United We Stand or Divided We Stand? Strategic Supplier Alliances under Order Default Risk

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We study the alliance formation strategy among suppliers in a one downstream firm-n upstream suppliers framework. Each supplier faces an exogenous random shock that may result in an order default. Each of them also has access to a recourse fund that can mitigate this risk. The suppliers can share the fund resources within an alliance, but need to equitably allocate profits of the alliance among the partners. In this context, suppliers need to decide whether to join larger alliances that have better chances of order fulfillment or smaller ones that may grant them higher profit allocations. We first analytically characterize the exact coalition-proof Nash-stable coalition structures that would arise for symmetric complementary or substitutable suppliers. Our analysis reveals that it is the appeal of default risk mitigation, rather than competition-reduction, that motivates cooperation. In general, a more risky and/or less fragmented supply base favors larger alliances, whereas substitutable suppliers and customer demands with lower pass-through rates result in smaller ones. We then characterize the stable coalition structures for an asymmetric supplier base. We establish that grand coalition is more stable when the supplier base is more homogenous in terms of their risk levels, rather than divided among few highly risky suppliers and other low-risk ones. Going one step further, our investigation of endogenous recourse fund levels for the suppliers demonstrates how financing costs affect suppliers investment in risk-reducing resources, and consequently their coalition formation strategy. Lastly, we discuss model generalizations and show that, in general, our insights are quite robust.

Key words: Cooperation, Competition, Supply risk, Order default, Coalition stability, Supplier alliances

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1. Introduction

Strategic alliances, whereby independent but cooperating organizations pool specific resources and skills in order to achieve common and individual goals, have emerged as a popular strategy in the business world (Varadarajan and Cunningham 1995). While such arrangements can be between horizontal and/or vertical partners, we focus on horizontal alliances, which we will also term as "coalitions" interchangeably throughout the paper. Horizontal alliances are observed between complementary firms as well as between competitors selling substitutable products. Examples of the former include the ones between Caterpillar and Mitsubishi in the earthmoving equipment sector, and among component suppliers in automobile and electronics sectors (Nagarajan and Sošić 2009). On the other hand, substitutable-product coalitions include Renault and Nissan in the automobile industry (Yoshino and Fagan 2003), Takeda and Hoechst in the pharmaceutical sector (Garella and Peitz 2007), and collaborative organizations in marine transportation (Girotra and Netessine 2014) and agricultural sectors (Oxfam International 2010).

There is considerable amount of literature, especially in the strategy and organization area, analyzing alliances from multiple perspectives. They deal with issues like alliances' governance structures (Gulati and Singh 2008) as well as effects of alliances on firm performance (Singh and Mitchell 2005), innovation rate (Stuart 2000), knowledge transfer (Mowery et al. 1996), market access (Varadarajan and Cunningham 1995) and bargaining power (Hamel 1991). But, extant practitioner/academic literature suggests that an important reason behind alliance formation might be to deal with external business risks (e.g., Girotra and Netessine 2014, Oxfam International 2010). This risk-mitigating role is particularly relevant for supply chains given the exogenous perils they face from demand and/or supply side. Indeed, a number of operations management (OM) papers have studied coalition formation in the presence of demand-side risk (refer to §2 for details).

However, longer and more decentralized value chains are now increasingly exposing supply-side risks. Such risks put suppliers' order fullfillment ability in jeopardy and range from relatively minor ones (e.g., due to minor maintenance or inventory problems) to really catastrophic ones like frost wiping out most of California's citrus crops (Rimal and Schmitz 1999), the recent earthquake and tsunami disrupting supply from Japan (NYTimes 2011), and the default of more than 10,000 factories in China during the financial crisis of 2008 (USAToday 2008) (refer to Lynch 2011 for more examples). Different strategies have been proposed to deal with supply risks including diversification (Babich et al. 2007), subsidies (Wang et al. 2010, Babich 2010, Wadecki et al. 2012), guarantees (Gümüs et al. 2012), and contracting (Swinney and Netessine 2009, Yang et al. 2009).

A number of firms, in industries as diverse as marine transportation and agriculture, have also started using "alliances" to deal with supply risks. For instance, Tankers International (TI) has developed a commercial shipping alliance whereby they charter a pool of VLCCs (Very Large Crude Carriers) from individual owners and use it to deliver crude oil to refineries. Such pools of (substitutable) tankers help to deal with order default risks caused by weather problems, refinery closings, maintenance and port issues - all common problems in commercial maritime transportation (Girotra and Netessine 2014). TI acts as a single entity that makes the aggregate capacity available to customers, collects earnings from transportation activities and distributes them among individual owners under a pre-arranged allocation system (Packard 1989, Haralambides 1996). In the agriculture industry, producers selling substitutable and/or complementary products to retailers also form alliances, e.g., Farmer Producer Organizations (FPOs) in India. One of the main rationale for this is to share supply chain risk-management funds among partners to deal with fulfillment risks arising from factors like bad weather, transport losses and difficulty in accessing capital, and then to properly share the gains (Oxfam International 2010, Government of India 2013).

Although the above implies that alliances could be an effective measure in dealing with the risk of defaulting on a supply contract, firms are aware that such partnerships require equitable sharing of risks and benefits, and consequently are not easy to sustain (see Oxfam International 2010, Haralambides 1996). Interestingly, analytical investigation as to what types of stable coalitions (so that there is no incentive for partners to profitably deviate) will develop in the presence of such risk is sparse in the academic literature. This paper attempts to address this gap via a cooperative model framework. In particular, the model will capture three salient features of the above TI and FPO examples. First, exogenous events may result in order default risks on the part of the suppliers. Second, risk-mitigating resources can be pooled by suppliers to overcome such disruptions. Third, cohesive entities formed purely by self-incentivized individuals (i.e., supplier alliances) can make coordinated decisions and distribute earnings among their members based on certain allocation scheme.

We consider a bi-level supply chain model framework composed of n upstream suppliers and one downstream firm (henceforth referred to by masculine and feminine pronouns, respectively). The downstream firm faces a price-sensitive, deterministic demand that she needs to satisfy by procuring the required components - complements or substitutes - from the suppliers. On the other hand, there is an exogenous random shock faced by each upstream supplier that exposes him to the risk of complete order default. However, he also has access to a fund reserved by him ex-ante for risk-mitigation. Ex-post, this fund can be used as a recourse to deal with the shock and supply the entire order, as long as the value of the shock is lower than the reserve amount (otherwise, he still defaults). It is costly for the supplier to operate the reserve fund, and the fund is generic and liquid enough (e.g, cash) to be shared with other suppliers. The reserve amount and default risk are inversely related - the higher is the amount accessible to a supplier, the lower is his effective default risk, although the risk-mitigating benefit shows diminishing marginal returns. In this paper, we will use the fund amount and default risk interchangeably, keeping in mind their inverse relationship.

The suppliers first decide on their *cooperative* alliance structures by determining whether to join an alliance, and, if so, with how many other partners. Since the reserve funds are shareable, the alliance partners can pool them. The relative values of the total shock facing the supplier alliance and its total reserve amount then determines whether it will fully default or deliver the whole order (like in the individual supplier case above). Each alliance then announces its supply limit for the downstream firm and *competes* horizontally via wholesale prices. Subsequently, the downstream firm decides on the order quantity for each alliance and on the retail price. Finally, default risks and consequent profits (if any) are realized for the alliances. Each alliance then divides its profit among the partners following a pre-determined allocation rule that is proportional to the share of each partner in the pooled fund. We use the above framework to address the following.

- What is the equilibrium stable alliance that would arise under the risk of order default?
- What factors incentivize the suppliers to opt for larger (or smaller) alliances?
- How robust are these results with respect to model assumptions? For example:
  — What if the suppliers need to decide how much to ex-ante invest in the reserve fund?
  — What if the profit allocation is not proportional to the partners' shares in the pooled fund?

We first focus on the case of suppliers who are symmetric in terms of their risk (i.e., fund) levels and analytically characterize the number of stable alliances and their sizes through coalition-proof Nash equilibrium technique. While making partnership decisions, the suppliers need to trade-off the benefits of joining larger alliances that result in lower probability of order default against the benefits of greater profit allocation in smaller alliances. Alliance literature in supply chain management area until now focused primarily on the second factor and hence suppliers usually end up forming small coalitions (see Yin 2010 for discussion). In contrast, by incorporating supply-side risks and possible order default, we are able to identify a diverse set of stable coalition structures, both large and small, depending on the business environment.

Specifically, we identify a novel *risk-adjusted stability factor*, which encapsulates the characteristics of both the supply base and customer demand, to determine the stable alliances. Analysis of this factor shows that, in general, larger alliances (including a grand coalition of all n suppliers) are more likely to be formed when: i) the supplier base is more risky and/or relatively small, ii) suppliers are complementary, and iii) retail price is more sensitive to wholesale prices (higher pass through rates). On the other hand, antithetical business conditions (e.g., less risky suppliers or lower pass through rate) result in smaller alliances and might even incentivize suppliers to operate alone. Intuitively, one would expect that suppliers would prefer larger alliances since this reduces the competition within the supply base. However, this argument does not take into account the inherent difficulty in keeping larger alliances stable. Interestingly, our results show that it is *risk-reduction* through fund sharing that serves as the glue holding alliances together, rather than competition reduction through collaborative decision-making.

Subsequently, we generalize the above model to account for an asymmetric supplier base consisting of certain more risky suppliers and some less risky ones, and once again analytically characterize the sizes of stable coalitions. This characterization requires a modification of the stability factor to account for the asymmetry. Our previous insights still remain valid; in addition, we show that a more homogenous (resp., heterogenous) supplier base results in larger (resp., smaller) alliances.

Lastly, we test the robustness of the above qualitative insights through multiple generalizations of our modeling framework. First, we make the decision to invest in reserve fund to be endogenous, i.e., suppliers decide on their coalition partners as well as how much they want to invest in risk-reducing, but costly, reserves. If investment is quite costly, suppliers would like to take advantage of risk reduction through fund sharing by forming large alliances, whereas if the investment is cheap, they would like to go alone in order to have a higher profit allocation. The other two generalizations address: i) a profit allocation mechanism that is not proportional to shares in the pooled fund, and ii) a non-trivial default premium for the downstream firm in case of an order default. The main insight of the first generalization is that larger alliances are sustained for relatively fair allocations. Regarding the second generalization, all our previous insights hold as long as the premium is not too high. As one would expect, a higher default premium increases the sizes of the coalitions.

As regards the rest of the paper, §2 discusses the related literature, while §3 presents our basic modeling framework with exogenous reserve funds. The operational decisions are analyzed in §4, and §5 deals with alliance formation decisions for both symmetric and asymmetric supplier bases. §6 studies the three model generalizations. The concluding discussion is provided in §7.

2. Literature Review

There are two streams of literature most directly related to our work: research dealing with coalition formation but where supply side risk (or resource availability to reduce such risk) is not considered, and research studying measures to counteract supply default risk, but where suppliers only compete with each other (i.e., without any consideration for cooperation).

Our modeling framework of multiple-suppliers-one-downstream-firm channel has a long history in operations literature. Papers in this area traditionally had a competitive focus, e.g., Wang and Gerchak (2003), Jiang and Wang (2010) for complementary suppliers, Bernstein and Federgruen (2005), Yang et al. (2012) for substitutable suppliers, and Netessine and Zhang (2005) for both types of suppliers, to name a few. In these papers, suppliers make individual decisions that maximize their own profits taking into account the responses of competing firms. The possibility for suppliers to communicate and jointly set their prices and/or quantities, capacities and such, is not considered.

In recent years, a line of research studying the coalition structures that could arise among collaborating suppliers has emerged. For example, Nagarajan and Sošić (2007) investigate the stability of coalitions among suppliers selling substitutable products in a dynamic setting. Suppliers are assumed to be farsighted and take into account possible future defections when making any immediate decision. On the other hand, Nagarajan and Bassok (2008) study coalition stability among complementary suppliers when they can negotiate with the downstream assembler about profit allocations. They find that grand coalition (resp., no coalition) will emerge if the bargaining power of the assembler is weak (resp., strong). Also in the context of assembly systems, Granot and Yin (2008) find that coalitions are more likely to be formed in a pull system than a push one, and in the latter case whether the suppliers will form grand coalition or act independently depends on their perspective about cooperation (farsighted or myopic). Nagarajan and Sošić (2009) consider three modes of competition among complementary suppliers, and analyze stable coalitions as a function of power structure, demand structure, and the number of suppliers. Under a similar framework, Sošić (2011) studies the impact of demand uncertainty on the alliance structures. Lastly, for a quite general market condition, Yin (2010) explicitly characterizes stable coalition structures in assembly systems, and their dependence on demand conditions.

Note that all of the above papers deal only with demand-side risks. In general, the incentive for coalition formation in this literature has been attributed to the channel/market structure (Granot and Yin 2008, Nagarajan and Sošić 2009, Yin 2010), bargaining power (Nagarajan and Bassok 2008), the cooperative perspective of the players (Nagarajan and Sošić 2007, Granot and Yin 2008, Nagarajan and Sošić 2009) and the nature/extent of demand uncertainty (Yin 2010, Sošić 2011). We follow this literature by also investigating how the stability of coalitions among suppliers is affected by various business conditions, but complement it by showing that the possibility of order default risk itself can also be a significant incentive behind cooperation.

As regards the second stream, there is a vast literature related to exogenous supply risks. Although our paper deals with both minor and major supply shocks, it particularly emphasizes order default/disruption risk because of which a buyer may not receive anything from her suppliers (refer to Kleindorfer and Saad 2013 and Sodhi et al. 2011 for reviews about supply risk in general). Previous studies have taken a wide angle regarding this issue. Analyzing from the buyer's perspective, Tomlin (2006) considers several mitigation measures and contingency tools to hedge against a variety of disruptions. Babich et al. (2007) and Chopra et al. (2007) investigate the impact of risk correlation and the type of risk (recurring or disruption) facing the supplier community on optimal sourcing diversification decisions, respectively. Swinney and Netessine (2009) analyze the value of long-term vs. short-term contracts in the presence of a default risk, and Yang et al. (2009) derives the optimal contract when suppliers hold private information about their reliability. Chaturvedi and Martínez-de Albéniz (2011) extend Yang et al. (2009) by also including supplier's cost as private information. Lastly, Saghafian and Van Oyen (2012) show the value of having a flexible backup supplier in the presence of disruption risk and discuss capacity reservation issues in that context. Looking from the suppliers' perspective, Gümüs et al. (2012) study the impact of guarantees on risk mitigation and the ability of suppliers to signal their true risk levels. Wei et al. (2013) discuss the implications of default risk coming from uncertain market prices or valuations, when the buyer can use vertical subsidy as a strategic measure. There is also a vast OM literature exploring the impact of yield risk on operational decisions. Among the recent ones, Tomlin (2009), Kazaz and Webster (2011) and Gurnani et al. (2012) investigate the effects of learning, yield-dependent cost structure, and information asymmetry on operational/marketing decisions, respectively. In general, the papers in this research stream have had either a centralized or a competitive focus. To the best of our knowledge, the current paper is among the first that considers supply default risk in a cooperative context, and is able to establish its role in suppliers' coalition formation decisions.

Note that, although our focus is on horizontal collaboration, there is a rich stream of literature dealing with vertical collaboration (refer to Paulraj et al. 2008, Kim and Netessine 2013). There are also two other growing streams that are in spirit related to our work: i) Empirical analysis of the causes of horizontal alliance formation (e.g., Li and Netessine 2011), and ii) horizontal mergers in supply chains (e.g., Cho 2014). However, these streams differ from this paper in terms of methodology as well as model setting.

3. Model Framework

Consider a supply chain with a single downstream firm procuring components from n upstream suppliers and selling a final product to end consumers. Let $N = \{1, 2, \ldots, n\}$ denote the set of n suppliers. In our setting, the components can either be complements or substitutes. If they are *complements*, the final product is an assembly consisting of one component each from the n suppliers; if they are *substitutes*, the final product consists of only one component available from any of the n suppliers. The downstream firm - an assembler or a buyer depending on the component type - acquires the components and then (costlessly) assembles/produces the end product to satisfy customer demand (refer to Figure 1). Below, we describe the salient features of the stakeholders in our supply chain. A glossary of notations is provided in Table A1 of the Appendix.

