

where $A_{t,H}$, $A_{t,L}$, $B_{t,H}$, and $B_{t,L}$ are strictly positive constants given by

$$A_{t,L} = \frac{(1-\beta)X^2_{t+1}}{\Delta_{t+1}}, \quad A_{t,H} = \frac{A_{t,L} + 1 - \beta}{2},$$

$$B_{t,L} = \frac{A_{t,L}\Phi_{t+1}}{\Delta_{t+1}},$$

$$B_{t,H} = \frac{(A_{t,L} + 1 - \beta)^2}{4(1-\beta)} + \alpha B_{t+1,H}\left(\frac{A_{t,L}}{X_{t+1}}\right)^2.$$

Proposition 2 fully characterizes the MPE by explicit recursive equations, which enable us to compute equilibrium prices in the following fashion. The coefficients $A_{t,i}$ and $B_{t,i}$ can be calculated backward for $i = H, L$. Let θ^*_t be the state visited in period t in equilibrium. Clearly, $\theta^*_1 = 1$. For $t = 2, \ldots, T$, θ^*_t can be determined forward, based on the equilibrium prices in the previous period $(p^*_{t-1,H}(\theta^*_{t-1}), p^*_{t-1,L}(\theta^*_{t-1}))$.

[5] Here we use the inductive hypothesis in period $t+1$.

With a slight abuse of notation, let $p^*_{t,i} \equiv p^*_{t,i}(\theta^*_t)$ denote the equilibrium price for firm i in period t, $i = H, L$. In the MPE, we expect $p^*_{t,i}$ to decrease in time t. Intuitively, all customers who purchased in period $t+1$ would have purchased in period t if they anticipate a price increase in period $t+1$. The following proposition provides a formal statement of this result.

Proposition 3. *The unique pure-strategy MPE has the following properties*:

(i) *Equilibrium prices decrease over time for each firm; that is, $p^*_{t,i} \geq p^*_{t+1,i}$ for $1 \leq t \leq T-1$ and $i = H, L$.*

(ii) *As β approaches 1, $p^*_{t,H}$ approaches c and $p^*_{t,L}$ approaches βc for $1 \leq t \leq T$, and the equilibrium profit of each firm approaches zero.*

Part (i) in Proposition 3 can be viewed as a generalization of Proposition 2 in Besanko and Winston (1990), which established the same result in a monopoly market. We conclude that price skimming arises under competition as well. Part (ii) is fairly intuitive. When the two firms offer almost identical products, perfect competition leads to market prices equal to marginal costs, and hence, zero profits in equilibrium.

How do equilibrium profits change with the number of periods T? As T increases, firms have more opportunities to change prices. Our numerical experiment shows that the profit coefficient $\{B_{t,i}\}$, for $i = H, L$, is a monotonically increasing and convergent sequence, except when γ is sufficiently small. This implies that the total equilibrium profit of each firm, in general, decreases in the number of periods T. We hence conclude that firms can suffer from dynamic pricing competition when faced with strategic customers. This result is consistent with the observation in a monopoly market studied by Besanko and Winston (1990). However, we also find that the negative impact of dynamic pricing tends to be larger in the presence of competition. Both firms suffer even when there is only one price change under competition except for a sufficiently small γ. In contrast, a monopoly may benefit from price adjustments for a wider range of γ. In other words, there exist γ values for which a firm benefits from price adjustments under monopoly but not duopoly.

To further develop insights into the impact of strategic customer behavior, we also investigate firms' equilibrium pricing policies when the number of periods T is sufficiently large.

Proposition 4. *When the number of periods T goes to infinity, the equilibrium price of each firm can be approximated by*

$$p^*_{t,H}(\theta_t) = \frac{2A^*(1-\beta)}{4A^*-1}(\theta_t - c) + c,$$

$$p^*_{t,L}(\theta_t) = \frac{1-\beta}{4A^*-1}(\theta_t - c) + \beta c,$$

where A^* is the unique real-valued solution to

$$A = 1 + \frac{(4A-1)(1-\beta)}{2A(2\beta-\gamma-\beta\gamma)-(\beta-\gamma)} - \frac{\alpha A(1-\beta)^2}{[2A(2\beta-\gamma-\beta\gamma)-(\beta-\gamma)]^2}.$$

We numerically find that $\partial A^*/\partial \gamma > 0$; together with the fact that $2A(1-\beta)/(4A^*-1)$ and $(1-\beta)/(4A^*-1)$ both decrease in A^*, we conclude that the equilibrium prices of both firms decrease in γ for any given state θ_t. This result is consistent with our intuition that firms have to reduce selling prices as customers become more strategic (and thus more willing to wait). Notice also that the ratio of profit margins of the two firms in period t, $(p^*_{t,H} - c)/(p^*_{t,L} - \beta c)$, is equal to $2A^*$. As customers behave more strategically, this ratio becomes larger, implying that strategic purchase behavior further exacerbates the quality disadvantage of firm L.

5. Equilibrium Results When Customers Are Myopic ($\gamma = 0$)

In contrast to strategic customers who take into account the entire price path when making purchase decisions, myopic customers ignore future purchase opportunities and make purchase decisions based on current prices only. The following proposition shows that there always exists a unique MPE with myopic customers. The equilibrium results can be derived directly from Proposition 2. However, we adopt a slightly different form for the recursive expressions to facilitate the proof of monotonicity properties stated in Proposition 5.

Proposition 5. *With myopic customers, the unique pure-strategy MPE in each period t, $t = 1, \ldots, T$, is characterized by*

$$\tilde{p}^*_{t,H}(\theta_t) = \frac{2\tilde{A}_t(1-\beta)}{4\tilde{A}_t - 1}(\theta_t - c) + c,$$

$$\tilde{r}^*_{t,H}(\theta_t) = \frac{4(1-\beta)\tilde{B}_t}{(4\tilde{A}_t - 1)^2}(\theta_t - c)^2,$$

$$\tilde{p}^*_{t,L}(\theta_t) = \frac{(1-\beta)}{4\tilde{A}_t - 1}(\theta_t - c) + \beta c,$$

$$\tilde{r}^*_{t,L}(\theta_t) = \frac{\tilde{A}_t(1-\beta)}{(4\tilde{A}_t - 1)^2}(\theta_t - c)^2,$$

where

$$\tilde{A}_t = \frac{1}{\beta} - \alpha\left(\frac{1}{\beta} - 1\right)^2 \frac{\tilde{A}_{t+1}}{(4\tilde{A}_{t+1} - 1)^2}, \quad \forall\, t = 1, \ldots, T-1,\ \tilde{A}_T = \frac{1}{\beta},$$

$$\tilde{B}_t = \tilde{A}_t^2 + \alpha\left(\frac{1}{\beta} - 1\right)^2 \frac{\tilde{B}_{t+1}}{(4\tilde{A}_{t+1} - 1)^2},$$
$$\forall\, t = 1, \ldots, T-1, \; \tilde{B}_T = \frac{1}{\beta^2}.$$

Furthermore, we have the following:

(i) *Equilibrium prices decrease monotonically over time; that is,* $\tilde{p}^*_{t,i} \geq \tilde{p}^*_{t+1,i}$, $i = H, L$, *for* $1 \leq t \leq T-1$.

(ii) *The coefficients of equilibrium profits decrease monotonically over time; that is, for a given* $\theta \geq c$, $\tilde{r}^*_{t,i}(\theta) > \tilde{r}^*_{t+1,i}(\theta)$, $i = H, L$, *for* $1 \leq t \leq T-1$.

Not surprisingly, equilibrium prices decrease over time, consistent with the earlier result in Proposition 3. An important fact revealed by the proof of the above proposition is that $\tilde{r}^*_{t,i}(\theta)$, $i = H, L$, is decreasing in t for any given state θ. This immediately implies that the equilibrium profits of both firms increase in the number of periods. In other words, firms benefit from more opportunities to adjust prices faced with myopic customers. In a monopolistic setting, Besanko and Winston (1990) illustrated the benefit of price skimming as a way to extract consumer surplus. Our result here confirms that price skimming is still effective in a competitive market, given that customers act myopically. However, this is, in general, not true when customers are strategic (i.e., $\gamma > 0$). Recall that, with strategic customers, the equilibrium profits decrease in the number of periods, provided that customers are not almost myopic. Price skimming generally hurts firms' profits in the presence of strategic customers. This is because the benefit of price skimming has to be weighted against the profit loss resulting from strategic customer waiting. This highlights the importance of understanding strategic consumer behavior in a competitive market.

6. Unilateral Commitment to Static Pricing

Dynamic pricing incentivizes strategic customers to postpone purchases and wait for lower prices. One way to thwart strategic waiting behavior is to charge a single price over the entire selling horizon. Indeed, Zara, the Spanish fashion retailer, often promotes affordable full prices, informing their customers that items are not eligible for future discounts. A commitment to static pricing eliminates customers' incentive to wait, and is therefore potentially beneficial. Note that price commitment may not be credible because the firm can deviate to a different price given the residual demand. However, it can be enforced through certain commitment devices, such as supply chain contracts (Su and Zhang 2008) and best price provisions (Butz 1990).

This section explores the value of commitment to static pricing (or price commitment, for short) in a duopoly pricing competition. When both firms commit to static pricing, the game reduces to the last-period problem with $\theta_T = 1$ analyzed in §4.1. Hence, in what follows, we confine ourselves to the situation where one firm commits to static pricing, and the other firm dynamically adjusts prices over time. Although there are several plausible assumptions on the sequence of the game (e.g., the commitment firm is a Stackelberg leader and the other firm acts as a Stackelberg follower), we focus on the case where both firms set prices simultaneously in the first period, and the noncommitment firm then dynamically changes prices in remaining periods. This is mainly for a fair comparison with the case of dynamic pricing competition analyzed in §4.2 where both firms move simultaneously in each period.