Downstream firm: We model the end product demand facing the downstream firm as a price-sensitive deterministic function $D(p)$, where p is the retail price set by the firm. We assume that $D(p)$ is positive and decreasing in p, and its price elasticity satisfies the following form:

$$\eta(p) = -\frac{D'(p)}{D(p)/p} = \frac{p}{\alpha + \beta p}, \tag{1}$$

where $\alpha \geq 0$ and $\beta \leq 1$ (as in Song et al. 2007). Note that the class of demand functions that satisfy (1) is quite general and subsumes most of the specific demand forms assumed in the related literature, such as iso-elastic (Wang 2006), linear (Nagarajan and Sošić 2007, Nagarajan and Sošić 2009) as well as linear-power and exponential (Yin 2010); see Table A2 in the Appendix for details.

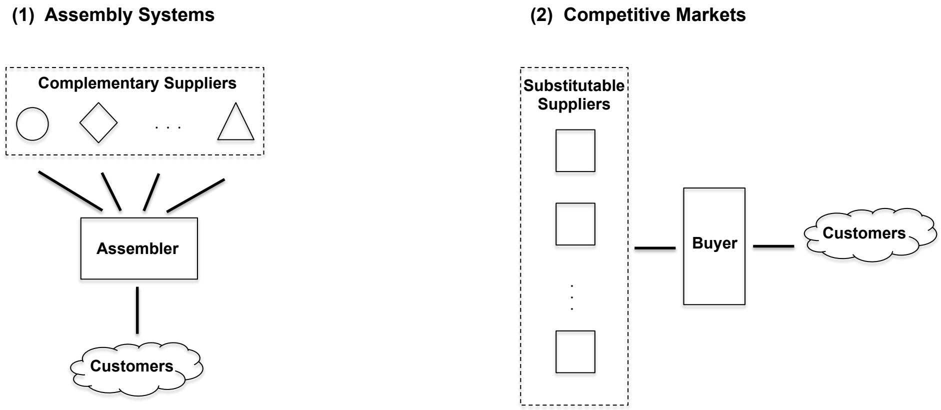

Figure 1 **Channel Structures with Complementary and Substitutable Suppliers**

Upstream suppliers: The n upstream suppliers in our model decide whether to operate independently or join a coalition. Each coalition acts as one entity in the context of our interest where the partners coordinate their pricing and capacity decisions. This aligns with the description of shipping pools in Haralambides (1996), and that of certain FPOs with the farmers as the shareholders (Government of India 2013). So, all upstream entities, independent suppliers or coalitions, set a single wholesale price and a single capacity limit that is available for the buyer. The buyer then decides on the order quantities from each entity (and the retail price, as described above).

Each of the n suppliers is also subject to an exogenous environmental shock. These shocks open them up to the risk of entirely defaulting on the buyer's order. For example, the buyer might specify in the contract that she requires the whole order by a certain date, and if it is not available by then, she will procure it from an outside source, and not accept supply of a partial order. Some of the shocks, e.g., equipment problem, might be small in impact, whereas some others, e.g., natural disasters and financial crisis, might be catastrophic. The suppliers then require funds to recover to their normal state from the shock. We use the random variable ξ_i to denote the shock, or, equivalently, the (minimum) amount of funds needed by supplier i to recover from the shock and fully satisfy the buyer's contract (Lynch 2011, SCDigest 2012). We assume that $\{\xi_i\}_1^n$ are i.i.d. with c.d.f. $G(\cdot)$ with $E[\xi_i] = \mu$.

Furthermore, suppose that each supplier has ex-ante reserved a risk-management fund F_i that he can use as a recourse for the purpose of recovery after a shock (SCDigest 2012). As long as the reserve amount is more than the funds needed to recover, the supplier can fulfill the entire order ("survive") and gain the associated profits. Otherwise, he "defaults", earning zero profits as well as incuring the monetary loss from the shock. The supplier also incurs costs to maintain the fund ($c_f(F_i)$), irrespective of the magnitude of the shock. The above principle holds true even for a coalition after accounting for the pooling of reserve funds and shocks for the partners (see below). We model the net cash flow of a supply entity, a single supplier or a coalition, as follows:

$$\text{Net cash flow for a supply entity} = \overbrace{-\text{Operating Expenses of Reserve Fund}}^{\text{Cost of ex-ante recourse investment}} \\ + \\ \underbrace{\begin{cases} -\text{ Shock} + \text{ Profits} & \text{if Shock} \leq \text{Reserve Fund} \quad \text{(survive state)} \\ -\text{ Shock} & \text{if Shock} > \text{Reserve Fund} \quad \text{(default state)} \end{cases}}_{\text{Ex-post net earnings}} \tag{2}$$

The above cash flow model follows Swinney and Netessine (2009). Similar to their paper, we assume that (existing capital + loans − interest payment − fixed operating expenses=0) for activities beyond our context, and only the damage ξ_i (loss on existing capital), the reserve fund operating cost $c_f(F_i)$ (operating expenses) and survival-state profit (revenue minus production expenses) are considered in cash flows. Unlike Swinney and Netessine (2009), however, suppliers or coalitions in our paper are all self-sustained, i.e., no external creditor or lender is involved in financing. Thus a positive/negative cash flow implies an increase/reduction in existing equity, respectively. Next we describe the expected cash flows for an individual supplier and that of a coalition in more details.

Individual supplier: Following (2), if an individual supplier's reserve fund is more than the shock (i.e., $\xi_i < F_i$), he delivers the full order as per contract and receives the profit associated with the order, π_i. Otherwise, if $\xi_i \geq F_i$, then he defaults, i.e., his profit from the order is zero. Thus, the *expected net cash flow for supplier* $i = -c_f(F_i) - \mathbf{E}[\xi_i] + G(F_i)\pi_i$.

Coalitions: Suppose a set of suppliers $S \subseteq N$ forms a coalition. Recall that we focus on risks that would inhibit a coalition from fulfilling even a partial order. Since the coalition acts as one entity, the reserve funds of the partners are pooled ($F_S = \sum_{i\in S} F_i$) and the coalition faces the total exogenous shock of its partners ($\xi_S = \sum_{i\in S} \xi_i$). Denote by $|S|$ the number of suppliers in S, and by $G_{|S|}(\cdot)$ the c.d.f. of $\sum_{i\in S} \xi_i$. If the coalition can mitigate the damage from the total shock by using the pooled fund, i.e., if $\xi_S \leq F_S$, then it survives and receives the payment for the order from the downstream firm. This happens with probability $G_{|S|}(F_S)$. Otherwise, if $\xi_S > F_S$, the coalition defaults, which takes place with probability $\bar{G}_{|S|}(F_S) = 1 - G_{|S|}(F_S)$, and receives no payment. In other words, in order for a coalition to fail, the amount of shock that they collectively face needs to be more than their total reserve funds, irrespective of whether individual shocks are more or less than individual reserve funds. Following the same logic as before, *expected net cash flow for the coalition* $= -\sum_{i\in S} c_f(F_i) - \xi_S + G_{|S|}(F_S)\pi_S$.

Since suppliers' contribution to the pooled fund F_i's directly affect the coalition's ability to deliver the order ($G_{|S|}(\sum_{i\in S} F_i)$), we allocate the expected profit for the coalition among the partners proportional to these contributions. This mechanism has real-life support in the weighing system used for shipping pools (Haralambides 1996[1]). We also discuss more general profit allocation rules in §6. Note that, although pooled funds can be transferred among partners for recovery purpose providing increased accessibility to resources, each supplier retains the financial ownership of his reserve fund throughout the process; so, the reserve fund amount itself does not need to be included in the cash flows for individual suppliers. However, the operating cost of maintaining the reserve fund, $c_f(F_i)$, is a sunk cost borne by individual suppliers and needs to be accounted for. In summary, given a structure of m coalitions $\{S_1, S_2, ..., S_m\}$ among n suppliers, where $\bigcup_{k=1}^m S_k = N$ and $S_k \cap S_{k'} = \emptyset$ for any $1 \leq k < k' \leq m$,

$$\begin{aligned}&\text{Expected net cash flow for supplier } i \text{ in coalition } k = -\text{Operating expenses } c_f(F_i)\\&-\text{Expected shock } \mathbf{E}[\xi_i] + \frac{F_i}{\sum_{j\in S_k} F_j} \times (\text{Expected profit for coalition } k).\end{aligned} \tag{3}$$

Each supplier will independently make his expected-cash-flow-maximizing decision about whether to join a coalition, and if so with how many partners, based on the above expression.

There are two other elements of the model - a technical assumption about the survival probability of an entity and the sequence of game among the stakeholders - that we discuss below. **Assumption:** $G_l(F)$ represents the survival probability for a coalition of l suppliers if their total risk-management fund is F, where l is a positive integer. Thus, for individual suppliers, we have $G_1(\cdot) = G(\cdot)$. Furthermore, denote by $V_l(F) = G_l(F)/F$ the *survival probability per unit of fund for a coalition of l suppliers with reserve fund F*. We restrict our attention to funds that are at least sufficient to cover the expected loss. That is, for each supplier i, $F_i \geq \mathbf{E}[\xi_i] = \mu$. For technical tractability, we also use the following assumption for values of F and l of our interest (refer to the Appendix for details) throughout the paper:

Assumption 1.
(a) $V_l(lF)$ decreases in F for $l = 1$, and decreases in l for any given F.
(b) $G_l(lF)/G_1(F)$ is unimodal in F for any given l.

As shown in the Appendix, Assumption 1 holds for several commonly used distributions, including exponential, Erlang and normal. Specifically, Assumption 1(a) posits how the reserve fund level F and the size of the coalition l affect the survival probability. F has a diminishing rate of impact on the survival probability of an individual supplier, and, for symmetric suppliers each holding F, the impact of the fund level on the survival probability of a coalition also diminishes in the

[1] In tanker pools like TI of §1, the net revenue of the total pool is allocated among the partner tankers proportional to their shares of the effective cargo carrying capacity (which takes into account factors like the base capacity, hiring period, tankers' efficiency and suitability for the pool's main trades and operations and fuel consumption) in the total pool's carrying capacity. For the exact formula used for allocation of net revenues in the case of Tanker pools, please refer to Haralambides (1996).

scale of the coalition (i.e., the survival probability per unit fund is higher for a smaller coalition). For symmetric suppliers, Assumption 1(b) concerns the benefit in terms of survival probability of joining a l-supplier coalition with $l > 1$ versus that of operating alone. It requires that such benefit increases in the average fund level F on $[\mu, F_0]$ for some $F_0 \geq \mu$, and decreases thereafter. Therefore, the relative benefit of joining an alliance increases when the reserve fund level is relatively low, but does not further increase beyond a threshold reserve fund level.

Game sequence: The sequence of the events in our framework is as follows (refer to Figure 2).

Stage 1: n upstream suppliers strategically form m coalitions $\mathscr{S} = \{S_1, S_2, ..., S_m\}$, by playing a *cooperative* game among themselves.

Stage 2: Each coalition S_k commits to its supply limit Q_{S_k}, which caps the amount it will produce for the downstream firm. For exposition purpose, we assume zero commitment cost, which, as shown in the Appendix, is without loss of generality. Each coalition then determines the wholesale price w_{S_k} it will charge to the downstream firm. This stage involves *competitive* decision-making.

Stage 3: The downstream firm maximizes her profit by determining the retail price p for the final product and the order quantities $\{q_{S_k}\}$, $q_{S_k} \leq Q_{S_k}$, from each coalition S_k.

Stage 4: Exogenous supply shock resolves. If a coalition S_k survives, the entire order from the downstream firm is delivered. Each unit produced by coalition S_k incurs a marginal cost c_{S_k}, where c_{S_k} is equal to $|S_k|c$ and c for complementary and substitutable cases, respectively. The resulting profit is shared among the coalition members based on their respective contributions to the pooled fund. Otherwise, in case of an order default, the coalition receives no payment, and the downstream firm has to utilize an emergency source for the shortfall that charges the firm a premium δ_{S_k} on top of the coalition's wholesale price w_{S_k}. When the components are complementary, there is a unit premium $\delta \geq 0$ for each component; so, $\delta_{S_k} = |S_k|\delta$. If they are substitutes, there is only one component from the coalition; so, $\delta_{S_k} = \delta$. Similar premium emergency sourcing option has been used before in the related literature, e.g., Dong and Tomlin (2012). Subsequently, the downstream firm sells the final product to the end customers at price p and collects her revenue.

4. Operational Decisions under a Given Coalition Structure

In this section, using backward induction, we characterize the equilibrium operational decisions of each coalition and the downstream firm (i.e., stages 2 and 3 of the game) for a given coalition structure $\mathscr{S} = \{S_1, ..., S_m\}$, where a coalition of size 1 represents an individual supplier.

We start with optimal ordering and pricing decisions of the downstream firm (i.e., stage 3). To make her ordering and pricing decisions, the downstream firm will take into account the possibility that each coalition S_k might default with certain probability, in which case she would have to pay a premium δ_{S_k} to ensure supply as discussed in the last section. This allows us to determine the

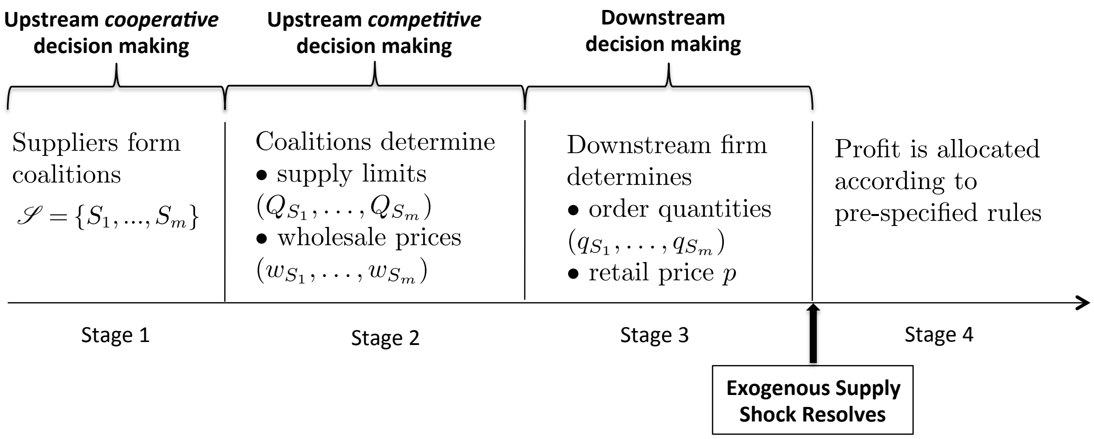

Figure 2 Game Sequence of the Model Framework

expected wholesale price $\widetilde{w}_{S_k} = w_{S_k} + \bar{G}_{|S_k|}(F_{S_k})\delta_{S_k}$ paid by the downstream party to coalition S_k for both complementary and substitutable cases. Based on the expected wholesale prices, the downstream firm then determines the expected-profit-maximizing order quantities from the coalitions (which determine the retail price based on the inverse demand function). Specifically,

• for an assembler dealing with complementary suppliers, the same order quantity applies to all coalitions and $q_{S_1} = q_{S_2} = ... = q_{S_m} = q$. The assembler's expected profit Π_0 is then

$$\Pi_0(q) = \max_{0 \le q \le \min\{Q_{S_k}\}} q\left(D^{-1}(q) - \sum_{k=1}^{m} \widetilde{w}_{S_k}\right)$$

• for a buyer dealing with substitutable suppliers, the total order to place with the suppliers is $q = \sum_{k=1}^{m} q_{S_k}$. The buyer's expected profit Π_0 can be expressed as

$$\Pi_0(q_{S_k},\, k = 1, 2, ..., m) = \max_{0 \le q_{S_k} \le Q_{S_k},\, k=1,2,...,m} \sum_{k=1}^{m} q_{S_k}\left(D^{-1}(q) - \widetilde{w}_{S_k}\right)$$

In either case, let $q^*_{S_k}(\mathbf{w}, \mathbf{Q})$ be the optimal order quantity that solves the optimization problems. Given $(w_{S_{-k}}, Q_{S_{-k}})$, coalition structure $\mathscr{S} = \{S_1, ..., S_m\}$ and provided that coalition S_k survives, the profit expressions for coalition S_k (denoted by π_{S_k}) and a supplier i within that alliance (denoted by π_i) can be written as follows:

$$\begin{aligned}
\pi_{S_k}(\mathscr{S}|w_{S_{-k}}, Q_{S_{-k}}) &= \max_{0 \le w_{S_k}, 0 \le Q_{S_k}} (w_{S_k} - c_{S_k}) q^*_{S_k}(\mathbf{w}, \mathbf{Q}), \\
\pi_i(\mathscr{S}|w_{S_{-k}}, Q_{S_{-k}}) &= \frac{F_i}{F_{S_k}} \pi_{S_k}(\mathscr{S}|w_{S_{-k}}, Q_{S_{-k}}).
\end{aligned}$$

The equilibrium pricing and ordering decisions for the suppliers and the buyer are given in Proposition 1. All the technical proofs are provided in the Appendix.

Proposition 1. **(Equilibrium Operational Decisions)** *Given a coalition structure* $\mathscr{S} = \{S_1, ..., S_m\}$*, the equilibrium decisions for stages 2-3 are as shown in Table 1.*

	Complementary suppliers	**Substitutable suppliers**
Retail Price (p^*)	$\dfrac{m\alpha+\widetilde{C}}{(1-m\beta)(1-\beta)}+\dfrac{\alpha}{1-\beta}$	$\dfrac{\alpha+m\tilde{c}}{(m-\beta)(1-\beta)}+\dfrac{\alpha}{1-\beta}$
Order Quantity ($q^*_{S_k}=Q^*_{S_k}$)	$D(p^*)$	$D(p^*)\left(\dfrac{1}{m}+\dfrac{m-\beta}{m}\dfrac{\left(\tilde{c}-\hat{c}_{S_k}\right)}{\alpha+\beta\tilde{c}}\right)$
Wholesale Price ($w^*_{S_k}$)	$\dfrac{\alpha+\beta\widetilde{C}}{1-m\beta}+c_{S_k}$	$\dfrac{\alpha+\beta\tilde{c}}{m-\beta}+c+\tilde{c}-\hat{c}_{S_k}$
In the above, $\widetilde{C}=nc+\sum_{k=1}^{m}\bar{G}_{\lvert S_k\rvert}(F_{S_k})\delta_{S_k}$, $\tilde{c}=c+\sum_{k=1}^{m}\bar{G}_{\lvert S_k\rvert}(F_{S_k})\delta/m$, $\hat{c}_{S_k}=c+\bar{G}_{\lvert S_k\rvert}(F_{S_k})\delta$ and $\bar{G}_{\lvert S_k\rvert}(F_{S_k})=1-G_{\lvert S_k\rvert}(F_{S_k})$		

Table 1 Equilibrium Operational Decisions for Stages 2 and 3

The equilibrium solution inherits a structure similar to those without supply risk (e.g., see Yin 2010 for the complementary case) with adjustments to account for the risk-reducing funds F_{S_k} of the coalitions, i.e., their effective supply risks. Indeed, F_{S_k} significantly impacts the decisions for both complementary and substitutable suppliers, albeit somewhat differently. For example, in the complementary case, the equilibrium coalition production quantity $q^*_{S_k}=Q^*_{S_k}$ is affected by the effective *total* production cost $\widetilde{C}$ reflecting the aggregated effect of all reserve funds, $\{F_{S_k}\}_{k=1}^{m}$. Thus, higher reserve levels result in higher order quantities for every complementary coalition ($\widetilde{C}$ and $q^*_{S_k}$ are inversely related). But, under the substitutable case, $q^*_{S_k}$ is affected via both effective *average* production cost $\widetilde{c}$ (another representation of $\{F_{S_k}\}_{k=1}^{m}$) and effective *individual* production cost $\hat{c}_{S_k}$ (related to F_{S_k} only), which determine the total order quantity and the order allocation among coalitions. In this case, although higher reserves still result in total quantity expansion, coalition S_k's order also depends on its own resource level compared to its competitors.

The sensitivity of the profits with respect to coalition structures is also different for upstream and downstream firms. For complementary suppliers, as the number of coalitions m decreases (i.e., larger coalitions), the ex-post profit of each coalition increases, and the expected profit of the downstream firm (i.e., assembler) and the consumer surplus increase as well. For substitutable suppliers, however, this might not be true. Particularly, when the default premium δ is small, while the the ex-post profit of each coalition still increases with the formation of larger coalitions (smaller m), both the expected profit of the downstream firm and consumer surplus may actually decrease. In essence, while upstream suppliers are always better off with more co-operation, the same may not be true for the downstream firm. Notably, a downstream firm that deals with complementary suppliers would generally prefer larger coalitions (e.g, highly integrated sub-assemblies) because they indirectly reduce the cost of default risk for the suppliers as well as the intensity of indirect competition (Jiang and Wang 2010). This enables the coalitions to sustain lower equilibrium wholesale prices. On the other hand, for substitutable suppliers a buyer would prefer smaller coalitions (e.g., highly fragmented market) in order to strengthen the competition that would depress the equilibrium wholesale prices. Obviously, lower wholesale prices result in lower retail prices to improve consumer surplus (and vice versa). Given the optimal decisions for a given coalition structure (Stages 2-3), we next analyze how the individual suppliers strategically decide on their equilibrium coalition structures (Stage 1).

5. Coalition Structure and Stability

In this section, we focus on characterizing the stable equilibrium coalition structures among upstream suppliers for the scenario where the amount invested in risk-mitigating reserve fund by each of them is exogenously known (we discuss the endogenous case in §6.1). In this context, we first analyze the case of symmetric fund amounts for all suppliers and then the asymmetric case. We initiate our analysis by first discussing the stability concept used in this paper.

5.1. Coalition-Proof Nash Equilibrium (CPNE)

An often-used concept to characterize coalition stability is the Nash equilibrium (NE). A Nash stable coalition is defined as the one in which there is no *individual* profitable deviation for any party. Even though NE allows for a simple verification, it does not account for profitable deviations as a *group*. As a remedy, the concept of Strong Nash Equilibrium (SNE) has been proposed, which requires that a coalition structure is immune to deviation by any arbitrary set of suppliers. However, this concept suffers from imposing too strong conditions on the coalitions and lacks consistency in definition (Bernheim et al. 1987). As an alternative, this paper adopts a refined stability concept, called *coalition-proof Nash equilibrium* (CPNE, refer to Bernheim et al. 1987 for details). In general,

- a CPNE must be *self-enforcing* and not strictly dominated by another *self-enforcing* strategy;
- a strategy among a group of players is *self-enforcing* if every subgroup plays CPNE strategy in its component game.

Therefore, unlike NE, CPNE allows suppliers to communicate and deviate as a group. However, CPNE does not consider *all* potential deviations as in SNE, but only the valid (by definition, *self-enforcing*) deviations — that no proper subset of the defecting players can reach a mutually beneficial agreement to deviate from the deviation. In this sense, CPNE is more consistent and forward-looking than SNE. In particular, it allows for explicit characterization of all stable outcomes. Due to these reasons, CPNE has been used in the literature for analyzing coalition formation in a number of settings, including some involving generic risks (Bernheim and Whinston 1987; Genicot and Ray 2003). Throughout the paper, we use "stable" to abbreviate "coalition-proof Nash stable", unless specified otherwise.

5.2. Key Trade-off in Coalition Formation

For a given coalition structure $\mathscr{S} = \{S_1, ..., S_m\}$, the expected profit of supplier i in a coalitionn alliance S_k is given by

$$\Pi_i(\mathscr{S}) = \pi_{S_k}(\mathscr{S}) G_{|S_k|}(F_{S_k}) \frac{F_i}{F_{S_k}}, \qquad \forall i \in S_k. \tag{4}$$

When the reserve fund amount is known, based on (3), we know that supplier i should make his decision about whether or not to join a coalition based on the above profit expression. The expression involves three terms. The first term $\pi_{S_k}(\mathscr{S})$ denotes the profit for coalition S_k, if it survives, and can be derived from Proposition 1. The second term, $G_{|S_k|}(F_{S_k})$, measures the survival probability of a coalition, i.e., its probability of successfully fulfilling the order. So, $\pi_{S_k}(\mathscr{S}) G_{|S_k|}(F_{S_k})$ is the expected profit of coalition S_k taking into account the order default risk. Lastly, F_i/F_{S_k} denotes the share of coalition profit allocated to supplier i.

The above profit function reveals the key trade-off between joining large or small coalitions for supplier i. On one hand, joining a large coalition has the *strategic benefit* of increasing his survival probability $G_{|S_k|}(F_{S_k})$. On the other hand, a large coalition may not yield a satisfying profit share for him, as $\pi_{S_k}(\mathscr{S}) F_i/F_{S_k}$ may decrease in $|S_k|$. This represents the *operational dis-benefit* of a large coalition. By focussing mainly on the operational disadvantage, existing literature (e.g., Granot and Yin 2008, Yin 2010) provides evidence that large coalitions are often not sustainable among firms where there is no supply risk. Taking into account also the strategic benefit, our model provides a more comprehensive account that generates new insights. The key lies in how suppliers leverage the above two opposing forces during the course of coalition-formation decision.

In order to focus on the key trade-off, for now, we assume that the price premium δ that the downstream firm has to pay in case of order default is minimal, i.e., $\delta = 0$. So, our basic framework represents a scenario where, if a supply entity fails, then there are plenty of external options available to "match" his price. This assumption is reasonable as long as the components are relatively commoditized. Nevertheless, we discuss the impact of $\delta > 0$ in §6.3.

5.3. Symmetric Suppliers

Suppose that all n suppliers have the same amount of fund F for dealing with order default risk (so they are equally risky). We first characterize the stable equilibrium coalition structure that will develop in this case and then discuss the factors that shape the coalition formation decision.

Our first result states that possible stable coalitions should be of similar sizes. In particular, stability disallows any two coalitions to differ by more than one supplier. Therefore, if the suppliers form m coalitions, there is only one configuration that is possibly stable. It is the one that has $n - m\lfloor n/m \rfloor$ coalitions of size $\lceil n/m \rceil$, and $m + m\lfloor n/m \rfloor - n$ coalitions of size $\lfloor n/m \rfloor$, where $\lfloor n/m \rfloor$ and $\lceil n/m \rceil$ denote the nearest integers that are (weakly) smaller and larger than n/m, respectively.

Proposition 2. *Consider n suppliers with identical reserve fund levels. Then, if CPNE contains m coalitions, the size of each coalition should be equal to either $\lceil n/m \rceil$ or $\lfloor n/m \rfloor$.*

Example 1. Suppose there are $n = 5$ suppliers. Let I_k represent a set of k identical suppliers. Then, Proposition 2 states that the possible stable coalition structures among the suppliers can only be one of the following forms: $\{I_5\}$, $\{I_2, I_3\}$, $\{I_1, I_2, I_2\}$, $\{I_1, I_1, I_1, I_2\}$, or $\{I_1, I_1, I_1, I_1, I_1\}$. □

To determine which one of the above structures will be stable, we need to verify the following conditions characterized by $U(m)$, where $U(m)$ is the ratio of ex-post payoff for m-supplier-coalition versus $(m+1)$-supplier-coalition (see Table A4 in the Appendix for detailed $U(m)$ expressions).

Theorem 1. *(**CPNE with n identical suppliers**) For n suppliers with identical reserve fund levels F, there exists a unique CPNE with m^* coalitions. In particular, the suppliers*

(i) will form a grand coalition ($m^* = 1$) if $U(1) \geq \dfrac{V_1(F)}{V_n(nF)}$;

(ii) will act independently ($m^* = n$) if $U(m) < \dfrac{V_1(F)}{V_{\lceil n/m \rceil}(\lceil n/m \rceil F)}$ for any $1 \leq m \leq n-1$;

(iii) will form m^* coalitions if $U(m) < \dfrac{V_1(F)}{V_{\lceil n/m \rceil}(\lceil n/m \rceil F)}$ for all $1 \leq m \leq m^* - 1$, and $U(m^*) \geq \dfrac{V_1(F)}{V_{\lceil n/m^* \rceil}(\lceil n/m^* \rceil F)}$, where $1 < m^* < n$.

Theorem 1 suggests an algorithm to find the number of stable coalitions among n suppliers. Specifically, the number of stable coalitions among n suppliers is determined by the smallest m (where m is between 1 and $n-1$) at which $U(m)$ exceeds $\dfrac{V_1(F)}{V_{\lceil n/m \rceil}(\lceil n/m \rceil F)}$. Equivalently, we can define for each $m \in \{1, 2, ..., n-1\}$ a Risk Adjusted Stability Factor ($RASF_m$) as follows:

$$RASF_m = U(m) - \frac{V_1(F)}{V_{\lceil n/m \rceil}(\lceil n/m \rceil F)} \tag{5}$$

and then characterize the number of stable coalitions by searching for the smallest m where $RASF_m$ becomes non-negative. This algorithm is illustrated via the following example.

Example 2. Suppose that there are $n = 3$ complementary suppliers, each facing i.i.d. risk $\xi_i \sim N(5, 1)$, and market demand is $D(p) = ap^{-b}$ with $b = 8$. This results in, by Table A4, $U(m) = (1 + \dfrac{1}{7-m})^7$. We can then plot RASF_m as function of F as shown in Figure 3.[2]

Clearly, for relatively low values of F (between 5.04 and 7.06), $m = 1$ is the smallest index that makes $RASF_m$ non-negative. Therefore $m^* = 1$ and the grand coalition $\{I_3\}$ is uniquely stable. As F increases, e.g., $F > 7.06$ (more than 98.03% individual survival probability), $RASF_2$ becomes the first non-negative RASF, hence $m^* = 2$ (i.e., two stable coalitions $\{I_1, I_2\}$). By similar argument,

[2] As indicated before, the reserve fund amount F of an individual supplier directly corresponds to his survival probability (in this case, it is $\Phi(F-5)$ for each supplier, where $\Phi(\cdot)$ is the CDF of standard normal random variable) and inversely to his default risk. For all our numericals we report both F and the corresponding survival probabilities.

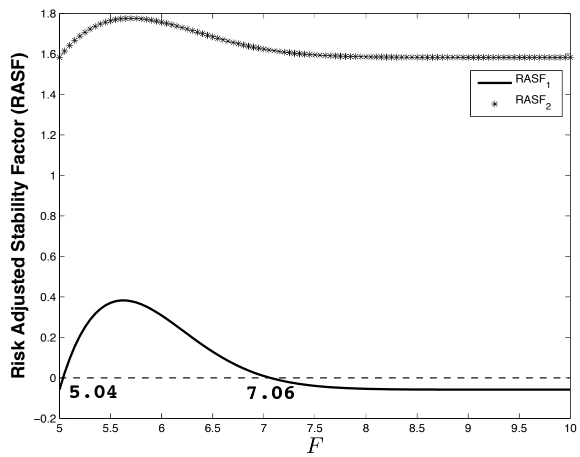

Figure 3 **Risk adjusted stability factor (RASF) in a three-supplier system with iso-elastic demand**

when F is extremely close to 5 (less than 5.04, i.e., 51.6% individual survival probability), two coalitions will be formed and $\{I_1, I_2\}$ is stable. Note that independent structure $\{I_1, I_1, I_1\}$ is never stable because $RASF_2$ is always non-negative for all values of F. $\square$

Based on above, it is clear that the coalition formation decision depends on RASF_m in (5). The second component of RASF_m is a function of the reserve fund level F (or, equivalently, risk level) of the suppliers and the size of the supplier base n, while the type of supplier (complements or substitutes) and the form of the demand function determine the first component $U(m)$ of RASF. We discuss the detailed effects of the above four factors on the incentive to form coalitions below.

Risk level of the suppliers (F): Example 2 illustrates that when the suppliers themselves have access to significant amount of funds to reduce their order default risks, they will prefer to form coalitions with a small number of other suppliers (i.e., large m^*). On the other hand, if the suppliers do not have such access, they would like to take advantage of resource-sharing by forming large coalitions. The only exception is when the fund is quite limited (F extremely small, very close to μ) and suppliers cannot garner much risk-reduction benefit even by pooling their funds. In that case, the suppliers will opt for small coalitions (i.e., large m^*) to get better profit allocations.