6.1. Firm H Commits to Static Pricing

Suppose the price charged by firm H is p_H, which is fixed over the entire horizon, and firm L charges a price $p_{t,L}$ in period t, $t = 1, \ldots, T$. Because a customer discounts her utility over time, she would purchase in the first period if she decides to purchase product H. As such, firm H incurs positive sales in the first period only. The game between H and L is equivalent to a two-player game, where the strategy of firm H is p_H, and the strategy of firm L is $\vec{p}_L = (p_{1,L}, \ldots, p_{T,L})$. The equilibrium prices $\vec{p}_L$ are chosen to be subgame perfect in the sense that $(p_{t,L}, \ldots, p_{T,L})$ is firm L's optimal price strategy from periods t to T, given the residual demand.

The payoff function of firm L is given by

$$r_L^{\mathrm{HC}}(p_H, \vec{p}_L) = \sum_{t=1}^{T} \alpha^{t-1}(p_{t,L} - \beta c)(\hat{\theta}_t - \hat{\theta}_{t+1}), \tag{2}$$

$$\text{s.t.} \quad \beta\hat{\theta}_1 - p_{1,L} = \hat{\theta}_1 - p_H, \tag{3}$$

$$\beta\hat{\theta}_t - p_{t-1,L} = \gamma(\beta\hat{\theta}_t - p_{t,L}), \quad \text{if } t = 2, \ldots, T, \tag{4}$$

$$\beta\hat{\theta}_{T+1} - p_{T,L} = 0. \tag{5}$$

Constraint (3) defines the marginal valuation $\hat{\theta}_1$ at which a customer is indifferent between purchasing products H and L in period 1; constraint (4) defines the marginal valuation $\hat{\theta}_t$ at which a customer is indifferent between purchasing product L in period $t-1$ and period t for $t = 2, \ldots, T$; for $t = T+1$, constraint (5) ensures the remaining customers with nonnegative surpluses purchase in the last period T.

The payoff function of firm H is then given by

$$r_H^{\mathrm{HC}}(p_H, \vec{p}_L) = (p_H - c)\left[1 - \frac{p_H - p_{1,L}}{1-\beta}\right]. \tag{6}$$

Intuitively, the highest-valuation segment is covered by firm H in the first period, and the remaining customers determine when to purchase product L so as to maximize each individual's own intertemporal utility. Proposition 6 fully characterizes the equilibrium pricing policy for the game. To facilitate the presentation, we define constants $\{C_t\}_{t=1}^T$ and $\{D_t\}_{t=2}^T$ as follows:

$$C_1 = \frac{(1-\beta)D_2^2}{3D_2^2 + 4(1-\beta)D_2 - 2\alpha(1-\beta)C_2},$$

$$C_t = \frac{D_{t+1}^2}{2D_{t+1} - \alpha C_{t+1}}, \quad \forall t = 2, \ldots, T-1, \qquad C_T = \frac{\beta}{2},$$

$$D_t = \beta - \gamma(\beta - C_t), \quad \forall t = 2, \ldots, T.$$

Proposition 6. *Suppose firm H commits to static pricing, and firm L dynamically changes prices over T discrete periods. A unique Markov perfect equilibrium for the game exists and can be described as follows*:

$$p_H^* = \frac{(1-\beta+C_1)(1-c)}{2} + c,$$

$$p_{1,L}^* = C_1(1-c) + \beta c,$$

$$p_{t,L}^* = C_t(\theta_t^* - c) + \beta c, \quad \forall t = 2, \ldots, T, \tag{7}$$

$$\theta_t^* = \frac{p_{t-1,L}^* - \beta c}{\beta - \beta\gamma + \gamma C_t} + c, \quad \forall t = 2, \ldots, T. \tag{8}$$

The equilibrium prices for H and L are p_H^* and $\vec{p}_L^*$, respectively, and θ_t^* is the maximum valuation of customers remaining in period t in equilibrium for $t = 2, \ldots, T$. The equilibrium profits for H and L are given by $r_H^{\mathrm{HC}}(p_H^*, \vec{p}_L^*)$ and $r_L^{\mathrm{HC}}(p_H^*, \vec{p}_L^*)$, respectively.

The recursive expressions shown in Proposition 6 allow us to calculate the equilibrium prices and profits for both firms. A comparison with the case where both firms change prices dynamically would reveal the value of price commitment. We provide more detailed discussions in §7.

6.2. Firm L Commits to Static Pricing

We now consider the case in which firm L commits to a static price p_L, and firm H changes prices dynamically over time and charges a price $p_{t,H}$ in period t, $t = 1, \ldots, T$. Following the same argument as when firm H commits, we conclude that firm L incurs sales only in the first period. Furthermore, because $\beta > \gamma$, customers prefer to buy product L in the first period than to wait for product H in the later period provided that the two products are equally priced at the net present value. Therefore, it must be, in equilibrium, that the highest-valuation segment purchases product H in the first period, the second highest-valuation segment purchases product L in the first period, and the rest of the customers delay their purchases to later periods.

We can then formulate a two-player game between firms H and L, where firm H's strategy specifies the price vector $\vec{p}_H = (p_{1,H}, \ldots, p_{T,H})$, and firm L's strategy prescribes a fixed price p_L over the selling horizon. The payoff function of H is given by

$$r_H^{\mathrm{LC}}(\vec{p}_H, p_L) = (p_{1,H} - c)\left(1 - \frac{p_{1,H} - p_L}{1-\beta}\right) + \sum_{t=1}^{T-1} \alpha^t (p_{t+1,H} - c)(\hat{\theta}_t - \hat{\theta}_{t+1}), \tag{9}$$

$$\text{s.t.} \quad \beta\hat{\theta}_1 - p_L = \gamma(\hat{\theta}_1 - p_{2,H}), \tag{10}$$

$$\hat{\theta}_t - p_{t,H} = \gamma(\hat{\theta}_t - p_{t+1,H}), \quad \text{if } t = 2, \ldots, T-1, \tag{11}$$

$$\hat{\theta}_T - p_{T,H} = 0. \tag{12}$$

Constraint (10) defines the marginal valuation $\hat{\theta}_1$ at which a customer is indifferent between purchasing product L in the first period and purchasing product H in the second period; constraint (11) defines $\hat{\theta}_t$ as the valuation of a marginal customer who is indifferent between purchasing H in period t and period $t+1$ for $t = 2, \ldots, T-1$; and constraint (12) ensures that the remaining customers with nonnegative surpluses buy product H in the last period T.

Then firm L's payoff function is given by

$$r_L^{\mathrm{LC}}(\vec{p}_H, p_L) = (p_L - \beta c)\left(\frac{p_{1,H} - p_L}{1-\beta} - \frac{p_L - \gamma p_{2,H}}{\beta - \gamma}\right). \tag{13}$$

The following proposition shows that there exists a unique MPE in the above game. We define constants $\{E_t\}_{t=2}^T$ to facilitate the presentation:

$$E_T = \frac{1}{2}, \quad E_t = \frac{(1-\gamma+\gamma E_{t+1})^2}{2(1-\gamma) + (2\gamma - \alpha)E_{t+1}}, \quad \forall t = 2, \ldots, T.$$

Proposition 7. *Suppose firm L commits to static pricing, and firm H dynamically changes prices over T discrete periods. A unique MPE for the game exists when $\beta > \gamma$. The equilibrium prices for H and L are, respectively, $\vec{p}_H^*$ and p_L^*, in the following*:

$$p_{1,H}^* = \frac{(1-\beta)(1-c) + p_L^* - \beta c}{2} + c, \tag{14}$$

$$p_{t,H}^* = E_t(\theta_t^* - c) + c, \quad \forall t = 2, \ldots, T, \tag{15}$$

$$p_L^* = \frac{(1-\beta)(\beta - \gamma + \gamma E_2)(1-c)}{4 - \beta - 3\gamma(1-E_2)} + \beta c, \tag{16}$$

$$\theta_2^* = \frac{p_L^* - \beta c}{\beta - \gamma(1-E_2)} + c, \tag{17}$$

$$\theta_t^* = \frac{p_{t-1,H}^* - c}{1 - \gamma(1-E_t)} + c, \quad \forall t = 3, \ldots, T, \tag{18}$$

where θ_t^* is the maximum valuation of customers remaining in period t in equilibrium. The equilibrium profits for H and L are given by $r_H^{\mathrm{LC}}(\vec{p}_H^*, p_L^*)$ and $r_L^{\mathrm{LC}}(\vec{p}_H^*, p_L^*)$, respectively.

These results can be immediately applied to calculate the equilibrium prices and profits in numerical experiments. We will illustrate the results and compare them with the case without price commitment in §7.

7. Numerical Study and Managerial Insights

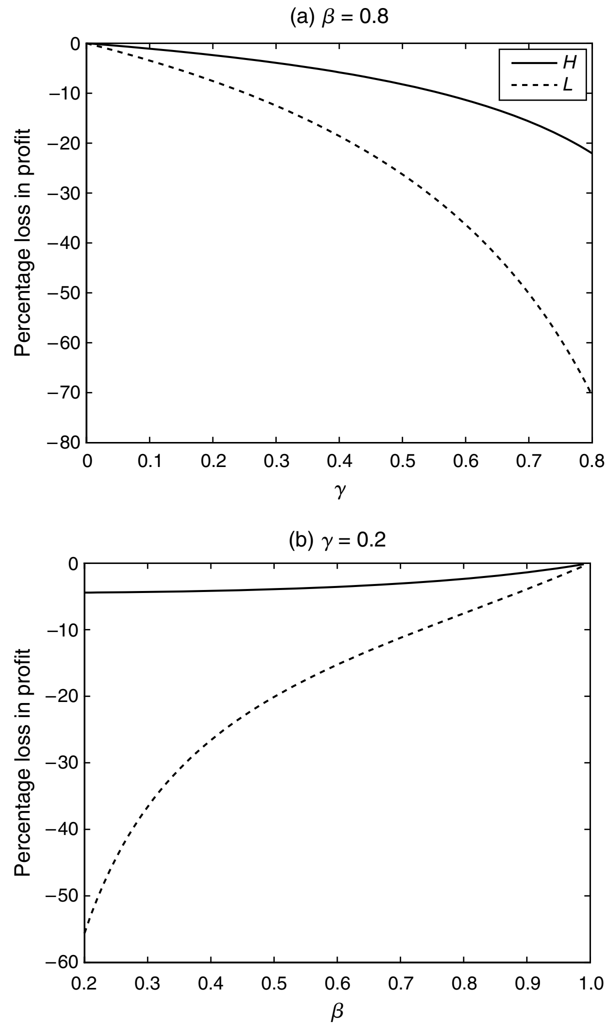
In this section, we conduct extensive numerical experiments and discuss the managerial implications of our models and results. Our experimental design centers around two main contributing factors in the model, including vertical product differentiation summarized by the parameter β and strategic customer behavior summarized by the parameter γ. We consider different combinations of β and γ while fixing $\alpha = 1$ and $c = 0$. We have tried different α and c values and found qualitatively similar results. We consider cases where $\beta \in \{0.01, 0.02, \ldots, 0.99\}$ and $\gamma \in \{0, 0.01, \ldots, 0.98\}$ with the requirement that $\beta > \gamma$. In total, we consider 4,851 cases with different combinations of β and γ values. Of course, the equilibrium profits for H and L also depend on the number of periods T. In almost all cases, however, we observe that the equilibrium profits converge quite quickly as T increases. In the following numerical studies, we take $T = 20$, at which we obtain the limiting equilibrium profits.