Size of the supplier base (n): Larger coalitions are more achievable when the supplier base is smaller (lower n), i.e., when the intensity of direct competition among substitutable suppliers or indirect competition among complementary suppliers (Jiang and Wang 2010) is lower. This implies that in assembly systems, low-modularization design, which would involve small number of suppliers delivering these modules, facilitates cooperative decision making, whereas in the substitutable case, cartels are more likely to be formed when there are less number of suppliers in the market. To illustrate this, consider Figure 4, which is based on Example 2. When $n = 3$, the grand coalition is achieved for $5.04 \leq F \leq 7.06$. When the number of suppliers increases to 6 or more, however, grand coalition is never stable. The above two effects are summarized in Proposition 3 below.

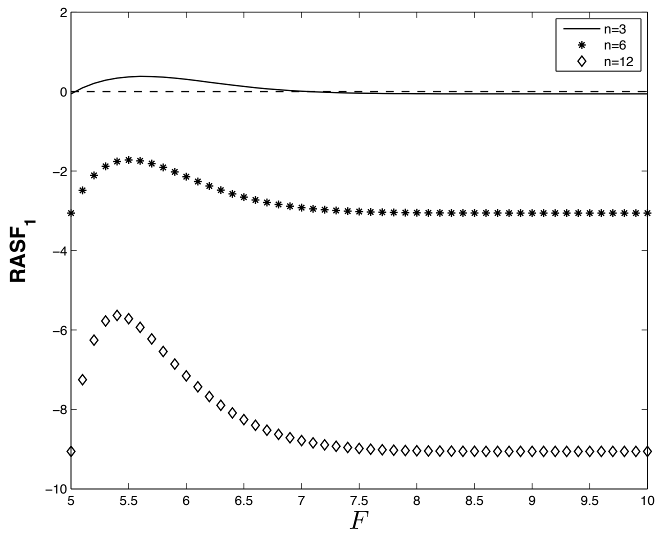

Figure 4 Risk adjusted stability factor with index 1 ($RASF_1$) with respect to the size of the supplier base n

PROPOSITION 3. *In general, larger coalitions are more likely to be stable*

(i) as the suppliers become more risky (or, equivalently, are endowed with lower amount of reserve fund F);

(ii) when the number of suppliers in the supply base is relatively small (small n).

As discussed before, the component type and the demand function shape coalition incentive through their effects on $U(m)$. In order to understand this better, for the rest of this section we restrict our attention to the three most commonly assumed demand functions in the related literature — iso-elastic, linear-power (linear is a special case) and exponential (refer to Table A2 in the Appendix). Also, suppose that the *pass-through rates* of these functions, defined as the ratio of retail price change to the wholesale price change (dp/dW) (Tyagi 1999, Moorthy 2005), are greater than 50%. Note that this assumption implies that the consumers will shoulder more of the change in wholesale prices than the retailer does; this is natural in many industries (Besanko et al. 2005).

Structure of customer demand: Analysis of the demand functions suggests that higher pass-through rate promotes the formation of larger coalitions. As illustrated in the Appendix, as the pass-through rate increases, the resource requirement in order for grand coalition to be the stable equilibrium also increases. Recall that smaller coalitions will most likely lead to higher wholesale prices and consequently higher retail prices (Proposition 1), and the pass-through rate measures the ratio of change in retail price over the change in wholesale price. Higher pass-through rate then implies a more sensitive vertical structure in which inefficient upstream decisions (e.g., small alliances, high wholesale prices) will lead to higher downstream retail prices. Thus, the jeopardy of small alliances is amplified as the pass-through rate increases, and hence large-alliance structure becomes more rewarding and stable. Based on Table A2 in the Appendix we can then conclude that iso-elastic demands with lower levels of elasticity or linear-power demands that are more price-sensitive are more conducive to larger coalitions.

Component/supplier type: It can also be shown that $U(m)$ for the substitutable case is smaller than the complementary one implying that large coalitions are more achievable among complementary than substitutable suppliers. We illustrate this in the example below. This result is somewhat intuitive — since complementary suppliers are competing indirectly (rather than direct competition faced by substitutable ones), there is less reluctance on their part to enter into partnerships.

EXAMPLE 3. Consider the same supplier set and market demand as in Example 2. For complementary suppliers, $U^C(m) = (\frac{8-m}{7-m})^7$. Theorem 1 shows that the grand coalition $\{I_3\}$ will be formed if $5.04 \le F \le 7.06$; otherwise, two coalitions $\{I_1, I_2\}$ will be formed. For substitutable suppliers, $U^S(m) = (\frac{m+1}{m})^2(\frac{8-1/m}{8-1/(m+1)})^7$. If $5.13 \le F \le 6.72$ (i.e., individual survival probability is between 55.17% and 95.82%), two coalitions $\{I_1, I_2\}$ will be formed; otherwise, independent coalitions $\{I_1, I_1, I_1\}$ will be formed. Note that, for any given F, substitutable suppliers yield smaller coalitions than complementary ones. $\square$

The above two effects are summarized in the proposition below.

PROPOSITION 4. *In general, larger coalitions are more likely to be stable*

(i) for supplier bases facing end customer demands with higher pass-through rates;

(ii) among complementary than substitutable suppliers.

Risk-reduction vs competition-reduction: Until now, we have focussed on characterizing the conditions under which coalitions would be formed and whether it will be a large or a small one. A natural question that would arise is whether coalition formation is driven by suppliers' desire to reduce their risks or by the lure to reduce competition among themselves through cooperative decision-making. Indeed, we can answer this question by characterizing the stable coalition structures under a risk-less environment. Specifically:

PROPOSITION 5. **(CPNE in riskless environment)** *If the suppliers are effectively riskless (i.e., F is sufficiently large), they will form the maximum number of possible coalitions with the lowest possible sizes.*

In our model, coalitions can provide two kinds of benefits — risk-mitigation and competition-reduction. If the primary goal of coalition formation is competition reduction, suppliers should do so even when they have large amount of funds available for dealing with order defaults and are effectively riskless. But, the above proposition suggests quite the contrary. Indeed, the least cooperative structure will arise in this case suggesting competition-reduction incentive alone does not lead to large coalitions.

When there are larger coalitions, the number of competing forces goes down and each coalition as a whole is able to obtain a higher profit from the downstream firm; so, one would expect that the suppliers would be better off with larger coalitions. However, this argument does not take into account the *stability* of such configurations. In particular, for larger coalitions to be stable, the benefit of adding one more supplier must be commensurate with the allocation that he will take away, and this might not be the case. Since suppliers individually decide on coalition-formation in Stage 1 to maximize their own expected profits, very often the equilibrium structure does not reflect the optimum for the entire supply base (i.e., large coalitions). Therefore, it becomes more difficult to keep larger alliances stable in equilibrium under a riskless environment.

Note that this result conforms with literature that studies coalition formation using different stability concepts than us in a riskless environment. For example, Yin (2010) applies the NE stability concept and derives a similar insight for complementary suppliers. In analyzing the CPNE coalition structure, we are able to extend this insight to both complementary and substitutable suppliers. Specifically, in the context of substitutable suppliers, we can show that *an independent structure is the stable equilibrium for high values of* F, as long as there are more than two suppliers in the supply base (refer to the Appendix). So, clearly, the incentive for coalition formation does not lie in competition reduction. Rather, it is the risk of order default and the impetus to mitigate that risk through resource sharing that holds the coalitions together.

5.4. Asymmetric Suppliers

In the previous section, we consider the case where all suppliers have identical amounts of risk-management funds F and so are equally risky. In this section, we extend our analysis to the case when suppliers face the same i.i.d. exogenous shocks but hold different level of reserve funds to deal with order default risks. The survival probability therefore varies across the suppliers.

For the sake of analytical tractability, we assume that there are two possible reserve fund levels for the suppliers. Specifically, there are n_L suppliers with low level of reserve funds L and $n_H = n - n_L$ suppliers with high levels of reserve funds $H(> L)$; i.e., there are n_L high-risk and n_H low-risk suppliers. We need the following definition in characterizing stable coalition structures.

Definition 1. Supplier coalitions $\{S_1, ..., S_m\}$ are *V-similar* if $V_{|S_k|}(F_{S_k}) \geq V_{|S_{k'}|+1}(F_{S_{k'}} + F_s)$ $\forall k, k^{'} \in \{1, 2, ..., m\}$ and $s \in S_k$.

Intuitively, the above definition states that the difference between the survival probabilities of any pair of coalitions S_k and $S_{k'}$ in a V-similar coalition structure $\mathscr{S}$ should not be very large. Using this definition, we first identify a condition for stable coalitions that is quite similar to the one with identical suppliers in the last section, except that there is an adjustment in the $RASF_m$ expression of (5) to account for the asymmetry. Specifically, in this case, $RASF_m = U(m) - T_m$, where $\{T_m\}_{m=1}^{n-1}$ depends on both risk and demographic profiles of the supplier base (see below).

THEOREM 2. (**CPNE among asymmetric suppliers**) *There exists a unique CPNE with m^* coalitions. In particular, the suppliers*

(i) will form a grand coalition ($m^* = 1$) if $U(1) \geq T_1$,

(ii) will act independently ($m^* = n$) if $U(m) \leq T_m \;\; \forall 1 \leq m \leq n-1$;

(iii) will form m^* coalitions if $U(m) \leq T_m \;\; \forall 1 \leq m \leq m^* - 1$, $U(m^*) \geq T_{m^*}$, and $1 < m^* < n$, where $T_m = \min_{\mathscr{S}}\{\max_{1\leq k\leq m, s\in S_k}\{\frac{V_1(s)}{V_{|S_k|}(F_{S_k})}\} : \mathscr{S}$ is a V-similar m-partition of the supply base $N\}$ and the CPNE is the structure $\mathscr{S}$ that yields T_{m^*}.

As in the symmetric case, Theorem 2 also suggests an algorithm to find the stable coalitions by searching for the smallest m at which $U(m)$ exceeds T_m, or equivalently, $RASF_m$ becomes non-negative. Unfortunately, $RASF_m$ is not straightforward to graph in the asymmetric case because of the difficulty in graphically expressing T_m, which involves two reserve fund levels. Therefore, we provide an example to illustrate Theorem 2.

EXAMPLE 4. Consider the demand $D(p) = ap^{-7}$, which yields $U(m) = (\frac{7-m}{6-m})^6$ for complementary suppliers. Suppose that $\xi_i \sim Exp(1/2)$, $L = 3$ (approximately 77.69% individual survival probability), $H = 9$ (approximately 98.89% individual survival probability), $n_L = 2$ and $n_H = 3$. Thus, $F_N = 33$, where $F_N = n_L L + n_H H$ is the total fund level for the supplier base.

There is only one 1-partition of N, hence $T_1 = \frac{V_1(3)}{V_5(33)} \approx 8.54 > U(1) = 2.99$. Thus, grand coalition is not stable. By Definition 1, the set of V-similar 2-partitions of 5 suppliers contains only $\{L_2H_1, H_2\}$, where L_xH_y denotes a set of x low-reserve suppliers and y high-reserve suppliers. By using the expression for T_m in Theorem 2, we can find $T_2 \approx 3.96 > U(2) = 3.81$. Hence, two-coalition structure is not a CPNE neither. Similarly, the set of V-similar 3-partitions contains only $\{L_1H_1, L_1H_1, H_1\}$. It can be calculated that $T_3 \approx 3.16 < U(3) = 5.62$. Therefore, the CPNE contains three coalitions $\{L_1H_1, L_1H_1, H_1\}$. $\square$

Based on the expression of $RASF_m$ in the asymmetric case, it is evident that the effects of supplier type and the customer demand on the coalition sizes noted in Proposition 4 for symmetric suppliers remain valid since these two factors affect only $U(m)$ and they affect it in the same way as before. In fact, even the effects of the size of the supplier base for identical suppliers also carry over to the asymmetric case. Beyond these, the main new insight we gain from this section pertains to the effects of two types of asymmetries (for large enough H and L) as discussed below.

Risk level asymmetry: The degree of asymmetry between risk levels in the supplier base can be characterized by H/L. The following example illustrates how this ratio affects the coalition structure.

EXAMPLE 5. In Example 4 above, $H/L = 3$. Keeping the total reserve fund at the same level, i.e., $F_N = 33$, we can reduce H/L by increasing L to 4.5, and decreasing H to 8, which approximately corresponds to 89.46% and 98.17% individual survival probabilities for L- and H-type suppliers, respectively. The ratio H/L is now around 1.78. In that case, $T_1 \approx 6.56 > U(1) = 2.99$, $T_2 \approx 3.41$ for coalitions $\{L_2H_1, H_2\}$, and $U(2) = 3.81 \geq T_2$. The CPNE now contains only 2 coalitions (compared to 3 for Example 4), and the stable structure can be identified as $\{L_2H_1, H_2\}$. □

The takeaway from the above example is that for large coalitions to be stable, the reserve fund levels (or, equivalently, order default risks) of the supplier base cannot be too different from each other. Indeed, we formalize the above by establishing the following for grand coalition.

PROPOSITION 6. *In general, given a demographic profile (n_H/n_L) and a total reserve fund amount F_N of the supplier base, a grand coalition is more likely to be sustained by suppliers with similar reserve fund levels (i.e., H/L closer to 1).*

Demographic Asymmetry: This asymmetry comes from the number of suppliers in low- and high-funded group. Consider two sets of suppliers with the same total resource level F_N and risk level asymmetry H/L, but the demographic distribution in one set is more skewed towards low-funded suppliers, e.g., $n_H/n_L > n'_H/n'_L$. The following example shows that larger alliances are more likely to be stable in the latter one.

EXAMPLE 6. Consider Example 4 again with $F_N = 33$ and $H/L = 3$, but suppose now we have three low-funded and two high-funded suppliers. Specifically, $n_L = 3$, $n_H = 2$, $L = 11/3$ and $H = 11$. We can then show CPNE in this case contains two coalitions $\{L_3, H_2\}$ compared to $\{L_1H_1, L_1H_1, H_1\}$ in Example 4. □

We can again formalize the above insight in the following proposition about grand coalitions.

PROPOSITION 7. *In general, given a risk profile H/L and a total reserve fund amount F_N of the supplier base, a grand coalition is more likely to be sustained when the supply base has relatively more suppliers with low reserve fund levels (i.e., n_H/n_L closer to 0).*

In summary, the main conclusion of the analysis in this section is that homogeneity in terms of risk profile and an abundance of suppliers with limited amount of funds available for risk-mitigation in case of an order default incentivize the formation of large coalitions, and vice versa.

6. Robustness Analysis

To focus on the core issues related to supplier coalitions, we made several assumptions in our analysis in §5. The major ones are: i) the amount of reserve fund F is an exogenously given parameter, ii) the profit allocation among the coalition partners is proportional to their shares in the pooled reserve fund, and iii) in case of an order default, the downstream firm can procure the component(s) without paying a premium. In this section, we briefly discuss the implications of relaxing the above assumptions and identify to what extent the insights of §5 are affected.

6.1. Endogenous Reseve Fund Investment Decision

Until now we have assumed that ex-ante investments by the suppliers in their reserve funds for risk-mitigation is exogenous. However, such investments can be expensive due to cost of capital (e.g., marginal return of capital, loss of other investment opportunities). So, the suppliers need to counterbalance the costs and benefits of coalition formation (see §5.2) against the costs and benefits of this investment. Keeping this in mind, in this subsection, we address the issue of upstream suppliers making their investment decisions in reserve funds before the coalition formation decision in Figure 3 (say, Stage 0), thus endogenizing the risk levels of the suppliers. Suppliers make these decisions competitively, and we characterize the resulting Nash equilibrium investment levels.

Suppose that a supplier's cost of investing at level F is $c_f(F) = vF$, where $v \geq 0$. We prove in the following that there is a unique level F^I (symmetric equilibrium) that each supplier would invest in. The corresponding stable CPNE coalition structure can then be identified via Theorem 1, as $m^I = m^*(F^I)$.