7.1. Impact of Strategic Customer Behavior in Dynamic Pricing Competition

We numerically verified that the equilibrium profit for each firm decreases as customers become more strategic (i.e., γ increases) for any given quality ratio β. The benchmark is each firm's equilibrium profit with myopic customers (i.e., $\gamma = 0$). Figure 2(a) illustrates the result for $\beta = 0.8$; the percentage loss in profit relative to the case of myopic customers increases in γ. The profit loss due to strategic customer behavior is not surprising, because demand becomes more price elastic when customers can delay their purchases resulting in intertemporal substitution. This observation corroborates a similar finding in the monopolistic setting (Besanko and Winston 1990).

Interestingly, Figure 2(a) also reveals that the profit loss from strategic customer behavior can be quite significant, with the low-quality firm incurring much larger profit loss than the high-quality firm. In the illustrative example, the percentage profit loss can reach up to 70% for firm L, but is less than 25% for firm H. To develop further insights into the role of product quality, we fix customer discount factor γ and compute the percentage profit loss relative to the case of myopic customers under different values of β. Figure 2(b) demonstrates the result when $\gamma = 0.2$.

Figure 2 Percentage Loss in Profits Relative to the Case of Myopic Customers



We observe that for a fixed value of γ, the percentage profit loss becomes larger as the quality difference increases (i.e., β decreases). When the quality difference is minimal (i.e., β is close to 1), price competition is extremely intense; as shown in Proposition 3, both firms price almost at their costs, respectively, and obtain near-zero profits. However, as the two firms differ more in quality, the negative consequence of strategic customer behavior becomes severer.

The intuition is the following: When customers behave myopically, they decide only which product to purchase in each period as they ignore the option of future purchases. Accordingly, firm L competes with firm H in each period ignoring intertemporal demand substitution. Difference emerges when customers act strategically. Within each period, strategic customers decide not only where to purchase, but also when to purchase because they may delay purchases in anticipation of future price discounts. Hence, product L is sandwiched between the offerings of product H

in both the current period and the immediate future period. As customers become more strategic (i.e., γ increases), the impact of intertemporal demand substitution across periods is even stronger, resulting in much larger profit loss for firm L than for firm H.

7.2. Value of Unilateral Commitment to Static Pricing

Building on the equilibrium results in §6, we numerically study the value of unilateral price commitment by comparing the equilibrium profits with or without price commitment. The benchmark is each firm's equilibrium profit when both dynamically change prices over time. Hence, each firm's percentage gain (loss) in profit when firm H (or L) commits to static pricing is calculated as the percentage difference from its equilibrium profit in the dynamic pricing competition without price commitment. Figure 3 illustrates the percentage gain (loss) in profit when either H or L commits to static pricing for different β and γ values.

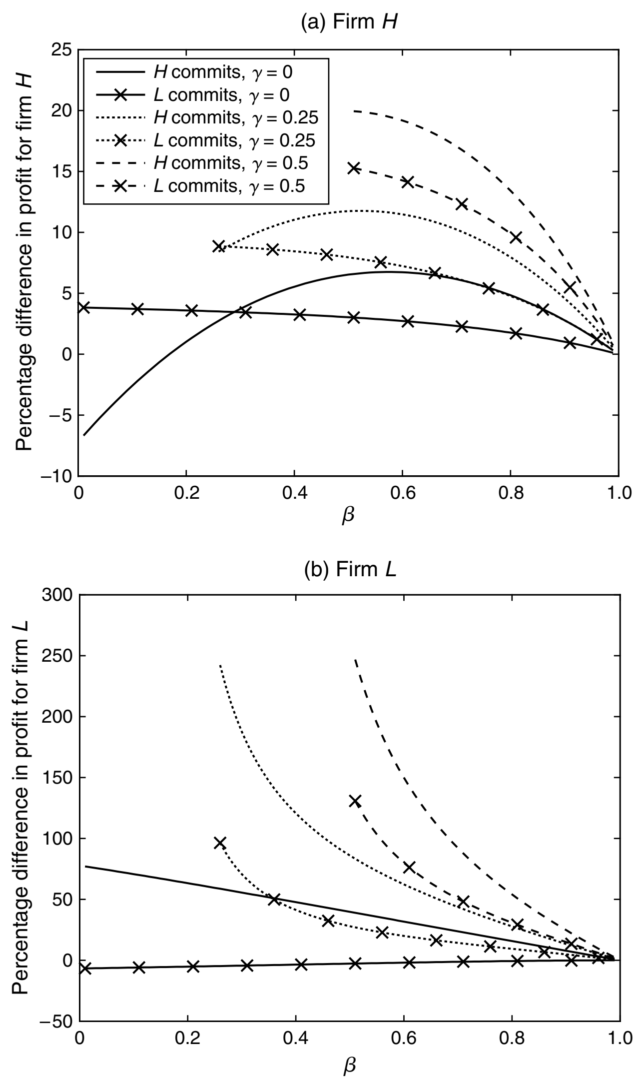
Our main observations can be summarized as follows. (i) Unilateral price commitment of either firm, in general, leads to significant profit gains for both firms, except when customers are more or less myopic (i.e., γ is quite small). (ii) The more strategic the customers, the larger the profit gain for each firm. (iii) Remarkably, the price commitment of the high-quality firm usually brings more benefits for both firms, compared with the commitment of the low-quality firm.

To better understand those observations, we note that a firm's unilateral price commitment has three immediate consequences. First, committing to static pricing eliminates the incentive to wait for customers who choose the committing firm, and allows the firm to charge a higher price than the prices when dynamic pricing is adopted. Second, price commitment reduces the market competition as the two firms compete only in the first period (because customers who choose the committing firm purchase in the first period only), leaving the other firm to act as if it were a monopolistic seller in future periods. Finally, by committing to static pricing, a firm loses pricing flexibility, and hence, cannot benefit from price discrimination among heterogeneous customers. The relative gain/loss of each of the three effects above together determines the value of unilateral price commitment for the committing firm and its competitor.

Given the trade-off analysis above, it is straightforward to see that a firm's price commitment always benefits its competitor, who can dynamically change prices, because of less competition in later periods. The committing firm is also generally better off, implying that the benefits of eliminating strategic customer behavior can more than compensate for the cost of price inflexibility. Exceptions occur when γ is small. Intuitively, when customers are nearly myopic, the cost of price inflexibility dominates, and it is more important to price discriminate among the customers. As γ increases, customers have more incentive to wait; thus, the net benefit of price discrimination diminishes for the firms, and hence, price commitment is more favored. This is, indeed, the case shown in Figure 3. Our results confirm that price commitment is an effective approach to mitigate the negative effects of strategic customer behavior.

One salient observation from Figure 3 is the asymmetric effects of price commitment on the two firms. Specifically, firm L always gains more from the price commitment of firm H than its own commitment, whereas firm H usually gains more from its own commitment than firm L's commitment, except when both β and γ are sufficiently small. In other words, the commitment of H in general dominates that of L.

Figure 3 Percentage Difference in Profit Relative to Dynamic Pricing Competition for Different γ Values



The reason is the following: Because of firm L's quality disadvantage, the simultaneous competition with firm H in the first period results in a lower profit for firm L. Hence, the potential profits extracted from the remaining customers are comparably important to L, which can more than offset the negative consequence of strategic waiting behavior. However, the trade-off is quite different for firm H; the benefits of eliminating strategic waiting and lessening competition usually dominate the benefits from dynamic pricing because firm H is able to capture a sizable high-valuation customer base and earn a larger profit in one period. Only when both β and γ are small are the benefits of dynamic pricing significant for firm H, because a highly differentiated market (i.e., small β) leads to a considerably large residual market after the first period. For example, when $\gamma = 0.1$ and $\beta = 0.2$, the remaining market size after the first period is about 35%; however, it shrinks to 7% when $\beta = 0.8$. The results suggest that a high-end business limit price markdowns; in so doing, it not only benefits itself but also the low-end competitor. This seems to be consistent with our observations in practice. For example, high-end retailers such as Neiman Marcus and Nordstrom sell most of their goods at full price (O'Donnell 2006).