Proposition 8. *Assume that* $\frac{V_1(\hat{F})}{V_k(\hat{F}+(k-1)F)}$ *decreases in* $\hat{F}$ *for any given* F. *For* n *suppliers each with investment cost rate* v, *there exists an* $F^I > 0$ *and* $1 \leq m^I \leq n$ *such that it is a Nash equilibrium for each supplier to ex-ante invest at level* F^I *in their reserve funds and suppliers form* m^I *coalitions of similar sizes. In particular,*

$$F^I = \max\{F : \frac{v}{\pi(m^*)} \leq g_{\lceil \frac{n}{m^*} \rceil}(\lceil \frac{n}{m^*} \rceil F)\frac{m^*}{n} + G_{\lceil \frac{n}{m^*} \rceil}(\lceil \frac{n}{m^*} \rceil F)\frac{n-m^*}{n}\},\ m^I = m^*(F^I), \tag{6}$$

where $m^*(\cdot)$ *follows Theorem 1.*

Note that for technical tractability we need to enforce some assumption as stated in the proposition. This assumption states that the survival probability per unit of fund when operating alone versus joining another alliance should be high (resp. low) when the fund level is low (resp. high), which holds for any exponential distribution.

In general, the equilibrium investment level F^I increases and the number of stable coalition m^I decreases with the production and the investment costs. As the production cost increases, the margin becomes slimmer, making the loss due to order default more significant, and hence collaboration more attractive. The investment cost v also has a similar impact. An environment where getting credit is costly (high v) incentivizes suppliers towards more resource pooling, yielding lean investment levels and more cooperation in equilibrium. On the other hand, a lower cost of capital hinders the formation of coalitions. Indeed, in the extreme case when the cost of capital is negligible, there is no incentive for the suppliers to form a coalition. This again supports our previous assertion that risk reduction is the primary motive for coalition formation.

6.2. Generalization of the Profit Allocation Rule

Our results so far are based on profits being allocated among the coalition partners proportional to their shares in the pooled coalition reserve fund. The existing cooperative game literature in supply chain management area mostly uses allocations based on Shapley values (equal allocation in absence of default risk; see Nagarajan and Sošić 2007 and Granot and Yin 2008). Our proportional allocation indeed turns out to be equivalent to Shapley allocations when suppliers are symmetric.

Now consider a generalization of our allocation scheme in which the profits are shared according to $\gamma_{i,S_k} = \dfrac{F_i^u}{\sum_{j\in S_k} F_j^u}$ for some $u \geq 0$. This family of allocation rules is quite general. In particular, through such mechanisms, the profit can be allocated proportionally ($u=1$, as in the main analysis), in favor of high-reserve suppliers (when $u>1$) or in favor of low-reserve suppliers (when $0<u<1$). Even in this case, we can characterize the condition for grand coalition to be stable as shown below.

Proposition 9. (**Grand coalition among asymmetric suppliers under general allocations**) *For a set of n_L suppliers with reserve levels L and n_H suppliers with reserve levels H, where $n_L+n_H=n$, $H>L$ and $F_N=n_LL+n_HH$,*

(i) grand coalition is a CPNE if and only if $U(1) \geq \max\{\dfrac{G_1(L)}{L^u}, \dfrac{G_1(H)}{H^u}\}\dfrac{n_LL^u+n_HH^u}{G_n(F_N)}$.

(ii) grand coalition is more likely to be stable when u is closer to $u_f = \dfrac{\ln G_1(L)/G_1(H)}{\ln L/H}$.

Clearly, the grand coalition characterization is similar to that for the proportional allocation rule (Theorem 1) with adjustments to account for varying u. The most interesting new insight is that grand coalition will most likely be induced by a "fair" allocation $u_f = \dfrac{\ln G_1(L)/G_1(H)}{\ln L/H} \in [0,1]$, where u_f incorporates the degree of heterogeneity in survival probabilities as well as reserve funds among the supply base. Moreover, even when the allocation rule is not "fair," grand coalition can still be stable provided that suitable values of u make allocations relatively fairer, i.e., make u close to u_f. Since the "fair" allocation u_f is always smaller than 1, it implies that in order to induce grand coalition, the allocation scheme needs to properly favour the more risky suppliers in the supply base (i.e., the suppliers with lower levels of reserve funds).

Lastly, note that as the suppliers become more homogenous, i.e., $L/H \to 1$, the "fair" allocation approaches the proportional rule used in the main analysis, i.e., $u_f \to 1$. Assuming the partners in the shipping pool to be relatively homogenous, this suggests that the proportional profit allocation system used in that industry (as indicated in Haralambides 1996) might indeed be appropriate since such a system makes it more likely that a large coalition would be stable.

6.3. Positive Default Premium on Wholesale Prices

In analyzing the key trade-off of joining small versus large alliances, we assumed that in case of an order default, the component can be procured from an alternate source in a frictionless manner (i.e., $\delta = 0$). While such an assumption maybe realistic under certain scenarios, in this section we allow $\delta > 0$ and investigate how it may affect coalition structures. That is, the downstream firm will take into account the possibility that each coalition might default with certain probability, in which case she would have to pay a premium δ_{S_k} to ensure supply, where $\delta_{S_k} = |S_k|\delta$ for complementary components and $\delta_{S_k} = \delta$ for substitutable ones.

In the Appendix we show that for symmetric suppliers the alliance structure in §5.2 remains valid as long as the premium δ is not too large. Specifically, we can use a set of refined risk adjusted stability factors (RASFs), which now depend also on δ, to identify the stable coalition structures. Rather than going into the details, we illustrate it with the following example.

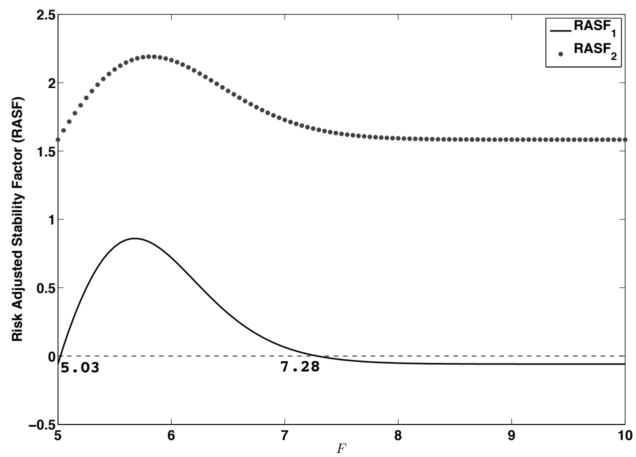

Figure 5 **Risk adjusted stability factor among complementary suppliers with** $D = ap^{-8}$**,** $n = 3$**,** $C = 1$ **and** $\delta = 0.1$

Example 7. Consider the same scenario as in Example 2, i.e., an assembly system with $D(p) = ap^{-8}$, $C = 1$, and $n = 3$. According to Lemma A9 in the Appendix, in general, the structure in Theorem 1 will hold as long as $\delta \leq 3.18$. Suppose that $\delta = 0.1$. Based on Figure 5, we can then deduce if the reserve fund level $F \in [5.03, 7.28]$, i.e., individual surival probability is between 51.2% and 98.87%, respectively, then grand coalition is uniquely stable. If $F < 5.03$ or $F > 7.28$, then a two coalition structure $\{I_1, I_2\}$ is stable. An independent structure is never stable[3]. □

The coalition structures in the above example with $\delta > 0$ are quite similar to Example 2 with $\delta = 0$, except for the threshold reserve fund levels. Although the applicability of Theorem 1 requires the default premium δ to be below a threshold value, our numericals have shown this condition to be not at all restrictive. For instance, in the above example with threshold $\delta = 3.18$, the equilibrium

[3] When the default premium is above a threshold value, e.g., $\delta > 3.18$ for Example 7, we cannot analytically guarantee that Theorem 1 structure will still hold. However, the CPNE can still be verified numerically for small numbers of suppliers. For Example 7, when $\delta = 4$, grand coalition will be formed among three suppliers when $F \in [5, 8.25]$; otherwise if $F > 8.25$, the two coalition structure $\{I_1, I_2\}$ is uniquely stable.

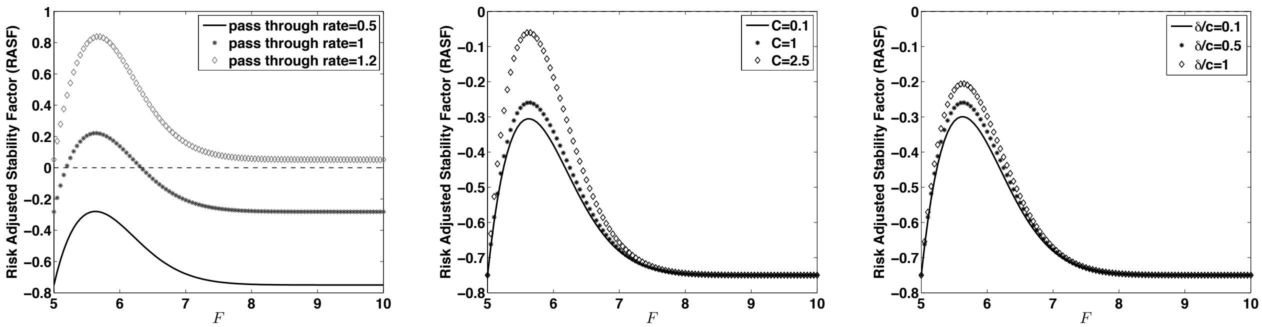

Figure 6 $RASF_1$ **for complementary suppliers with variation in (i) pass through rates, (ii) total production costs, (iii) default premium/component cost ratios**

wholesale prices are between 0.53 and 1.48. These values are significantly less than the threshold default premium.

Also, the range of F that induces grand coalition is wider for $\delta > 0$, implying that *suppliers will be more incentivized to join larger coalitions and therefore, smaller number of coalitions should be observed in equilibrium for a positive default premium.* Our numericals suggest this insight to be quite robust. They also suggest that the curvature of RASF with $\delta > 0$ is robust to problem parameters. Figure 6 shows the change of $RASF_1$ with respect to pass-through rate, total production cost ($C = nc$) and the default premium to component cost ratio δ/c. In general, the curves shift rightwards/upwards as these factors become more significant — indicating that stable large alliances are more achievable when the downstream is more sensitive to upstream price changes, when the raw material/labor is more costly, and when the default penalty is more severe. We again wish to point out that the threshold δ for our results to be valid is not too restrictive indicating the generality of our results. For the examples shown in Figure 6, the maximum δ goes beyond the total production cost $C = cn = 1$.

Lastly, we comment that we assume a per unit premium δ in our analysis. An alternative assumption is a lump sum payment, reflecting the fixed searching/expediting fees of emergency sourcing. That is, if a coalition S_k defaults, the downstream firm can still procure at the pre-announced wholesale price w_{S_k}, yet it costs her Δ_{S_k} to seek new suppliers. All results in §4-§5 hold unconditionally for this alternative assumption.

7. Concluding Remarks

Forming cooperative alliances/coalitions with other firms is an important lever that an organization may seek beyond its internal measures to deal with business risks. Such an arrangement equips its members with better resources and opportunities, and also changes the way they operate and envision themselves. This phenomenon is observed in a number of industries including agriculture, marine transportation and manufacturing. Alliances can be especially effective in dealing with external risks that a firm might be facing. However, given the individual and collective profit motives that an alliance must satisfy, the incentives to form them and their stability are issues of research interest. So far, supply chain management literature has studied alliance-formation focusing primarily on demand-side risks. However, one of the salient features of the recent business environment has been a significant increase in the supply-side risk. The objective of this paper is to understand what types of stable supplier alliances will develop in the presence of the risk of supply (order) default, and how alliance formation incentives are shaped by the business environment.

In order to achieve our objective, we use a channel framework consisting of n upstream suppliers and one downstream firm where the suppliers face the risk of completely defaulting in fulfilling their orders and can form alliances to counteract such risk. Our framework is applicable for both complementary and substitutable suppliers and has a number of other characteristics that distinguishes it from the existing literature. Specifically, each supplier faces an exogenous random shock that creates the default risk. Each of them also incurs operating cost to maintain a fund that has been reserved to deal with the shock, provided the fund amount is large enough. Since the funds are generic and shareable, entering into alliances can further reduce the default risk through fund sharing among partners, although the risk-mitigating benefit of such funds exhibits diminishing returns. Also, the profit allocation mechanism among the partners in an alliance is proportional to their shares in the pooled fund (equivalent to Shapley value based allocation for symmetric suppliers). The above enables us to deal with an important trade-off a supplier faces while deciding on whether to join an alliance not captured before in the literature — doing so decreases a supplier's default risk but also might have adverse implications in terms of his profit share.

We first focus on the scenario where all suppliers are symmetric in terms of fund (or, risk) levels and fully characterize the stable alliances that will develop among them in equilibrium. It turns out that the sizes and number of stable alliances depend primarily on a measure, termed Risk Adjusted Stability Factor (RASF), that succinctly captures the business environment of the suppliers. Further analysis of this measure reveals that larger alliances are stable when: i) the suppliers are more risky (i.e., their fund levels are lower), ii) the supplier base is smaller, iii) the suppliers are complements (rather than substitutes), and iv) the pass through rate of the customer demand is higher. On the other hand, the converse business conditions result in smaller alliances. One of our most important insights is that it is the need to reduce the risk of order default through resource sharing, rather than to reduce competition, that encourages and provides stability to large alliances. Consequently, if suppliers are risk-free, they tend to shy away from forming alliances and mostly operate independently, and when they are very risky, they tend to form grand coalitions. We also characterize the exact composition of stable alliances that develops even when the suppliers are asymmetric in terms of their fund levels. Most of the above insights continue to hold, except when the supplier base is diverse in terms of their reserve fund levels or contains too many suppliers with high amounts of reserve funds.

Traditionally, stability of grand coalitions has been important to anti-trust authorities because of its implications for monopoly power (although many industries are exempted from such laws under a variety of instances as exemplified in Government of Canada 2002). Our context brings another aspect of such large alliances to light; they are less prone to order default. So, conditions for stable large alliances result in less risky supply chains. Given the importance of reliable supply chains in world commerce, anti-trust authorities need to keep this impact in mind while evaluating them (especially in industries where supply risks are of concern). For example, stability of alliances like TI and FPOs discussed in the paper has far reaching implications. Any disruption in marine oil supply, which accounts for the majority of oil transportation, can be devastating for world economy (The Economist 2012), while disruption in food supply in countries like India has major food security ramifications (EIU 2012). Moreover, our analysis also suggests certain rationale as to why we see more alliances in industries like automobile (Geneva 2005) and agriculture (Oxfam International 2010). The former may be attributed to the complementary nature of the components while the latter may arise from the fact that most of the members of organizations like FPOs in India are small farmers who face significant amount of risks and have low amount of reserve funds accessible to them.

We also consider the case where the suppliers first decide on how much they want to invest in their costly risk-management reserve funds before their alliance formation strategy. It turns out that if the investment cost is relatively low (e.g., in the present interest rate environment), they will invest significantly and not form alliances so as not to share the profits. On the other hand, if resource investment is costly, each of them will not invest much and depend on resource sharing in large alliances to reduce their risks. Interestingly, the results of this paper are quite robust. For example, they hold true irrespective of whether or not a default premium is to be paid in addition to the wholesale price in case of a supply default, as long as the premium is not too high. The primary effect of a positive default premium is that it increases the sizes of the alliances and reduces their number. We also generalize our analysis by considering non-proportional profit allocations. Specifically, we demonstrate what form of “fair” allocations (maybe non-proportional) can provide stability to alliances by showing disproportionate favoritism towards risky suppliers.

While this paper tackles how order default risk would affect alliance/coalition formation and how suppliers trade off the pros and cons of such a strategy, there are certainly many ways to extend this line of work. A more in-depth study would call for further differentiation among the suppliers

– more than the two levels considered in this paper, with more refined characterization of the risks they are exposed to. Extending the analysis to more general supply chain networks and partial order default settings would also be interesting. One could also consider other types of supply-side risks, including random yield and fluctuating raw material costs, possibly bringing risk correlation into the picture. Another possible avenue for future research is to focus on resources that are product- or relation-specific, e.g., inventory or capacity, and hence can only be shared under certain circumstances. This will possibly require separate analysis for complementary and substitutable cases. Last but not least, this paper focuses on understanding the interplay between two driving forces of forming alliances: supply risk mitigation and competition reduction. Factors such as demand risk and negotiation power are left outside the scope of the study. One possibility is to extend the model to incorporate some other potential drivers of alliance formation such as demand risk mitigation, benefits due to better access to markets, higher bargaining power and economies of scale. This would require significantly different modelling and analysis frameworks. However, our conjecture is that some of these factors would result in further adjustments in RASFs developed in this paper that in turn would affect the incentives for alliance formation. Another possibility is to find proxies for these drivers and empirically test and evaluate their relative importances in alliance decisions.