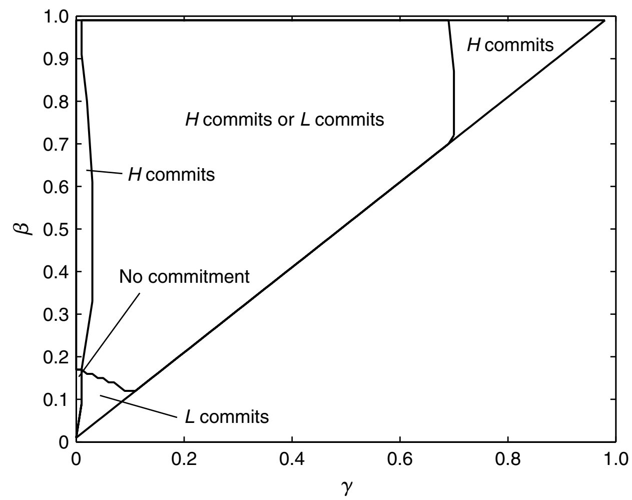
7.3. The Commitment Game

Given that commitment to static pricing often leads to profit improvements for both firms, it is natural to ask whether such commitment can be achieved in equilibrium in a properly augmented commitment game. To this end, we consider a two-stage commitment game in which firms decide whether to commit in the first stage, and compete on prices in the second stage. When both firms choose to commit in the first stage, the game in the second stage becomes a static pricing competition game analyzed in §4.1. When only one firm commits, the second-stage game is a unilateral price commitment game introduced in §6. When both firms do not commit, the second-stage game is a dynamic pricing game in §4.2. Because we have derived the equilibrium results under each possible scenario in the second stage, we are fully equipped to analyze the commitment game in the first stage.

Figure 4 plots in the β-γ plane the regions for different equilibrium outcomes in the first stage. We make a few observations. First, in equilibrium, at most one firm would commit. Therefore, static pricing competition would never arise as an equilibrium outcome. Second, only in a tiny region where γ is close to 0 and β is small, does neither firm commit to static pricing in equilibrium. Hence, dynamic pricing competition rarely emerges as an equilibrium.

The results have several implications. First, a dynamic pricing competition (neither firm commits) equilibrium requires two conditions: (i) a sufficiently small γ implying customers are nearly myopic, and (ii) a small β implying the market is highly differentiated. Second, a unilateral price commitment can be largely sustained as an equilibrium outcome. Furthermore, when unilateral commitment of either firm arises as an equilibrium, the price commitment of H is shown to be a Pareto equilibrium unless both β and γ are quite small. Last, we caution that the commitment game we analyze implicitly assumes that both firms can commit credibly to static pricing, which may require certain commitment devices to enforce it.

Figure 4 **Nash Equilibrium in the Commitment Game**



7.4. Other Insights

Our numerical studies also yield several other interesting conclusions. As revealed earlier, dynamic pricing tends to reduce firms' profits in the presence of strategic customers and competition. We further find that the detrimental effect of dynamic pricing is almost fully realized, even when each firm changes price only once. The result is somewhat negative from a practical standpoint and suggests that firms be careful when adopting dynamic pricing strategy, even when prices are changed infrequently.

We also numerically investigate the cost of ignoring strategic customer behavior. In particular, we find that the firm that ignores strategic customer behavior incurs profit loss, while the other, which recognizes strategic customer behavior, enjoys profit lift. This, in a way, suggests that firms would be unwilling to share information on strategic customer behavior. In addition, the low-quality firm tends to suffer more by wrongly assuming myopic customers than the high-quality firm, implying that it is even more important for the low-quality firm to manage strategic customer behavior. Our results echo those of Levin et al. (2009) in a related, but different, context.

8. Summary

We consider the dynamic pricing competition between two vertically differentiated firms when customers are strategic. We characterize a unique pure-strategy MPE by explicit recursive expressions for the game under certain conditions. Price skimming naturally arises as a subgame perfect equilibrium in our model, paralleling the same result in a monopoly market in Besanko and Winston (1990). By virtue of a vertical differentiation setup, we emphasize the asymmetric impact of strategic customer behavior on quality-differentiated firms, with the low-quality firm suffering much more than the high-quality firm. A broad message from our work is that the disadvantage of offering inferior products is exacerbated by strategic customer behavior.

To counteract negative effects of strategic customer behavior, we investigate a unilateral price commitment game in which one firm commits to static pricing and the other firm dynamically changes prices over time. We show that unilateral price commitment usually benefits both firms compared with dynamic pricing competition, revealing the value of price commitment in a competitive market. Moreover, our results highlight the difference between price commitment of the high-quality firm and that of the low-quality firm. We show that price commitment of the high-quality firm generally results in larger profit improvements for both firms. Via a study of an extended game where firms first choose whether to commit to static pricing, we find that unilateral commitment almost always emerges as an equilibrium; in addition, price commitment of the high-quality firm tends to Pareto dominate that of the low-quality firm. Our results suggest that high-end businesses charging constant prices is frequently a desirable market outcome for sellers, which seems to be supported by casual observations in markets.

Our stylized model assumes unlimited capacity and deterministic demand. Although we do believe that relaxing these assumptions is of general interest and can sharpen our insights, our initial attempt suggests that it often leads to significant analytical challenges and, therefore, presents directions for future research.

Acknowledgments

The authors are grateful to conference attendees of the 10th Annual INFORMS Revenue Management and Pricing Section Conference at Cornell University for many helpful comments and suggestions. The authors thank department editor Yossi Aviv, an anonymous associate editor, and three anonymous referees for constructive comments that led to significant improvements in the paper. The work of Qian Liu was supported by the Hong Kong Research Grant Council [Grants RGC616807, RGC616308]. The work of Dan Zhang was supported in part by research grants from the Natural Sciences and Engineering Research Council of Canada, the Social Sciences and Humanities Research Council of Canada, and Fonds de recherche sur la société et la culture of Quebec.

Appendix. Proofs

Proof of Lemma 1. Fix period $t < T$. Suppose a customer with valuation v' purchases in period t. She purchases product H in period t if and only if

$$v' - p_{t,H} \geq \max\{\gamma^k(v' - p_{t+k,H}), \gamma^k(\beta v' - p_{t+k,L}), \beta v' - p_{t,L}, 0\},\quad k = 1, \ldots, T-t. \tag{19}$$

The above inequalities are equivalent to

$$v' \geq \max\left\{\frac{p_{t,H} - \gamma^k p_{t+k,H}}{1-\gamma^k}, \frac{p_{t,H} - \gamma^k p_{t+k,L}}{1-\beta\gamma^k}, \frac{p_{t,H} - p_{t,L}}{1-\beta}, p_{t,H}\right\},\quad \forall k = 1, \ldots, T-t.$$

It is then straightforward to see that (19) holds for any $v > v'$. Hence, all customers with $v > v'$ will also purchase product H in period t.

Similarly, a customer with valuation v' purchases product L in period t if and only if

$$v' - p_{t,L} \geq \max\{\gamma^k(v' - p_{t+k,H}), \gamma^k(\beta v' - p_{t+k,L}), \beta v' - p_{t,H}, 0\},\quad \forall k = 1, \ldots, T-t. \tag{20}$$

Because $\beta > \gamma$, solving the system of the above inequalities yields

$$\frac{p_{t,H} - p_{t,L}}{1-\beta} \geq v' \geq \max\left\{\frac{p_{t,L} - \gamma^k p_{t+k,H}}{\beta - \gamma^k}, \frac{p_{t,L} - \gamma^k p_{t+k,L}}{\beta(1-\gamma^k)}, \frac{p_{t,L}}{\beta}\right\}.$$

It can be verified that for any v such that $(p_{t,H} - p_{t,L})/(1-\beta) \geq v > v'$, (20) holds, and hence these customers purchase product L in period t. On the other hand, if $v > (p_{t,H} - p_{t,L})/(1-\beta)$, then $v - p_{t,H} > \beta v - p_{t,L} \geq 0$, and thus v satisfies (20). Those customers therefore purchase product H in period t. □

Proof of Proposition 1. When $p_{T,H} \geq p_{T,L} + (1-\beta)\theta_T$ or $p_{T,H} < p_{T,L}/\beta$, one of the firms receives zero profit; therefore, a Nash equilibrium cannot be sustained. This means that a Nash equilibrium price pair must satisfy $p_{T,L}/\beta \leq p_{T,H} < p_{T,L} + (1-\beta)\theta_T$, in which case the first-order conditions lead to the equilibrium prices and corresponding profits given in the proposition.

To show that the price pair $(p^*_{T,H}(\theta_T), p^*_{T,L}(\theta_T))$ is a Nash equilibrium, we need to show that both firms do not have incentive to deviate from this solution. First observe that $p^*_{T,H}(\theta_T) < (1-\beta)\theta_T + \beta c$; hence, firm L cannot deviate to improve his profit. For firm H, it can be shown that the optimal price in the region $p_{T,H} \leq p^*_{T,L}(\theta_T)/\beta$ is given by $p^*_{T,L}(\theta_T)/\beta = (1-\beta)(\theta_T - c)/(4-\beta) + c$ with corresponding profit $3(1-\beta)(\theta_T - c)^2/(4-\beta)^2 < r^*_{1,H}(\theta_T)$. Thus, firm H does not have an incentive to deviate. This establishes the price pair $(p^*_{T,H}(\theta_T), p^*_{T,L}(\theta_T))$ as the unique Nash equilibrium. □

Proof of Proposition 2. The result in period T is given in Proposition 1. We use backward induction to establish a unique pure-strategy MPE in periods 1 to $T-1$.

The payoff function of firm H is

$$r_{t,H}(\theta_t, m_t) = \begin{cases} \alpha B_{t+1,H}\left(\dfrac{m_{t,L}}{X_{t+1}}\right)^2 & \text{region I}, \\ m_{t,H}\left(\theta_t - \dfrac{m_{t,H}}{1-\beta+X_{t+1}} - c\right) & \\ \quad + \alpha B_{t+1,H}\left(\dfrac{m_{t,H}}{1-\beta+X_{t+1}}\right)^2 & \text{region II}, \\ m_{t,H}\left(\theta_t - \dfrac{m_{t,H}-m_{t,L}}{1-\beta} - c\right) & \\ \quad + \alpha B_{t+1,H}\left(\dfrac{m_{t,L}}{X_{t+1}}\right)^2 & \text{region III}, \\ \alpha B_{t+1,H}(\theta_t - c)^2 & \text{region IV}. \end{cases}$$

Similarly, the payoff function of firm L is

$$r_{t,L}(\theta_t, m_t) = \begin{cases} m_{t,L}\left(\theta_t - \dfrac{m_{t,L}}{X_{t+1}} - c\right) + \alpha B_{t+1,L}\left(\dfrac{m_{t,L}}{X_{t+1}}\right)^2 & \text{region I}, \\ \alpha B_{t+1,L}\left(\dfrac{m_{t,H}}{1-\beta+X_{t+1}}\right)^2 & \text{region II}, \\ m_{t,L}\left(\dfrac{m_{t,H}-m_{t,L}}{1-\beta} - \dfrac{m_{t,L}}{X_{t+1}}\right) & \\ \quad + \alpha B_{t+1,L}\left(\dfrac{m_{t,L}}{X_{t+1}}\right)^2 & \text{region III}, \\ \alpha B_{t+1,L}(\theta_t - c)^2 & \text{region IV}. \end{cases}$$

Taking partial derivatives, we have

$$\frac{\partial r_{t,H}(\theta_t, m_t)}{\partial m_{t,H}} = \begin{cases} 0 & \text{region I}, \\ \theta_t - c - \dfrac{2m_{t,H}}{1-\beta+X_{t+1}} + \dfrac{2\alpha B_{t+1,H} m_{t,H}}{(1-\beta+X_{t+1})^2} & \text{region II}, \\ \theta_t - c - \dfrac{2m_{t,H}-m_{t,L}}{1-\beta} & \text{region III}, \\ 0 & \text{region IV}, \end{cases}$$

and

$$\frac{\partial r_{t,L}(\theta_t, m_t)}{\partial m_{t,L}} = \begin{cases} \theta_t - c - \dfrac{2m_{t,L}}{X_{t+1}} + \dfrac{2\alpha B_{t+1,L} m_{t,L}}{X_{t+1}^2} & \text{region I}, \\ 0 & \text{region II}, \\ \dfrac{m_{t,H}-2m_{t,L}}{1-\beta} - \dfrac{2m_{t,L}}{X_{t+1}} + \dfrac{2\alpha B_{t+1,L} m_{t,L}}{X_{t+1}^2} & \text{region III}, \\ 0 & \text{region IV}. \end{cases}$$

We next show that there exists a unique Nash equilibrium in period t in region III. The proof proceeds in several steps. We first solve the first-order conditions in each region ignoring boundary conditions, and verify that the solution in region III is valid. We then show that the solution in region III is a Nash equilibrium by demonstrating that both firms have no incentive to deviate from this solution. Finally, we show that the solutions in regions I, II, and IV cannot sustain as Nash equilibria, establishing the uniqueness of the equilibrium.

Step 1: *Solve the first-order conditions in each region*. We first solve the first-order conditions in each region, ignoring the boundary conditions. We obtain the following solutions for H and L, respectively:

$$\hat{m}_{t,H}(\theta_t) = \begin{cases} \text{any value in range} & \text{if solution in region I}, \\ \dfrac{[1-\beta+X_{t+1}]^2(\theta_t - c)}{2[1-\beta+X_{t+1}-\alpha B_{t+1,H}]} & \text{if solution in region II}, \\ \dfrac{\hat{m}_{t,L}(\theta_t) + (1-\beta)(\theta_t - c)}{2} & \text{if solution in region III}, \\ \text{any value in range} & \text{if solution in region IV}, \end{cases}$$

$$\hat{m}_{t,L}(\theta_t) = \begin{cases} \dfrac{X_{t+1}^2(\theta_t - c)}{2[X_{t+1}-\alpha B_{t+1,L}]} & \text{if solution in region I}, \\ \text{any value in range} & \text{if solution in region II}, \\ \dfrac{(1-\beta)X_{t+1}^2(\theta_t - c)}{3X_{t+1}^2 + 4(1-\beta)X_{t+1} - 4\alpha(1-\beta)B_{t+1,L}} & \\ & \text{if solution in region III}, \\ \text{any value in range} & \text{if solution in region IV}. \end{cases}$$

We next show that the solution is valid in region III. Note that the best response functions in region III for H and L are, respectively,

$$m_{t,H}(\theta_t, m_{t,L}) = \frac{m_{t,L} + (1-\beta)(\theta_t - c)}{2},$$
$$m_{t,L}(\theta_t, m_{t,H}) = \frac{X_{t+1}^2 m_{t,H}}{2[X_{t+1}^2 + (1-\beta)X_{t+1} - \alpha(1-\beta)B_{t+1,L}]}. \quad (21)$$

The solution of first-order conditions in region III is the intersection of the two best response functions. Because the response function $m_{t,H}(\theta_t, m_{t,L})$ is below the boundary of regions I and III in Figure 1, we only need to show the response function $m_{t,L}(\theta_t, m_{t,H})$ is above the boundary of regions II and III, requiring that $X_{t+1}^2/(2[X_{t+1}^2 + (1-\beta)X_{t+1} - \alpha(1-\beta)B_{t+1,L}]) < X_{t+1}/(1-\beta+X_{t+1})$. The condition above is equivalent to $[\beta-\gamma(1-A_{t,H})]^2 + (1-\beta)[\beta-\gamma(1-A_{t,H})] - 2\alpha(1-\beta)B_{t,L} > 0$, which readily follows if $\beta A_{t,H} - 2\alpha B_{t,L} \geq 0$ because $\gamma < \beta$. We now show this result by induction.

When $t = T$, we have $A_{T,H} = 2(1-\beta)/(4-\beta)$ and $B_{T,L} = \beta(1-\beta)/(4-\beta)^2$. Then it is easy to verify that $\beta A_{T,H} - 2\alpha B_{T,L} \geq 0$. Now assume $\beta A_{t+1,H} - 2\alpha B_{t+1,L} \geq 0$, $t \leq T-1$. In period t,

$$\frac{A_{t,H}}{B_{t,L}} = \frac{2\left(3[\beta-\gamma(1-A_{t+1,H})]^2 + 4(1-\beta)[\beta-\gamma(1-A_{t+1,H})] - 4\alpha(1-\beta)B_{t+1,L}\right)}{[\beta-\gamma(1-A_{t+1,H})]^2}$$

$$\geq \frac{2(3[\beta-\gamma(1-A_{t+1,H})]^2+2(1-\beta)[\beta-\gamma(1-A_{t+1,H})])}{[\beta-\gamma(1-A_{t+1,H})]^2}$$
$$=2\left(3+\frac{2(1-\beta)}{\beta-\gamma(1-A_{t+1,H})}\right)\geq 2\left(3+\frac{2(1-\beta)}{\beta}\right)>\frac{2\alpha}{\beta}.$$

Therefore, $\beta A_{t,H}-2\alpha B_{t,L}\geq 0$. We conclude that the solution derived from first-order conditions in region III is valid.

Step 2: *Show that the solution in region* III *is a Nash equilibrium*. To check whether the solution in region III is a Nash equilibrium, we show that neither firm has an incentive to deviate. We first show that firm L has no incentive to deviate. Note that firm L cannot increase profit by deviating to region II, where only H incurs positive demand in period t. This follows immediately from the fact that $\hat{m}_{t,L}(\theta_t)$ is the optimal solution for L in region III, which is necessarily better than the solution on the boundary of regions III and II when $\hat{m}_{t,H}(\theta_t)$ stays constant. Now we show that L cannot deviate to region I. To show this, note that

$$\hat{m}_{t,L}(\theta_t)\leq \frac{(1-\beta)X_{t+1}^2(\theta_t-c)}{X_{t+1}^2+2(1-\beta)X_{t+1}}\leq (1-\beta)(\theta_t-c)$$

under the assumption $\Phi_{t+1}-\alpha(1-\beta)B_{t+1,L}>0$. It then follows that

$$\hat{m}_{t,H}(\theta_t)=\frac{\hat{m}_{t,L}(\theta_t)+(1-\beta)(\theta_t-c)}{2}\leq (1-\beta)(\theta_t-c).$$

It is immediate to see from Figure 1 that L cannot deviate to region I profitably when $\hat{m}_{t,H}(\theta_t)\leq(1-\beta)(\theta_t-c)$.

We next show that firm H has no incentive to deviate. Notice that the valuation of a marginal customer, who is indifferent between purchasing in periods t and $t+1$, is determined by the margin of L in period t in both region I and region III. Should Firm H deviate from region III to region I, it earns no profit in period t and the same profit in the future. Hence, firm H will not deviate from region III to region I. Next we show that firm H cannot deviate profitably from region III to region II. By the optimality of $\hat{m}_{t,H}(\theta_t)$ in region III, firm H has no incentive to deviate from $\hat{m}_{t,H}(\theta_t)$ to the boundary of region II and region III unilaterally. In region II, the margin of H determined by first-order conditions is given by

$$\tilde{m}_{t,H}(\theta_t)=\frac{[1-\beta+X_{t+1}]^2(\theta_t-c)}{2[1-\beta+X_{t+1}-\alpha B_{t+1,H}]}\geq\frac{[1-\beta+X_{t+1}](\theta_t-c)}{2}.$$

On the other hand,

$$\hat{m}_{t,L}(\theta_t)\leq\frac{(1-\beta)X_{t+1}^2(\theta_t-c)}{X_{t+1}^2+2(1-\beta)X_{t+1}}\leq\frac{X_{t+1}(\theta_t-c)}{2}.$$

It then follows that

$$\tilde{m}_{t,H}(\theta_t)\geq\frac{[1-\beta+X_{t+1}]\hat{m}_{t,L}(\theta_t)}{X_{t+1}}.$$

Therefore, the point $(\tilde{m}_{t,H}(\theta_t),\hat{m}_{t,L}(\theta_t))$ lies above region II, implying that the profit of firm H is optimized at the boundary of regions II and III within region II. Combining the arguments above shows that firm H has no incentive to deviate from region III to region II.