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Appendix

Common Notation	n	Number of suppliers
	N	The set of all suppliers
	$\mathscr{S}$	Alliance structure: $\mathscr{S}=\{S_1,...,S_m\}$
	m	Number of coalitions: $m=\lvert\mathscr{S}\rvert$
	S_k	The set of suppliers in coalition k, $k=1,2,..,m$
	F_i	Reserve fund of supplier i, $i=1,2,...n$
	F_S	Reserve fund of coalition S: $F_S=\sum_{i\in S}F_i$
	p	Retail price of the assembler/buyer
	c	Production cost for each supplier
	δ	Outside option penalty for the assembler/buyer if some supplier defaults
	Q_{S_k}	Supply limit of coalition S_k
	q_{S_k}	Order quantity of the downstream firm from coalition S_k
	w_{S_k}	Wholesale price of coalition S_k
	$\widetilde{w}_{S_k}$	Expected wholesale price for the buyer purchasing from coalition S_k
Complementary Suppliers	W	Wholesale price for the final product: $W=\sum w_{S_k}$
	q	Order quantity of the assembler, $q=q_{S_k}$, $k=1,2,...,m$
	c_{S_k}	Production cost for coalition S_k: $c_{S_k}=\lvert S_k\rvert c$
	C	Total production cost for the final product: $C=nc$
Substitutable Suppliers	Q	Total supply limit: $Q=\sum Q_{S_k}$
	q	Order quantity of the buyer, $q=\sum_{k=1}^{m}q_{S_k}$
	$\widetilde{w}$	Market-clearance expected wholesale price

Table A1 Notations

$D(p)$	Parameter Range	α	β	Pass-Through Rate
Linear-power Demand $(a-bp)^\theta$	$a,b>0,\theta\geq 1$	$a/(b\theta)$	$-1/\theta$	$\theta/(\theta+1)$
Exponential Demand ae^{-bp}	$a,b>0$	$1/b$	0	1
Iso-elastic Demand ap^{-b}	$a>0,b>n$	0	$1/b$	$b/(b-1)$

Table A2 Demand Functions $D(p)$

Proof of Assumption 1 for Commonly Used Distributions

We prove that Assumption 1 holds for normal, exponential, and Erlang distribution respectively. Consider the minimum reserve fund level among all suppliers $\underline{F} = \min_i\{F_i\} > \mathbf{E}[\xi_i]$. Then the reserve fund level for any individual suppliers or coalitions, F_S where $S \in N$, can only take the value on $\{\underline{F}\} \cup [2\underline{F}, \infty]$. We thereby denote $\mathbb{F}(\underline{F}) = \{\underline{F}\} \cup [2\underline{F}, \infty]$ the set of value reserve fund can set foot on given the minimum individual fund level $\underline{F}$. Then Assumption 1 essentially states that, for any $F \in \mathbb{F}$ and positive integer k, there are

(i) $V_1(F)$ decreases in F.

(ii) $V_k(kF)$ decreases in k for any given F.

(iii) $G_k(kF)/G_1(F)$ is a unimodal in F for any given k.

Lemma A1. *Assumption 1 holds when ξ_i's are i.i.d. on $N(\mu, \sigma^2)$.*

Proof of Lemma A1.

(i). First of all, we prove that $\dfrac{\partial G(F)/F}{\partial F} = \dfrac{g(F)F - G(F)}{F^2} \le 0$ for $F \ge 2\mu$. Since $\dfrac{\partial g(F)F - G(F)}{\partial F} = g'(F) < 0$ for all the $F \ge \mu$, we only need to show that $g(F)F - G(F) \le 0$ at $F = 2\mu$. Note that $g(F)F - G(F)|_{F=2\mu} \le 2\mu\dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{\mu^2}{2\sigma}} - 1/2$ which is maximized at $\mu = \sigma$. Therefore, $g(F)F - G(F)|_{F=2\mu} \le \sqrt{\dfrac{2}{\pi e}} - \dfrac{1}{2} < 0$ hence $\dfrac{\partial V_1(F)}{\partial F} \le 0$ for $F \ge 2\mu$.

Finally, since $F > \mu$ it is obvious that $G_1(F) > 1/2 > G_1(2F) - G_1(F)$. Thus $2G_1(F) > G_1(2F)$ and $V_1(F) > V_1(2F)$ for any $F > \mu$.

These prove that $V_1(F)$ decreases in F on $\mathbb{F}_0 = \{F_0\} \cup [2F_0, \infty]$ for any $F_0 > \mu$.

For (ii) and (iii), without loss of generality, consider $G_1(\cdot)$ the c.d.f. of standardized normal distribution with mean 0 and standard deviation 1, and $G_k(\cdot)$ is the c.d.f. of normal distribution with mean 0 and standard deviation $\sqrt{k}$ (the case with general normal distribution could be proved in a similar fashion). It can be verified that

$$G_k(kF) = \int_{-\infty}^{\sqrt{k}F} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx. \tag{A1}$$

(ii). It is sufficient to show that $\dfrac{1}{k}G_k(kF) \ge \dfrac{1}{k+1}G_{k+1}((k+1)F)$ for any $F > 0$. By (A1),

$$\frac{1}{k}G_k(kF) = \frac{1}{k}\int_{-\infty}^{\sqrt{k}F} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \ge \frac{1}{2k}$$

and

$$\begin{aligned} G_{k+1}((k+1)F) - G_k(kF) &= \int_{\sqrt{k}x}^{\sqrt{k+1}F} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \le \frac{1}{\sqrt{2\pi}}(\sqrt{k+1} - \sqrt{k})Fe^{-\frac{kF^2}{2}} \\ &= \frac{1}{\sqrt{2\pi}} \frac{F}{\sqrt{k+1}+\sqrt{k}} e^{-\frac{kF^2}{2}} \le \frac{1}{\sqrt{2\pi}} \frac{1}{2\sqrt{k}} F e^{-\frac{kF^2}{2}} \end{aligned}$$

Note that $Fe^{-\frac{kF^2}{2}}$ is maximized at $F = 1/\sqrt{k}$, we have $G_{k+1}((k+1)F) - G_k(kF) \le \dfrac{1}{\sqrt{2\pi}}\dfrac{1}{2k}e^{-\frac{1}{2}}$. Thus

$$G_{k+1}((k+1)F) - G_k(kF) \le \frac{1}{\sqrt{2\pi e}}\frac{1}{2k} < \frac{1}{2k} \le \frac{1}{k}G_k(kF).$$

This proves that $\dfrac{1}{k+1}G_{k+1}((k+1)F) \le \dfrac{1}{k}G_k(kF)$. Taking $k = 1$ we have $V_1(F) \ge V_2(2F)$.

(iii). We prove that $\dfrac{\partial G_1(F)/G_k(kF)}{\partial F}=0$ for at most one $F>0$. By (A1),

$$\frac{\partial G_1(F)/G_k(kF)}{\partial F} = \frac{e^{-\frac{F^2}{2}}\int_{-\infty}^{\sqrt{k}F} e^{-\frac{x^2}{2}}dx - \sqrt{n}e^{-\frac{nF^2}{2}}\int_{-\infty}^{F} e^{-\frac{x^2}{2}}dx}{(\int_{-\infty}^{\sqrt{k}F} e^{-\frac{x^2}{2}}dx)^2}$$
$$= \frac{e^{\frac{(n-1)F^2}{2}}/\sqrt{n} - G_1(F)/G_k(kF)}{e^{\frac{nF^2}{2}}\int_{-\infty}^{\sqrt{k}F} e^{-\frac{x^2}{2}}dx/\sqrt{n}}$$

We will show that there exists a unique $F>0$ such that

$$\frac{e^{\frac{(n-1)F^2}{2}}}{\sqrt{n}} - \frac{G_1(F)}{G_k(kF)} = 0. \tag{A2}$$

Due to the nature of normal distribution, it is straightforward for any $F>0$ that (1) $G_1(F)/G_k(kF)<1$; (2) $\lim_{F\to 0} G_1(F)/G_k(kF) = \lim_{F\to\infty} G_1(F)/G_k(kF) = 1$. Therefore, the roots for $\dfrac{\partial G_1(F)/G_k(kF)}{\partial F}=0$ can only be an odd number.

Suppose the root is not unique, e.g., there exist $F_0<F_1<F_2$ satisfying $\dfrac{\partial G_1(F)/G_k(kF)}{\partial F}=0$. Moreover, there should be $\dfrac{\partial^2 G_1(F)/G_k(kF)}{\partial F^2}$ is positive at F_0 and F_2, and negative at F_1, implying that $\dfrac{G_1(F_1)}{G_k(kF_1)} > \dfrac{G_1(F_2)}{G_k(kF_2)}$. On the other hand, (A2) suggests that $\dfrac{G_1(F_1)}{G_k(kF_1)} = \dfrac{e^{\frac{(n-1)F_1^2}{2}}}{\sqrt{n}} < \dfrac{e^{\frac{(n-1)F_2^2}{2}}}{\sqrt{n}} = \dfrac{G_1(F_2)}{G_k(kF_2)}$. Contradiction. Thus there exists a unique root F_0 satisfying the first order condition, and $G_1(F)/G_k(kF)$ is a unimodal when $F>0$. □

Next consider ξ_i follows Erlang distribution (r,θ), which is the sum of r i.i.d. random variable following exponential distribution with mean θ. Since exponential distribution is achieved when $r=1$, we prove Assumption 1 for Erlang and exponential distributions together.

LEMMA A2. *Assumption 1 holds when ξ_i's follow i.i.d. exponential or Erlang distribution with shape r and inverse rate θ.*

Proof of Lemma A2. Note that the sum of any k ξ_i's will follow Erlang distribution (kr,θ) with c.d.f.

$$G_k(F) = 1 - \sum_{i=0}^{kr-1} \frac{(\frac{F}{\theta})^i}{i!} e^{-\frac{F}{\theta}} = \frac{\sum_{i=kr}^{\infty} \frac{(\frac{F}{\theta})^i}{i!}}{e^{\frac{F}{\theta}}} \tag{A3}$$

(i). We first prove that $\dfrac{\partial V_1(F)}{\partial F}<0$ when $F\geq 2\mathbf{E}[\xi_i]=2r\theta$. By (A3),

$$\frac{\partial V_1(F)}{\partial F} = \frac{\sum_{i=r}^{\infty}\frac{(F/\theta)^i}{(i-1)!}e^{F/\theta} - \sum_{i=r}^{\infty}\frac{(F/\theta)^i}{i!}e^{F/\theta} - \sum_{i=r}^{\infty}\frac{(F/\theta)^i}{i!}\frac{F}{\theta}e^{F/\theta}}{F^2e^{2F/\theta}} = \frac{\frac{(F/\theta)^r}{(r-1)!} - \sum_{i=r}^{\infty}\frac{(F/\theta)^i}{i!}}{F^2e^{F/\theta}}.$$

As $F\geq 2r\theta$, there is

$$\frac{(F/\theta)^r}{(r-1)!} - \sum_{i=r}^{\infty}\frac{(F/\theta)^i}{i!} < \frac{(F/\theta)^r}{(r-1)!} - \sum_{i=r}^{2r}\frac{(F/\theta)^i}{i!} = \frac{(F/\theta)^r}{(r-1)!}[1-\sum_{i=0}^{r}\frac{(F/\theta)^i(r-1)!}{(r+i)!}] < \frac{(F/\theta)^r}{(r-1)!}[1-\frac{1}{r}(r+1)]<0.$$

Thus $\partial V_1(F)/\partial F<0$.

We next show that $V_1(F)>V_1(2F)$, or equivalently, $2G_1(F)>G_1(2F)$, for any $F>r\theta$. Note that $G_1(2F)/G_1(F) = \dfrac{\sum_{i=r}^{\infty}\frac{(\frac{2F}{\theta})^i}{i!}}{e^{\frac{F}{\theta}}\sum_{i=r}^{\infty}\frac{(\frac{F}{\theta})^i}{i!}}$ increases with r. Thus it is suffice to show $2G_1(F)>G_1(2F)$ at $r\to\infty$. This is true as $\lim_{r\to\infty} 2G_1(F) = \lim_{r\to\infty} 2(1-\dfrac{\sum_{i=0}^{r-1}\frac{(\frac{F}{\theta})^i}{i!}}{e^{\frac{F}{\theta}}}) = 0 = \lim_{r\to\infty}[1-\dfrac{\sum_{i=0}^{r-1}\frac{(\frac{2F}{\theta})^i}{i!}}{e^{\frac{F}{\theta}}}] = \lim_{r\to\infty} G_1(2F)$.

These prove that $V_1(F)$ decreases in F on $\mathbb{F}_0$ for any $F_0 > \mathbf{E}[\theta_i] = r\theta$.

(ii). We will show that $V_k(kF) \geq V_{k+1}((k+1)F)$, or equivalently $\frac{G_k(kF)}{k} \geq \frac{G_{k+1}((k+1)F)}{k+1}$, for any $k \geq 1$ and $F \geq r\theta$. By (A3),

$$\Delta_k = \frac{\sum_{i=kr}^{\infty} \frac{(\frac{kF}{\theta})^i}{i!}}{ke^{\frac{kF}{\theta}}} - \frac{\sum_{i=(k+1)r}^{\infty} \frac{(\frac{(k+1)F}{\theta})^i}{i!}}{(k+1)e^{\frac{(k+1)F}{\theta}}}$$

and

$$\frac{\partial \Delta_k}{\partial F} = \frac{(kF/\theta)^{kr-1}}{e^{kF/\theta}(kr-1)!} - \frac{((k+1)F/\theta)^{kr+r-1}}{e^{(k+1)F/\theta}(kr+r-1)!} = \frac{k^{kr-1}(F/\theta)^{kr+r-1}}{e^{(k+1)F/\theta}(kr-1)!}[\frac{e^{F/\theta}}{(F/\theta)^r} - (\frac{k+1}{k})^{kr}\frac{(k+1)^r}{\prod_{j=1}^{r}(kr+j)}].$$

It is suffice to verify that the above is positive at $F/\theta = r$, or equivalently $\frac{e}{(1+1/k)^k} \geq [\frac{(k+1)^r}{\prod_{j=1}^{r}(k+j/r)}]^{1/r}$. Note that the RHS is increasing in r, and $\lim_{r\to\infty}[\frac{(k+1)^r}{\prod_{j=1}^{r}(k+j/r)}]^{1/r} = e^{\int_0^1 \ln\frac{k+1}{k+x}dx} = e^{1-k\ln\frac{k+1}{k}} = \frac{e}{(1+1/k)^k}$. Thus $\dfrac{\partial \Delta_k}{\partial F} \geq 0$.

We next show that $\Delta_k > 0$ at $F/\theta = r$, or essentially, $\dfrac{\sum_{i=kr}^{\infty}\frac{(kr)^i}{i!}}{kre^{kr}} - \dfrac{\sum_{i=(k+1)r}^{\infty}\frac{((k+1)r)^i}{i!}}{(k+1)re^{(k+1)r}} > 0$ for any $r \geq 1$. It is suffice to show that

$$\frac{\sum_{i=k}^{\infty}\frac{k^i}{i!}}{ke^k} - \frac{\sum_{i=k+1}^{\infty}\frac{(k+1)^i}{i!}}{(k+1)e^{k+1}} > 0. \tag{A4}$$

For any $s, x > 0$, denote $\Gamma(s,x) = \int_x^{\infty} t^{s-1}e^{-t}dt$ and $\gamma(s,x) = \int_0^x t^{s-1}e^{-t}dt$. Specifically when s and x are positive integers, there are $\Gamma(s,x) = (s-1)!e^{-x}\sum_{i=0}^{s-1}\frac{x^i}{i!}$ and $\gamma(s,x) = (s-1)!e^{-x}\sum_{i=s}^{\infty}\frac{x^i}{i!}$. Then (A4) is equivalent to

$$(k+1)\gamma(k,k) > \gamma(k+1,k+1), \tag{A5}$$

and it is suffice to prove the above for any $k > 0$. Note that integration by parts implies that $\gamma(k+1,k+1) = k\gamma(k,k+1) - (k+1)^k e^{k+1}$. Thus (A5) becomes $\gamma(k,k) + (k+1)^k e^{-(k+1)} > k(\gamma(k,k+1) - \gamma(k,k)) = k\int_k^{k+1} t^{k-1}e^{-t}dt = \int_k^{k+1} e^{-t}dt^k$. Integration by parts suggests that $\int_k^{k+1} e^{-t}dt^k = e^{-t}t^k|_k^{k+1} + \int_k^{k+1} t^k e^{-t}dt = e^{-(k+1)}(k+1)^k - e^{-k}k^k + \int_k^{k+1} t^k e^{-t}dt$. Since t^k/e^t decreases on t when $t \geq k$, there is $\int_k^{k+1} t^k e^{-t}dt < k^k/e^k$. Thus $\int_k^{k+1} e^{-t}dt^k < e^{-(k+1)}(k+1)^k < e^{-(k+1)}(k+1)^k + \gamma(k,k)$. (A5) holds for any $k > 0$ and consequently (A4) for any positive integer k.