Step 3: *Show that solutions in regions* I, II, *and* IV *cannot sustain as Nash equilibria*. The solution in region I cannot be a Nash equilibrium in period t. The reason is the following: firm H can always deviate from region I to region III by lowering its price. Because the marginal valuation, at which a customer is indifferent in purchasing in periods t and $t+1$, is determined by the margin of L in period t in this case, firm H can earn higher profit in period t and the same profit in the future by deviating to region III.

The solution in region II cannot be a Nash equilibrium because firm L can deviate profitably. For a fixed margin of firm H, firm L can always lower its margin and set it according to (21). Because the function (21) is above the boundary of regions II and III (see Figure 1), such a deviated solution always lie in region III. According to the proof in Step 2, firm L earns higher profit by such deviation than staying in region II.

The solution in region IV cannot be a Nash equilibrium. In region IV, both firms have zero sales, thus the number of remaining customers (i.e., state θ_t) is the same as in the next period $t+1$. Because both firms discount profits over time, they would incur sales in period t rather than in period $t+1$ unless there is no sales in the future anymore. But Proposition 1 shows that there are positive sales in the last period. Therefore, no Nash equilibrium exists in region IV by induction.

Summarizing the arguments above completes the proof. □

Lemma 2 is used in the proofs of Proposition 3 and Proposition 4.

Lemma 2. *The unique pure-strategy MPE in a multiperiod game shown in Proposition* 2 *can also be written as*

$$p_{t,H}^*(\theta_t)=\frac{2(1-\beta)A_t}{4A_t-1}(\theta_t-c)+c,$$
$$r_{t,H}^*(\theta_t)=\frac{4(1-\beta)B_t}{(4A_t-1)^2}(\theta_t-c)^2,$$
$$p_{t,L}^*(\theta_t)=\frac{(1-\beta)}{4A_t-1}(\theta_t-c)+\beta c,\quad r_{t,L}^*(\theta_t)=\frac{(1-\beta)A_t}{(4A_t-1)^2}(\theta_t-c)^2,$$

where

$$A_t=1+(4A_{t+1}-1)Y_{t+1}-\alpha A_{t+1}Y_{t+1}^2,\quad \forall\, t=1,\ldots,T-1,\ A_T=\frac{1}{\beta},\tag{22}$$

$$B_t=A_t^2+\alpha B_{t+1}Y_{t+1}^2,\quad \forall\, t=1,\ldots,T-1,\ B_T=\frac{1}{\beta^2},\tag{23}$$

$$Y_t=\frac{1-\beta}{(\beta-\gamma)(4A_t-1)+2\gamma(1-\beta)A_t},\quad \forall\, t=2,\ldots,T.\tag{24}$$

The sequence $\{A_t\}_{t=1}^T$ *is monotone in* t. *In particular, given* β, *there exists* $\hat{\gamma}\in(0,\beta)$ *such that* $\{A_t\}_{t=1}^T$ *is monotonically decreasing in* t *for* $\beta>\gamma\geq\hat{\gamma}$ *and monotonically increasing in* t *for* $0\leq\gamma<\hat{\gamma}$.

Proof. We prove the equilibrium result by induction. First the equilibrium result holds in period T following Proposition 1. For the inductive step, fix $t\leq T-1$ and assume the result holds in period $t+1$.

In period t, suppose the state is θ_t and the price pair is p_t. Let $\hat{\theta}_{t+1}$ be the valuation of a marginal customer who is indifferent between buying product L in period t and buying product H in period $t+1$. Then $\hat{\theta}_{t+1}$ should satisfy

$$\begin{aligned}\beta\hat{\theta}_{t+1}-p_{t,L} &= \gamma(\hat{\theta}_{t+1}-p^*_{t+1,H}(\hat{\theta}_{t+1}))\\ &= \gamma\left(\hat{\theta}_{t+1}-\frac{2(1-\beta)A_{t+1}}{4A_{t+1}-1}(\hat{\theta}_{t+1}-c)-c\right).\end{aligned}$$

Here, we implicitly use the equilibrium result of Proposition 2, in particular that both H and L are offered in each period. It follows that

$$\hat{\theta}_{t+1}=\frac{(4A_{t+1}-1)(p_{t,L}-\beta c)}{(\beta-\gamma)(4A_{t+1}-1)+2\gamma(1-\beta)A_{t+1}}+c.$$

The profit functions in period t are given by

$$r_{t,H}(\theta_t,p_t)=(p_{t,H}-c)\left(\theta_t-\frac{p_{t,H}-p_{t,L}}{1-\beta}\right)+\alpha r^*_{t+1,H}(\hat{\theta}_{t+1}), \quad (25)$$

$$r_{t,L}(\theta_t,p_t)=(p_{t,L}-\beta c)\left(\frac{p_{t,H}-p_{t,L}}{1-\beta}-\hat{\theta}_{t+1}\right)+\alpha r^*_{t+1,L}(\hat{\theta}_{t+1}). \quad (26)$$

Using the expressions of $r^*_{t+1,H}(\hat{\theta}_{t+1})$ and $r^*_{t+1,L}(\hat{\theta}_{t+1})$ from the induction hypothesis, we can show that $r_{t,H}(\theta_t,p_t)$ is concave in $p_{t,H}$, and $r_{t,L}(\theta_t,p_t)$ is concave in $p_{t,L}$. The equilibrium prices can be solved from the first-order conditions $\partial r_{t,H}(p_{t,H},p_{t,L})/\partial p_{t,H}=0$ and $\partial r_{t,L}(p_{t,H},p_{t,L})/\partial p_{t,L}=0$, leading to

$$\begin{aligned}p^*_{t,L}(\theta_t) &= \frac{(1-\beta)(\theta_t-c)}{3+4(4A_{t+1}-1)Y_{t+1}-4\alpha A_{t+1}Y^2_{t+1}}+\beta c\\ &= \frac{(1-\beta)}{4A_t-1}(\theta_t-c)+\beta c,\end{aligned}$$

$$p^*_{t,H}(\theta_t)=\frac{(1-\beta)\theta_t+p^*_{t,L}(\theta_t)+c}{2}=\frac{2(1-\beta)A_t}{4A_t-1}(\theta_t-c)+c,$$

where Y_{t+1} is given in (24).

Substituting the optimal prices $p^*_{t,H}(\theta_t)$ and $p^*_{t,L}(\theta_t)$ into (25) and (26) and simplifying leads to the expressions for $r^*_{t,H}(\theta_t)$ and $r^*_{t,L}(\theta_t)$.

From (22) and (24), we have $A_t=f(A_{t+1})$ for all $t<T$, where

$$f(x)=1+\frac{(4x-1)(1-\beta)}{(\beta-\gamma)(4x-1)+2\gamma(1-\beta)x}-\frac{\alpha(1-\beta)^2x}{[(\beta-\gamma)(4x-1)+2\gamma(1-\beta)x]^2}.$$

It can be shown that $f(x)$ is strictly increasing in x when $(\beta-\gamma)(4x-1)+2\gamma(1-\beta)x>0$.

Now we show the monotonicity of $\{A_t\}_{t=1}^T$ by induction. First, note that $A_{T-1}=f(A_T)=f(1/\beta)$. Hence,

$$A_{T-1}-A_T=\frac{(1-\beta)[\beta\gamma(4-\beta)(2+\beta-3\gamma)-4\gamma^2(1-\beta)^2-\alpha\beta^2(1-\beta)]}{\beta[(\beta-\gamma)(4-\beta)+2\gamma(1-\beta)]^2}.$$

The sign of the difference is determined by the numerator on the right-hand side, which we denote by $h(\gamma)$. It can be shown that $h(\gamma)$ is concave in γ and is maximized at $\gamma^*=\beta(4-\beta)/(2(\beta+2))<\beta$. Furthermore, $h(0)<0$ and $h(\beta)>0$. Therefore, there exists $0<\hat{\gamma}<\gamma^*$ such that $h(\gamma)<0$ for $0\le\gamma<\hat{\gamma}$ and $h(\gamma)\ge 0$ for $\hat{\gamma}\le\gamma\le\beta$, where $\hat{\gamma}$ is the unique solution of $h(\gamma)=0$. Hence, for any given β, $A_{T-1}<A_T$ when $0\le\gamma<\hat{\gamma}$ and $A_{T-1}\ge A_T$ when $\hat{\gamma}\le\gamma\le\beta$. Because $f(x)$ is strictly increasing in x, $A_{T-2}=f(A_{T-1})<f(A_T)=A_{T-1}$ when $A_{T-1}<A_T$ and $A_{T-2}=f(A_{T-1})>f(A_T)=A_{T-1}$ when $A_{T-1}>A_T$. Recursively applying this argument, we conclude that $\{A_t\}_{t=1}^T$ is monotonically increasing in t when $0\le\gamma<\hat{\gamma}$ whereas it is monotonically decreasing in t when $\beta\ge\gamma\ge\hat{\gamma}$. □

Proof of Proposition 3. Part (i): By Lemma 2, the sequence $\{A_t\}_{t=1}^T$ is monotonically increasing in t for small γ and monotonically decreasing in t otherwise. Note that the coefficients of the equilibrium expressions, $2A_t(1-\beta)/(4A_t-1)$ and $(1-\beta)/(4A_t-1)$, both decrease in A_t, and hence the equilibrium prices decrease over time for an increasing sequence of $\{A_t\}_{t=1}^T$.