The above shows that $\dfrac{\partial \Delta_k}{\partial F} \geq 0$ and $\Delta_k > 0$ at $F/\theta = r$, which together prove that $\Delta_k > 0$.

(iii). We prove that $\dfrac{\partial G_1(F)/G_k(kF)}{\partial F} = 0$ for at most one $F > 0$.

$$\begin{aligned}\frac{\partial G_1(F)/G_k(kF)}{\partial F} &= \frac{g_1(F)G_k(kF) - kg_k(kF)G_1(F)}{G_k^2(kF)} \\ &= \frac{g_k(kF)}{G_k(kF)}\left[\frac{g_1(F)}{g_k(kF)} - k\frac{G_1(F)}{G_k(kF)}\right]\end{aligned} \tag{A6}$$

Note that for exponential distribution, $g_1(F)/g_k(kF) = \dfrac{(kr-1)!}{(r-1)!}\dfrac{\theta^{(k-1)r}}{k^{kr-1}}\dfrac{e^{\frac{(k-1)F}{\theta}}}{F^{(k-1)r}}$ is convex in F on $[0,\infty]$. By (A3), there is $\dfrac{G_1(F)}{G_k(kF)} = \sum_{i=r}^{\infty}\dfrac{(\frac{F}{\theta})^i}{i!} / \sum_{i=kr}^{\infty}\dfrac{(\frac{kF}{\theta})^i}{i!}$. Thus $\lim_{F\to 0}\dfrac{G_1(F)}{G_k(kF)} = \infty$ and $\lim_{F\to\infty}\dfrac{G_1(F)}{G_k(kF)} = 1$. Together with (A6), it can be verified that $\lim_{F\to 0}\dfrac{\partial\frac{G_1(F)}{G_k(kF)}}{\partial F} < 0$ and $\lim_{F\to\infty}\dfrac{\partial\frac{G_1(F)}{G_k(kF)}}{\partial F} > 0$. Therefore, the roots for $\dfrac{\partial G_1(F)/G_k(kF)}{\partial F} = 0$ can only be an odd number.

Suppose the root is not unique, e.g., there exist $F_0 < F_1 < F_2$ satisfying $\frac{\partial G_1(F)/G_k(kF)}{\partial F} = 0$. Moreover, $\frac{\partial^2 G_1(F)/G_k(kF)}{\partial F^2}$ is positive at F_0 and F_2, and negative at F_1, implying that $\frac{G_1(F_1)}{G_k(kF_1)} > \frac{G_1(F_2)}{G_k(kF_2)}$ and $\frac{G_1(F_1)}{G_k(kF_1)} > \frac{G_1(F_0)}{G_k(kF_0)}$. On the other hand, $\frac{\partial G_1(F)/G_k(kF)}{\partial F} = 0$ at F_0, F_1 and F_2. Therefore, there should also be $\frac{g_1(F_1)}{g_k(kF_1)} > \frac{g_1(F_2)}{g_k(kF_2)}$ and $\frac{g_1(F_1)}{g_k(kF_1)} > \frac{g_1(F_0)}{g_k(kF_0)}$, which contradicts with the fact that $g_1(F)/g_k(kF)$ is a convex function. Thus there exists a unique root F_0 satisfying the first order condition and $G_1(F)/G_k(kF)$ is a unimodal. $\square$

Stage 2&3: Operational Decision Makings

Proof of Proposition 1 Denote the effective wholesale price of alliance S_k as $\widetilde{w}_{S_k} = w_{S_k} + \bar{G}_{|S_k|}(F_{S_k})\delta_{S_k}$ and $D^{-1}(\cdot)$ the inverse demand function. Then

- For the assembler dealing with complementary suppliers, the same order quantity applies to all alliances and $q_{S_1} = q_{S_2} = ... = q_{S_m} = q$. The assembler's expected profit Π_0 is then

$$\Pi_0(q) = \max_{q \le \min\{Q_{S_k}\}} q\left(D^{-1}(q) - \sum_{k=1}^{m} \widetilde{w}_{S_k}\right)$$

- for the buyer dealing with substitutable suppliers, the total order to place with the suppliers is $q = \sum_{k=1}^{m} q_{S_k}$. The buyer's expected profit Π_0 can be expressed as

$$\Pi_0(q_{S_k},\, k = 1,2,...,m) = \max_{q_{S_k} \le Q_{S_k},\, k=1,2,...,m} \sum_{k=1}^{m} q_{S_k}\left(D^{-1}(q) - \widetilde{w}_{S_k}\right)$$

In either case, let $\mathbf{q}^*(\mathbf{w},\mathbf{Q})$ be the optimal order quantity that solves the above optimization problems. Given $(w_{S_{-k}}, Q_{S_{-k}})$, supplier i in alliance S_k has the following profit if survived:

$$\pi_i = \max_{0 \le w_{S_k}, 0 \le Q_{S_k}} \frac{F_i}{F_{S_k}}(w_{S_k} - c_{S_k})q^*_{S_k}$$

The analysis of the above problems is as follows.

- **Assembly systems**. In this case the assembler will order the same quantity from each alliance $q = q_{S_1} = ... = q_{S_m} \le \min\{Q_{S_1}, ..., Q_{S_m}\}$. Therefore, no alliance is incentivized to commit more capacity than the others hence in equilibrium there should be $Q_{S_1} = ... = Q_{S_m} = Q$. Similarly, noting that the production amount q is also capped by the minimum capacity $\min\{Q_{S_1}, ..., Q_{S_m}\}$, it is also straightforward that alliances will not invest in excess capacities, i.e., $Q = q$. Thus it is equivalent to consider a problem in which alliances determine wholesale prices $\{w_{S_1}, ..., w_{S_k}\}$ first, and the assembler chooses order quantity q and retail price $p = D^{-1}(q)$ accordingly. First consider the assembler's problem. Taking into account of the *effective* expected wholesale price she has to pay, the assembler practically needs to choose a retail price that maximizes her expected profit $\max_p \Pi_0 = \max_p \left[p - \sum_{k=1}^{m}\left(w_{S_k} + \bar{G}_{|S_k|}(F_{S_k})\delta_{S_k}\right)\right] D(p)$. Solving the FOC gives

$$p^* = \frac{\alpha + \sum_{k=1}^{m}\left(w_{S_k} + \bar{G}_{|S_k|}(F_{S_k})\delta_{S_k}\right)}{1-\beta} \tag{A7}$$

Given the assembler's reaction in (A7), alliance k solves the following problem:

$$\max_{w_{S_k}} \pi_{S_k} = \max_{w_{S_k}} D(p^*)(w_{S_k} - c_{S_k})G_{|S_k|}(F_{S_k}) \tag{A8}$$

By (A7) we have

$$\frac{\partial D(p^*)}{\partial w_{S_k}} = D^{'}(p^*)\frac{\partial p^*}{\partial w_{S_k}} = D^{'}(p^*)\frac{1}{1-\beta} \tag{A9}$$

The FOC of (A8) implies that $\frac{\partial D(p^*)}{\partial w^*_{S_k}}(w^*_{S_k} - c_{S_k}) + D(p^*) = D^{'}(p^*)\frac{1}{1-\beta}(w^*_{S_k} - c_{S_k}) + D(p^*) = 0$ and by (1) and (A7) we have

$$w^*_{S_k} - c_{S_k} = -(1-\beta)\frac{D(p^*)}{D^{'}(p^*)} = (1-\beta)(\alpha+\beta p^*) = \alpha + \beta\sum_{k=1}^{m}\left(w^*_{S_k} + \bar{G}_{|S_k|}(F_{S_k})\delta_{S_k}\right), \qquad i = 1,2,...,m \tag{A10}$$

Adding the m equations together, there is $W^* - C = m\alpha + m\beta\sum_{k=1}^{m}\bar{G}_{|S_k|}(F_{S_k})\delta_{S_k} + m\beta W^*$. Hence

$$W^* = \frac{m\alpha + m\beta\sum_{k=1}^{m}\bar{G}_{|S_k|}(F_{S_k})\delta_{S_k} + C}{1-m\beta}$$

Substituting the above into (A7) and (A10) gives the optimal retail price p^* and equilibrium wholesale prices $w^*_{S_k}$ immediately.

Note that when capacity cost is nontrivial, e.g., $c_K > 0$, then similar results could be obtained by replacing c for $c + c_K$. Therefore the structure of the equilibrium stays the same and it is without loss of generality for us to assume $c_K = 0$.

• **Competitive Markets**. Assume that *effective* expected wholesale prices are ordered with their index, i.e., $\widetilde{w}_{S_i} \le \widetilde{w}_{S_j}$ if $i < j$. Basically, suppliers with smaller index are those more cost efficient under default risk. Then a profit-maximizing buyer should start ordering from lower-indexed suppliers to higher-indexed ones. That is, $q_{S_1} = Q_{S_1},..., q_{S_{l-1}} = Q_{S_{l-1}}$, $q_{S_l} < Q_{S_l}$ for some $1 \le l \le m$, and $q_{S_{l+1}} = ... = q_{S_m} = 0$. The buyer's expected profit Π_0 can then be expressed as

$$\Pi_0 = \max_{\sum_{k=1}^{l-1} Q_{S_k} \le q < \sum_{k=1}^{l} Q_{S_k}} \sum_{k=1}^{l-1} Q_{S_k}\left(D^{-1}(q) - \widetilde{w}_{S_k}\right) + \left(q - \sum_{k=1}^{l-1} Q_{S_k}\right)\left(D^{-1}(q) - \widetilde{w}_{S_l}\right)$$

In solving the buyer's problem, consider an *adjusted* demand function $\widetilde{D}(w)$ (its inverse function $\widetilde{D}^{-1}$ is shown in Table A3), where

$$\widetilde{D}(w) = \arg\max_q qD^{-1}(q) - wq$$

Then for a given l, the optimal solution to the buyer's problem is $q = \left(\widetilde{D}(w_{S_l}) \vee \sum_{k=1}^{l-1} Q_{S_k}\right) \wedge \sum_{k=1}^{l} Q_{S_k}$. Note that $\widetilde{D}(w_{S_l})$ is non-increasing in l. Then there exists a unique l^* such that $\sum_{k=1}^{l^*-1} Q_{S_k} \le \widetilde{D}(w_{S_{l^*}}) < \sum_{k=1}^{l^*} Q_{S_k}$ and $q^* = \widetilde{D}(w_{S_{l^*}})$.

Particularly when $m = 2$, the above problem falls the same as the capacity-constrained Bertrand problem studied by Kreps and Scheinkman (1983), under demand function $\widetilde{D}(\cdot)$. Many consequent papers (e.g., Boccard and Wauthy 2000, Moreno and Ubeda 2006, Lepore 2009) also prove the robustness of this result under oligopoly scenario ($m > 2$). We therefore adopt the finding that capacity-constrained pricing competition yields the same equilibrium as quantity competition, and consider a revised problem as follows:

The alliances determine the production quantities $\{q_{S_1},...,q_{S_m}\}$, being aware that the buyer will procures every unit that has been produced from alliance k at wholesale price $w_{S_k} = \widetilde{w} - \bar{G}_{|S_k|}(F_{S_k})\delta$ where $\widetilde{w} = \widetilde{D}^{-1}\left(\sum_{k=1}^{m} q_{S_k}\right)$. The retail price is set at $p = D^{-1}(q)$.

D(p)	Parameter Range	$\widetilde{D}^{-1}(Q)$
Linear-power Demand $(a-bp)^{\theta}$	$a,b>0,\theta\geq 0$	$a/b-Q^{1/\theta}(1+\theta)/(b\theta)$
Exponential Demand ae^{-bp}	$a,b>0$	$\ln(a/Q)^{1/b}-1/b$
Iso-elastic Demand ap^{-b}	$a>0,b>n$	$(a/Q)^{1/b}(b-1)/b$

Table A3 Adjusted demand function $\widetilde{D}^{-1}$

Note that the buyer's optimal reaction has already been counted in $\widetilde{D}$. Thus it is optimal for her to purchase all the available q units in the market at given *effective* expected wholesale price $\widetilde{w}$. For this reason, we refer $\widetilde{w}$ as the *market-clearance expected wholesale price*. The problem can be solved by analyzing the alliances' decisions directly. Basically, Alliance k solves

$$\max_{q_{S_k}} \pi_{S_k} = \max_{q_{S_k}} q_{S_k}(\widetilde{D}^{-1}(q) - \bar{G}_{|S_k|}(F_{S_k})\delta - c)G_{|S_k|}(F_{S_k}) \tag{A11}$$

The FOC is

$$\frac{\partial \widetilde{D}^{-1}(q)}{\partial q} q_{S_k} + \widetilde{D}^{-1}(q) = c + \bar{G}_{|S_k|}(F_{S_k})\delta \tag{A12}$$

We next prove the results based on linear-power demand. For other demand functions in Table A3, proofs can be done in a similar fashion. Consider linear-power demand $D(p)=(a-bp)^{\theta}$. By Table A3, $\frac{\partial \widetilde{D}^{-1}(q)}{\partial q} = -\frac{1+\theta}{b\theta^2}q^{1/\theta-1}$. Thus the FOC requires $-\frac{1+\theta}{b\theta^2}q^{-1+1/\theta}q_{S_k} + \frac{a}{b} - \frac{1+\theta}{b\theta}q^{1/\theta} = c + \bar{G}_{|S_k|}(F_{S_k})\delta$, $\forall k=1,..,m$. The sum of the m equations suggests that $-\frac{1+\theta}{b\theta^2}q^{1/\theta} + \frac{ma}{b} - \frac{1+\theta}{b\theta}mq^{1/\theta} = m\widetilde{c}$, $\forall k=1,..,m$. It can be solved that $q^* = \left[\frac{(a-b\widetilde{c})\theta}{1+\theta}\frac{m\theta}{1+m\theta}\right]^{\theta}$. Thus by Table A3 $p^* = D^{-1}(q^*) = a/b - (q^*)^{1/\theta}/b = \frac{a}{b}\frac{1+(m+1)\theta}{(1+\theta)(1+m\theta)} + \widetilde{c}\frac{\theta}{1+\theta}\frac{m\theta}{1+m\theta}$ and $\widetilde{w}^* = \widetilde{D}^{-1}(q^*) = a/b - (q^*)^{1/\theta}(1+\theta)/(b\theta) = \frac{a}{b(1+m\theta)} + \widetilde{c}\frac{m\theta}{1+m\theta}$.

By Table A2 we know that for linear-power demand, $\alpha = a/(b\theta)$ and $\beta = -1/\theta$. It can be verified that

$$p^* = \frac{\alpha + m\widetilde{c}}{(n-\beta)(1-\beta)} + \frac{\alpha}{1-\beta}, \quad \widetilde{w}^* = \frac{\alpha+m\widetilde{c}}{m-\beta}, \quad w^*_{S_k} = \frac{\alpha+m\widetilde{c}}{m-\beta} - \bar{G}_{|S_k|}(F_{S_k})\delta = \frac{\alpha+\beta\widetilde{c}}{m-\beta} + c + \widetilde{c} - \hat{c}_{S_k}.$$

In addition, by (A12) $q^*_{S_k}/q^* = \frac{1}{m} + \frac{1+m\theta}{m}\frac{(\sum_i \bar{G}_{|S_i|}(F_{S_i})\delta/m - \bar{G}_{|S_k|}(F_{S_k})\delta)}{a/b - c - \sum_k \bar{G}_{|S_k|}(F_{S_k})\delta_k/m} = \frac{1}{m} + \frac{1-m\beta}{m}\frac{\widetilde{c}-\hat{c}_{S_k}}{\alpha+\beta\widetilde{c}}$. □

Corollary 1. *As the number of coalitions m decreases,*

(i) when the suppliers are complementary: the ex-post profit of each coalition increases, the expected profit of the downstream firm (assembler) increases, and consumer surplus increases;

(ii) when the suppliers are substitutable and $\delta=0$: the ex-post profit of each coalition increases, the expected profit of the downstream firm (buyer) decreases, and consumer surplus decreases.