We next show this is also true when $\{A_t\}_{t=1}^T$ is decreasing in t; that is, $p^*_{t,i}(\theta_t)>p^*_{t+1,i}(\theta^*_{t+1}(\theta_t))$, for $t=1,\ldots,T-1$, $i=H,L$. The valuation $\theta^*_{t+1}(\theta_t)$ is the highest valuation that would purchase in period $t+1$ given the equilibrium price pair $(p^*_{t,H},p^*_{t,L})$ at state θ_t, satisfying $\beta\theta^*_{t+1}(\theta_t)-p^*_{t,L}(\theta_t)=\gamma[\theta^*_{t+1}(\theta_t)-p^*_{t+1,H}(\theta^*_{t+1}(\theta_t))]$, where

$$p^*_{t+1,H}(\theta^*_{t+1}(\theta_t))=\frac{2(1-\beta)A_{t+1}}{4A_{t+1}-1}(\theta^*_{t+1}(\theta_t)-c)+c$$

by Lemma 2. Hence, $\theta^*_{t+1}(\theta_t)$ is determined by

$$\theta^*_{t+1}(\theta_t)=\frac{(4A_{t+1}-1)(p^*_{t,L}(\theta_t)-\gamma c)+2\gamma c(1-\beta)A_{t+1}}{(\beta-\gamma)(4A_{t+1}-1)+2\gamma(1-\beta)A_{t+1}}.$$

For $i=H,L$ and $t=1,\ldots,T-1$, substituting $\theta^*_{t+1}(\theta_t)$ into $p^*_{t+1,i}(\theta^*_{t+1}(\theta_t))$ shows that $p^*_{t,H}(\theta_t)>p^*_{t+1,H}(\theta^*_{t+1}(\theta_t))$ is equivalent to $A_t>A_{t+1}Y_{t+1}$, and $p^*_{t,L}(\theta_t)>p^*_{t+1,L}(\theta^*_{t+1}(\theta_t))$ is equivalent to $Y_{t+1}<1$.

Notice that

$$Y_{t+1}=\frac{1-\beta}{(\beta-\gamma)(4A_{t+1}-1)+2\gamma(1-\beta)A_{t+1}}$$

is decreasing in A_{t+1}. Together with the fact

$$Y_T=\frac{\beta(1-\beta)}{(\beta-\gamma)(4-\beta)+2\gamma(1-\beta)}<1,$$

we can conclude that $Y_{t+1}<1$, for $t=0,\ldots,T-1$.

We next show $A_t>A_{t+1}Y_{t+1}$ by induction. In period $T-1$, we can easily verify $A_{T-1}>A_TY_T$. Assume this also holds in period $t\le T-1$. Because the function $A_{t+1}Y_{t+1}$ decreases in A_{t+1}, thus $A_tY_t<A_{t+1}Y_{t+1}$ as $A_t>A_{t+1}$. Therefore, $A_{t-1}>A_t>A_{t+1}Y_{t+1}>A_tY_t$. The first inequality holds because $\{A_t\}_{t=1}^T$ is decreasing over time, and the second inequality follows from induction hypothesis. This completes the proof of part (i).

Part (ii) is obvious because the coefficients of the equilibrium margins and profits for both firms approach zero as β goes to one. □

Proof of Proposition 4. By Lemma 2, the sequence $\{A_t\}_{t=1}^T$ is monotone in time index t. Thus, for any fixed t, A_t will approach a limit, denoted by A^*, as the number of periods T goes to infinity. And A^* is the unique real-value solution to

$$A = 1 + \frac{(4A-1)(1-\beta)}{2A(2\beta-\gamma-\beta\gamma)-(\beta-\gamma)} - \frac{\alpha A(1-\beta)^2}{[2A(2\beta-\gamma-\beta\gamma)-(\beta-\gamma)]^2}. \quad \square$$

Proof of Proposition 5. The equilibrium results immediately follow from Lemma 2 by setting $\gamma = 0$. Also, by Lemma 2, the sequence $\{\tilde{A}_t\}_{t=1}^T$ increases in t, and hence, the equilibrium prices decrease over time.

We now show that the equilibrium profits also decrease over time. For firm L, because the coefficients of equilibrium profits, $\tilde{A}_t(1-\beta)/(4\tilde{A}_t-1)^2$, decrease in $\tilde{A}_t$, its equilibrium profit decreases in t. We next demonstrate the sequence of equilibrium profits of firm H decreases in t, which is sufficient to show that $\{\tilde{B}_t/(4\tilde{A}_t-1)^2\}_{t=1}^T$ is a decreasing sequence in t by induction. In period $T-1$, $\tilde{A}_{T-1} = 1/\beta - \alpha(1-\beta)^2/(\beta(4-\beta)^2)$, and $\tilde{B}_{T-1} = 1/\beta^2 - (\alpha/\beta^2)((1-\beta)/(4-\beta))^2 + (\alpha^2/\beta^2)((1-\beta)/(4-\beta))^4$; and simple algebraic manipulations yield $\tilde{B}_{T-1}/(4\tilde{A}_{T-1}-1)^2 > \tilde{B}_T/(4\tilde{A}_T-1)^2$. Assume that $\tilde{B}_t/(4\tilde{A}_t-1)^2 > \tilde{B}_{t+1}/(4\tilde{A}_{t+1}-1)^2$, for $t \le T-1$. Now consider

$$\begin{aligned} &\frac{\tilde{B}_{t-1}}{(4\tilde{A}_{t-1}-1)^2} - \frac{\tilde{B}_t}{(4\tilde{A}_t-1)^2} \\ &\quad = \frac{\tilde{A}_{t-1}^2}{(4\tilde{A}_{t-1}-1)^2} - \frac{\tilde{A}_t^2}{(4\tilde{A}_t-1)^2} + \alpha\left(\frac{1}{\beta}-1\right)^2 \\ &\qquad \cdot \left(\frac{\tilde{B}_t}{(4\tilde{A}_t-1)^2(4\tilde{A}_{t-1}-1)^2} - \frac{\tilde{B}_{t+1}}{(4\tilde{A}_{t+1}-1)^2(4\tilde{A}_t-1)^2}\right). \end{aligned}$$

Because both $x^2/(4x-1)^2$ and $1/(4x-1)^2$ decrease in x when $4x-1>0$, together with the fact $\tilde{A}_{t-1} < \tilde{A}_t$ and $\tilde{B}_t/(4\tilde{A}_t-1)^2 > \tilde{B}_{t+1}/(4\tilde{A}_{t+1}-1)^2$ by induction hypothesis, we conclude $\tilde{B}_{t-1}/(4\tilde{A}_{t-1}-1)^2 - \tilde{B}_t/(4\tilde{A}_t-1)^2 > 0$. $\square$

Lemma 3 is used in the proof of Proposition 6.

Lemma 3. *For given p_H, firm L's best response can be reduced to a dynamic program where the state θ_t in period t represents the maximum valuation of remaining customers. In particular, the optimal prices and profits for $t = 2, \dots, T$ are given by*

$$p^*_{t,L}(\theta_t) = C_t(\theta_t - c) + \beta c, \quad r^{\mathrm{HC}}_{t,L}(\theta_t) = \frac{C_t(\theta_t-c)^2}{2}. \tag{27}$$

Proof. For a given p_H, the problem faced by firm L is to maximize $r^{\mathrm{HC}}_L(p_H, \vec{p}_L)$ over $\vec{p}_L$ subject to (3)–(5). This problem is equivalent to a dynamic program defined as follows. For $t = 2, \dots, T$, let the state in period t be θ_t, where $[0, \theta_t]$ is the valuation of remaining customers. Let $r^{\mathrm{HC}}_{t,L}(\theta_t)$ be the maximum profit that can be collected in the remaining periods. Then the dynamic programming equations for $t = 2, \dots, T$ can be written as

$$r^{\mathrm{HC}}_{t,L}(\theta_t) = \max_{p_{t,L}}\{(p_{t,L} - \beta c)(\theta_t - \theta_{t+1}) + \alpha r^{\mathrm{HC}}_{t+1,L}(\theta_{t+1})\}, \tag{28}$$

$$\text{s.t. } \beta\theta_{t+1} - p_{t,L} = \begin{cases} \gamma(\beta\theta_{t+1} - p^*_{t+1,L}(\theta_{t+1})) \\ \qquad \text{if } t = 2, \dots, T-1, \\ 0 \quad \text{if } t = T. \end{cases} \tag{29}$$

Here, $p^*_{t+1,L}(\cdot)$ is the optimal policy in period $t+1$. The boundary conditions are given by $r^{\mathrm{HC}}_{T+1,L}(\theta_{T+1}) = 0$ for all θ_{T+1}. The payoff function of L, $r^{\mathrm{HC}}_L(p_H, \vec{p}_L)$, i.e., the profit earned over the entire time horizon, can be rewritten as $r^{\mathrm{HC}}_L(p_H, \vec{p}_L) = (p_{1,L} - \beta c)(\theta_1 - \theta_2) + \alpha r^{\mathrm{HC}}_{2,L}(\theta_2)$, where $\theta_1 = (p_H - p_{1,L})/(1-\beta)$ and θ_2 satisfies $\beta\theta_2 - p_{1,L} = \gamma(\beta\theta_2 - p^*_{2,L}(\theta_2))$.

We solve firm L's dynamic program by backward induction. First consider the problem in period T with state θ_T. It can be shown that $p^*_{T,L}(\theta_T) = \beta(\theta_T - c)/2 + \beta c$ and $r^{\mathrm{HC}}_{T,L}(\theta_T) = \beta(\theta_T - c)^2/4$.

Now, assume firm L's optimal price in period t and associated profit from period $t+1$ to period T are given in (27). From (29), $\theta_{t+1} = (p_{t,L} - \gamma c(\beta - C_{t+1}))/(\beta - \gamma(\beta - C_{t+1}))$. Substituting it into (28), we have

$$\begin{aligned} r^{\mathrm{HC}}_{t,L}(\theta_t) = \max_{p_{t,L}} \Bigg\{ &(p_{t,L} - \beta c)\left(\theta_t - \frac{p_{t,L} - \gamma c(\beta - C_{t+1})}{\beta - \gamma(\beta - C_{t+1})}\right) \\ &+ \frac{\alpha C_{t+1}}{2}\left(\frac{p_{t,L} - \beta c}{\beta - \gamma(\beta - C_{t+1})}\right)^2\Bigg\}. \end{aligned}$$

We can easily check that the objective function on the right-hand side above is strictly concave and hence the optimal price $p^*_{t,L}(\theta_t)$ can be determined by first-order condition, leading to the optimal price and associated profit in (27). $\square$

Proof of Proposition 6. Using Lemma 3, we have

$$\begin{aligned} r^{\mathrm{HC}}_L(p_H, \vec{p}_L) = (p_{1,L} - \beta c)&\left(\frac{p_H - p_{1,L}}{1-\beta} - \frac{p_{1,L} - \gamma c(\beta - C_2)}{\beta - \gamma(\beta - C_2)}\right) \\ &+ \frac{\alpha C_2}{2}\left(\frac{p_{1,L} - \beta c}{\beta - \gamma(\beta - C_2)}\right)^2. \end{aligned}$$

Note that $r^{\mathrm{HC}}_L(p_H, \vec{p}_L)$ is a function of p_H and $p_{1,L}$ only. Solving first-order conditions leads to the equilibrium prices p^*_H and $p^*_{1,L}$. Expressions (7) and (8) can be obtained recursively, noting that $p^*_{t,L}$ corresponds to the prices charged on the equilibrium path. Expressions for equilibrium profits for H and L follow immediately. $\square$

Proof of Proposition 7. For given p_L, the problem faced by firm H is to maximize $r^{\mathrm{LC}}_H(\vec{p}_H, p_L)$ over $\vec{p}_H$, which is equivalent to a dynamic program. For $t = 1, \dots, T$, let the state in period t be θ_t, the maximum valuation of remaining customers in period t; let $r^{\mathrm{LC}}_{t,H}(\theta_t)$ be the maximum profit that can be collected in the remaining periods.

To solve the problem, we first consider the problem for $t \ge 2$. The optimality equations for $t \ge 2$ can be written as

$$r^{\mathrm{LC}}_{t,H}(\theta_t) = \max_{p_{t,H}}\{(p_{t,H} - c)(\theta_t - \theta_{t+1}) + \alpha r^{\mathrm{LC}}_{t+1,H}(\theta_{t+1})\},$$

$$\text{s.t. } \theta_{t+1} - p_{t,H} = \begin{cases} \gamma(\theta_{t+1} - p^*_{t+1,H}(\theta_{t+1})) & \text{if } 2 \le t \le T-1, \\ 0 & \text{if } t = T. \end{cases} \tag{30}$$

Here, $p^*_{t+1,H}(\cdot)$ is the optimal policy in period $t+1$. The boundary conditions are given by $r^{\mathrm{LC}}_{T+1,H}(\theta_{T+1}) = 0$ for all θ_{T+1}.

We now solve the problem for $t \ge 2$ by induction. It can be easily verified that $p^*_{T,H}(\theta_T) = E_T(\theta_T - c) + c$ and $r^{\mathrm{LC}}_{T,H}(\theta_T) = E_T(\theta_T - c)^2/2$. Suppose that for $t \le T-1$, we have $p^*_{t+1,H}(\theta_{t+1}) = E_{t+1}(\theta_{t+1} - c) + c$ and $r^{\mathrm{LC}}_{t+1,H}(\theta_{t+1}) = E_{t+1}(\theta_{t+1} - c)^2/2$. Then, by replacing θ_{t+1} derived from (30), we obtain

$$r^{\mathrm{LC}}_{t,H}(\theta_t) = \max_{p_{t,H}} \left\{ (p_{t,H}-c)\left(\theta_t - c - \frac{p_{t,H}-c}{1-\gamma+\gamma E_{t+1}}\right) + \frac{\alpha E_{t+1}}{2}\left(\frac{p_{t,H}-c}{1-\gamma+\gamma E_{t+1}}\right)^2 \right\}.$$

Because the objective function on the right-hand side is concave quadratic in $p_{t,H}$, the maximization can be solved from first-order conditions on $p_{t,H}$, leading to $p^*_{t,H}(\theta_t) = (1-\gamma+\gamma E_{t+1})^2(\theta_t - c)/(2(1-\gamma)+(2\gamma-\alpha)E_{t+1}) + c = E_t(\theta_t - c) + c$. The corresponding expression for the value function is $r^{\mathrm{LC}}_{t,H}(\theta_t) = E_t(\theta_t - c)^2/2$.

Next we turn the problem in period 1. A customer purchases H in period 1 if $\theta - p_{1,H} \geq \beta\theta - p_L$, i.e., $\theta \geq (p_{1,H} - p_L)/(1-\beta)$. On the other hand, a marginal customer who is indifferent between purchasing L in period 1 and purchasing H in period 2 must have valuation θ such that

$$\beta\theta - p_L = \gamma(\theta - p^*_{2,H}(\theta)) = \gamma(\theta - E_2(\theta - c) - c)$$
$$\Leftrightarrow \quad \theta = \frac{p_L - \beta c}{\beta - \gamma(1-E_2)} + c.$$

Therefore, the payoff functions can be written as

$$r^{\mathrm{LC}}_{1,H}(\vec{p}_H, p_L) = (p_{1,H}-c)\left(1 - \frac{p_{1,H}-p_L}{1-\beta}\right) + \frac{E_2}{2}\left(\frac{p_L - \beta c}{\beta - \gamma(1-E_2)}\right)^2,$$
$$r^{\mathrm{LC}}_{L}(\vec{p}_H, p_L) = (p_L - \beta c)\left(\frac{p_{1,H}-p_L}{1-\beta} - \frac{p_L - \beta c}{\beta - \gamma(1-E_2)} - c\right).$$

Solving first-order conditions leads to the equilibrium prices in period 1. The expression for θ^*_t can be determined recursively. This completes the proof. □

References

Aviv Y, Pazgal A (2008) Optimal pricing of seasonal products in the presence of forward-looking consumers. *Manufacturing Service Oper. Management* 10(3):339–359.

Besanko D, Winston WL (1990) Optimal price skimming by a monopolist facing rational customers. *Management Sci.* 36(5):555–567.

Bulow JI (1982) Durable-goods monopolists. *J. Political Econom.* 90(2):314–332.

Butz DA (1990) Durable-good monopoly and best-price provisions. *Amer. Econom. Rev.* 80(5):1062–1076.

Cachon GP, Feldman P (2010a) Dynamic versus static pricing in the presence of strategic customers. Working paper, University of Pennsylvania, Philadelphia.

Cachon GP, Feldman P (2010b) Is advance selling desirable with competition? Working paper, University of Pennsylvania, Philadelphia.

Cachon G, Swinney R (2009) Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Sci.* 55(3):497–511.

Coase RH (1972) Durability and monopoly. *J. Law Econom.* 15(1):143–149.

Dasu S, Tong C (2010) Dynamic pricing when consumers are strategic: Analysis of posted and contingent pricing schemes. *Eur. J. Oper. Res.* 204(3):662–671.

Fudenberg D, Tirole J (1991) *Game Theory* (MIT Press, Cambridge, MA).

Gallego G, Hu M (2006) Dynamic pricing of perishable assets under competition. Working paper, Columbia University, New York.

Gallego G, Phillips R, Şahin Ö (2008) Strategic management of distressed inventory. *Production Oper. Management* 17(4):402–415.

Jerath K, Netessine S, Veeraraghavan SK (2010) Revenue management with strategic customers: Last-minute selling and opaque selling. *Management Sci.* 56(3):430–448.

Lai G, Debo LG, Sycara K (2010) Buy now and match later: The impact of posterior price matching on profit with strategic consumers. *Manufacturing Service Oper. Management* 12(1):33–55.

Lazear EP (1986) Retail pricing and clearance sales. *Amer. Econom. Rev.* 76(1):14–32.

Levin Y, McGill J, Nediak M (2009) Dynamic pricing in the presence of strategic consumers and oligopolistic competition. *Management Sci.* 55(1):32–46.

Lin KY, Sibdari SY (2009) Dynamic price competition with discrete customer choices. *Eur. J. Oper. Res.* 197(3):969–980.

Liu Q, van Ryzin GJ (2008) Strategic capacity rationing to induce early purchases. *Management Sci.* 54(6):1115–1131.

Martínez-de-Albéniz V, Talluri K (2011) Dynamic price competition with fixed capacities. *Management Sci.* 57(6):1078–1093.

Motta M (1993) Endogenous quality choice: Price vs. quantity competition. *J. Indust. Econom.* 41(2):113–131.

Mussa M, Rosen S (1978) Monopoly and product quality. *J. Econom. Theory* 18(2):301–317.

Netessine N, Tang CS (2009) *Consumer-Driven Demand and Operations Management Models* (Springer, New York).

O'Donnell J (2006) Retailers try to train shoppers to buy now. *USA Today* (September 25), http://www.usatoday.com/money/industries/retail/2006-09-25-sales-shopping-usat_x.htm.

Ovchinnikov A, Milner JM (2012) Revenue management with end-of-period discounts in the presence of customer learning. *Production Oper. Management* 21(1):69–84.

Parlaktürk AK (2012) The value of product variety when selling to strategic consumers. *Manufacturing Service Oper. Management* 14(3):371–385.

Parlaktürk A, Kabul MO (2010) The value of commitments when selling to strategic customers: A supply chain perspective. Working paper, University of North Carolina at Chapel Hill, Chapel Hill.

Perakis G, Sood A (2006) Competitive multi-period pricing for perishable products: A robust optimization approach. *Mathematical Programming* 107(1):295–335.

Rozhon T (2003) Clothing retailers hope shoppers take holiday prices at face value. *New York Times* (November 22), http://www.nytimes.com/2003/11/22/business/clothing-retailers-hope-shoppers-take-holiday-prices-at-face-value.html.

Shen M, Su X (2007) Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production Oper. Management* 16(6):713–728.

Stokey N (1979) Intertemporal price discrimination. *Quart. J. Econom.* 93(3):355–371.

Su X, Zhang F (2008) Strategic customer behavior, commitment, and supply chain performance. *Management Sci.* 54(10):1759–1773.

Tirole J (1988) *The Theory of Industrial Organization* (MIT Press, Cambridge, MA).

Xu X, Hopp WJ (2006) A monopolistic and oligopolistic stochastic flow revenue management model. *Oper. Res.* 54(6):1098–1109.

Yin R, Aviv Y, Pazgal A, Tang C (2009) Optimal markdown pricing: Implications of inventory display formats in the presence of strategic customers. *Management Sci.* 55(8):1391–1408.