Proof of Corollary 1.

• **Assembly Systems**. It follows immediately from Proposition 1 that the total profit of coalition S_k if it survives is $\pi_{S_k} = (w^*_{S_k} - c_{S_k})Q^* = \frac{\alpha+\beta\widetilde{C}}{1-m\beta} D\left(\frac{m\alpha+\widetilde{C}}{(1-m\beta)(1-\beta)} + \frac{\alpha}{1-\beta}\right)$ and the expected profit for the assembler is $\Pi_0 = (p^* - \sum w^*_{S_k} - \bar{G}_{|S_k|}(F_{S_k})\delta_{S_k})Q^* = \left(\frac{m\alpha+\widetilde{C}}{(1-m\beta)}\frac{\beta}{1-\beta} + \frac{\alpha}{1-\beta}\right) D\left(\frac{m\alpha+\widetilde{C}}{(1-m\beta)(1-\beta)} + \frac{\alpha}{1-\beta}\right)$. The results that $\frac{\partial\pi_{S_k}}{\partial m} \le 0$ and $\frac{\partial\Pi_0}{\partial m} \le 0$ can be shown in a similar manner as Yin (2010) with C is substituted by $\widetilde{C}$.

In addition, if m is reduced due to a merger between two coalitions, it can be verified that $\widetilde{C}$ is reduced. As both π_{S_k} and Π_0 decrease with $\widetilde{C}$, the profits for both the supplier coalitions and the downstream firm increase as a result of a smaller m.

For $\widetilde{C}$,

$$\begin{aligned}\frac{\partial\pi_{S_k}}{\partial\widetilde{C}} &= \frac{\beta}{1-m\beta}D(p^*) + \frac{\alpha+\beta\widetilde{C}}{1-m\beta}D'(p^*)\frac{1}{(1-m\beta)(1-\beta)} \sim \beta + (\alpha+\beta\widetilde{C})\frac{D'(p^*)}{D(p^*)}\frac{1}{(1-m\beta)(1-\beta)}\\ &= \beta - (\alpha+\beta\widetilde{C})\frac{1}{\alpha+\beta p^*}\frac{1}{(1-m\beta)(1-\beta)} \sim \beta - 1 < 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\Pi_0}{\partial\widetilde{C}} &= \frac{\beta}{(1-m\beta)(1-\beta)}D(p^*) + \left(\frac{m\alpha+\widetilde{C}}{(1-m\beta)}\frac{\beta}{1-\beta} + \frac{\alpha}{1-\beta}\right)D'(p^*)\frac{1}{(1-m\beta)(1-\beta)}\\ &\sim \beta + \left(\frac{m\alpha+\widetilde{C}}{(1-m\beta)}\frac{\beta}{1-\beta} + \frac{\alpha}{1-\beta}\right)\frac{D'(p^*)}{D(p^*)} = \beta - \left(\frac{m\alpha+\widetilde{C}}{(1-m\beta)}\frac{\beta}{1-\beta} + \frac{\alpha}{1-\beta}\right)\frac{1}{\alpha+\beta p^*} \sim \beta - 1 < 0\end{aligned}$$

Obviously $\widetilde{C}$ increases with δ. So π_{S_k} and Π_0 also decrease with δ.

• **Competitive Markets**. It follows immediately from Proposition 1 that the total profit of coalition S_k if it survives is $\pi_{S_k} = (\widetilde{w}^* - \hat{c}_{S_k})Q^*_{S_k} = \left[\frac{\alpha+m\widetilde{c}-(m-\beta)\hat{c}_{S_k}}{m-\beta}\right]^2 \frac{1}{\alpha+\beta\widetilde{c}} D\left(\frac{\alpha+m\widetilde{c}}{(m-\beta)(1-\beta)} + \frac{\alpha}{1-\beta}\right)$. The expected profit for the buyer is $\Pi_0 = (p^* - \widetilde{w}^*)Q^* = \left(\frac{\alpha+m\widetilde{c}}{(m-\beta)}\frac{\beta}{1-\beta} + \frac{\alpha}{1-\beta}\right) D\left(\frac{\alpha+m\widetilde{c}}{(m-\beta)(1-\beta)} + \frac{\alpha}{1-\beta}\right)$. Note that $\partial\frac{\alpha+\beta\widetilde{c}}{m-\beta}/\partial m < 0$ and $\pi_{S_k} \sim \left[\frac{\alpha+\beta\widetilde{c}}{m-\beta} + \widetilde{c} - \hat{c}_{S_k}\right]^2 D\left(\frac{\alpha+\beta\widetilde{c}}{m-\beta}\frac{1}{1-\beta} + \frac{\alpha+\widetilde{c}}{1-\beta}\right)$, we can show

$$\begin{aligned}\frac{\partial\pi_{S_k}}{\partial m} &\sim 2\left[\frac{\alpha+\beta\widetilde{c}}{m-\beta} + \widetilde{c} - \hat{c}_{S_k}\right]\left(\partial\frac{\alpha+\beta\widetilde{c}}{m-\beta}/\partial m\right)D(p^*) + \left[\frac{\alpha+\beta\widetilde{c}}{m-\beta} + \widetilde{c} - \hat{c}_{S_k}\right]^2 D'(p^*)\frac{1}{1-\beta}\left(\partial\frac{\alpha+\beta\widetilde{c}}{m-\beta}/\partial m\right)\\ &\sim -2D(p^*) - \left[\frac{\alpha+\beta\widetilde{c}}{m-\beta} + \widetilde{c} - \hat{c}_{S_k}\right]D'(p^*)\frac{1}{1-\beta} \sim -2 + \frac{\frac{\alpha+\beta\widetilde{c}}{m-\beta} + \widetilde{c} - \hat{c}_{S_k}}{(\alpha+\beta p^*)(1-\beta)}\\ &= -2 + \frac{\frac{\alpha+\beta\widetilde{c}}{m-\beta} + \widetilde{c} - \hat{c}_{S_k}}{\frac{m}{m-\beta}(\alpha+\beta\widetilde{c})} = -2 + \frac{1 + (m-\beta)\frac{\widetilde{c} - \hat{c}_{S_k}}{\alpha+\beta\widetilde{c}}}{m} = -2 + Q^*_{S_k}/Q^* < 0\end{aligned}$$

Now consider $\frac{\partial\Pi_0}{\partial m}$. We find that if $D(p) = (a-bp)^\theta$, then $\Pi_0(m) \sim \left(\frac{m\theta}{1+m\theta}\right)^{\theta+1}$ which increases with m. If $D(p) = ae^{-bp}$, then $\Pi_0(m) \sim \frac{a}{be}e^{-1/m}$ which also increases with m. Finally when $D(p) = ap^{-b}$, there is also $\Pi_0(m) \sim \left(\frac{mb-1}{mb}\right)^{b-1}$ increasing with m. $\square$

$D(p)$	$U(m)$ Complementary Suppliers	$U(m)$ Substitutable Suppliers
Linear-power Demand $(a-bp)^\theta$	$\left(\frac{\theta+m+1}{\theta+m}\right)^{\theta+1}$	$\left(\frac{m+1}{m}\right)^2\left(\frac{\theta+1/(m+1)}{\theta+1/m}\right)^{\theta+1}$
Exponential Demand ae^{-bp}	e	$\left(\frac{m+1}{m}\right)^2 e^{-\frac{1}{m(m+1)}}$
Iso-elastic Demand ap^{-b}	$\left(\frac{b-m}{b-m-1}\right)^{b-1}$	$\left(\frac{m+1}{m}\right)^2\left(\frac{b-1/m}{b-1/(m+1)}\right)^{b-1}$

Table A4 $U(m)=\pi(m)/\pi(m+1)$ **for Complementary vs. Substitutable Suppliers**

Stage 1: Stable Coalition Structures

$U(m)=\pi(m)/\pi(m+1)$ where $\pi(m)$ is the ex-post profit each alliance receives if there is m coalitions in total. It has been shown in the proof of Corollary 1 that the ex post profit of any alliance, π_{S_k}, depends upon the number of coalitions that have been formed, m. Specifically when $\delta=0$, the ex-post profit for any alliance under an m-coalition structure is $\pi(m)=\frac{\alpha+\beta C}{1-m\beta}D\left(\frac{m\alpha+C}{(1-m\beta)(1-\beta)}+\frac{\alpha}{1-\beta}\right)$, which decreases with m.

The characterization for $U(m)$ with *complementary* suppliers appeared in Yin (2010). The extended framework in Huang et al (2012) developed the representations of $U(m)$ for *substitutable* suppliers. See Table A4 for detailed representation. The following definition will be used throughout the rest of the proof.

Definition 2. A coalition structured $\mathscr{K}=\{K_1,K_2,...,K_l\}$ is *ordered* if $|F_{K_t}|\le|F_{K_j}|$ for any $t<j$.

1. Symmetric Suppliers

Given the recursive nature of the definition of coalition-proof equilibrium, we first characterize self-enforcing strategies for one player, and then the coalition-proof Nash equilibrium (CPNE) for a one-player game. On top of these results, we can further derive self-enforcing strategies and CPNE two and more players.

Single-player game: For an arbitrary player, suppose the rest of the $n-1$ players form ordered coalitions $\{K_1,K_2,...,K_l\}$ where $n_i=|K_i|$ for $i=1,...,l$. The player has $l+1$ strategies: join coalition K_i (if K_i also accepts it) for $i\in\{1,2,...,l\}$, or be an independent player. The expected profit for the single player is $V_{n_i+1}(F)\pi(l)$ and $V_1(F)\pi(l+1)$ respectively. For each member in coalition K_i, the expected profit is $V_{n_i+1}(F)\pi(l)$ and $V_{n_i}(F)\pi(l+1)$ respectively. Apparently, if the player wishes to join K_i, the coalition would also accept it. CPNE are the ones that maximizes the player's expected profit:

Lemma A3. *The CPNE for the game with single-player s*
(i) $\{K_1\cup\{s\},K_2,...,K_l\}$, *that is, join* K_1, *if* $U(l)\ge\frac{V_1(F)}{V_{n_1+1}((n_1+1)F)}$

(ii) $\{\{s\}, K_1, K_2, ..., K_l\}$, *that is, be independent, if* $U(l) \leq \dfrac{V_1(F)}{V_{n_1+1}((n_1+1)F)}$
where ordered structure $\{K_1, K_2, ..., K_l\}$ *represents the strategy for the rest of the players* $N - \{s\}$.

y-player game: Consider an arbitrary y players and the rest $n-y$ players forming ordered structure $\mathscr{K} = \{K_1, K_2, ..., K_l\}$ and $n_i = |K_i|$ for $i = 1, 2, ..., l$. Then strategies of the y players will be in the form of

$$\{P_1, ..., P_a, K_1 \cup Q_1, ..., K_b \cup Q_b, K_{b+1}, ..., K_l\}$$

where $|P_i| = p_i > 0$, for $i = 1, ..., a$, $|Q_j| = q_j > 0$ for $j = 1, ..., b$ and $\sum_{i=1}^a p_i + \sum_{j=1}^b q_j = y$.

Consider assigning y players to the coalitions in $\mathscr{K}$. Naturally, a *feasible* assignment $\mathbf{Y} = (y_1, y_2, ..., y_l)$ should satisfy $\sum_{i=1}^l y_i = y$ and $y_i \geq 0$ for any $1 \leq i \leq l$. That is, allocate y_i players to coalition K_i where $i = 1, 2, ..., l$. Among the numerous feasible assignments, we further define the *efficient* assignment as follows:

Definition 3. A feasible assignment $\mathbf{Y}$ is efficient if

(i) $y_1 \geq y_2 \geq ... \geq y_l$

(ii) $n_1 + y_1 \leq n_2 + y_2 \leq ... \leq n_l + y_l$

(iii) for any feasible assignment $\mathbf{Y}'$, $b = \max\{i : y_i > 0, 1 \leq i \leq l\} \leq b' = \max\{i : y_i' > 0, 1 \leq i \leq l\}$ and $n_b + y_b \leq n_{b'} + y_{b'}'$.

Denote the resulting structure of allocating y players to coalitions in $\mathscr{K}$ via the *efficient assignment* as $\mathscr{K}^*(\mathscr{K}, y) = \{K_1^*, ..., K_l^*\}$, and $n^*(\mathscr{K}, y) = n_b + y_b$. We can prove the following results for the CPNE with y players:

Proposition A1. *For y players, given that the rest of the $n-y$ players form ordered structure $\mathscr{K} = \{K_1, K_2, ..., K_l\}$,*

(i) the CPNE satisfies $a = 0$ if and only if $U(l) \geq \dfrac{V_1(F)}{V_{n^*(\mathscr{K},y)}(n^*(\mathscr{K}, y)F)}$

(ii) the CPNE satisfies $0 < a < y$, if and only if

$$\begin{aligned}
U(l) &\leq \frac{V_1(F)}{V_{n^*(\mathscr{K},y)}(n^*(\mathscr{K}, y)F)}, \\
U(l+i) &\leq \frac{V_1(F)}{V_{n^*(\mathscr{I}_i \cup \mathscr{K}, y-i)}(n^*(\mathscr{I}_i \cup \mathscr{K}, y-i)F)} \text{ for } 1 \leq i \leq a-1, \\
U(l+a) &\geq \frac{V_1(F)}{V_{n^*(\mathscr{I}_a \cup \mathscr{K}, y-a)}(n^*(\mathscr{I}_a \cup \mathscr{K}, y-a)F)}
\end{aligned}$$

(iii) the CPNE is $a = y$ if and only if

$$\begin{aligned}
U(l) &\leq \frac{V_1(F)}{V_{n^*(\mathscr{K},y)}(n^*(\mathscr{K}, y)F)}, \\
U(l+i) &\leq \frac{V_1(F)}{V_{n^*(\mathscr{I}_i \cup \mathscr{K}, y-i)}(n^*(\mathscr{I}_i \cup \mathscr{K}, y-i)F)} \text{ for } 1 \leq i \leq y-1
\end{aligned}$$

where $\mathscr{I}_k$ is the coalition structure with k independent suppliers, i.e., $\mathscr{I}_k = \{P_1, ..., P_k\}$ where $|P_i| = 1$ for $i = 1, ..., k$.

Proof. We prove the proposition using math induction. By Lemma A5, the statements are true for $y = 1$. Suppose that the same holds for $y \leq Y - 1$, we now prove that they are also true for $y = Y$.

(i). We can show that the structure $\mathscr{K}^*(\mathscr{K},Y)$, which satisfies $a=0$, is the CPNE when

$$U(l) \geq \frac{V_1(F)}{V_{n^*(\mathscr{K},y)}(n^*(\mathscr{K},y)F)} \tag{A13}$$

First, we prove that the $\mathscr{K}^*$ is self-enforcing (i.e., every subgroup plays CPNE) under (A13). Consider an proper subset of the Y players, S. Let $\mathscr{K}^*_{-S}$ denote the strategy for the rest of the $n-|S|$ players in $\mathscr{K}^*$. Apparently, efficiently assigning S into $\mathscr{K}^*_{-S}$ results in the same structure $\mathscr{K}^*$. Hence $n^*(\mathscr{K}^*_{-S}, y-|S|) \leq n^*(\mathscr{K},y)$ and by (A13)

$$U(l) \geq \frac{V_1(F)}{V_{n^*(\mathscr{K}^*_{-S},y-|S|)}(n^*(\mathscr{K}^*_{-S},y-|S|)F)}, \quad \forall |S| < Y \tag{A14}$$

Applying Proposition A1 (i) for $y<Y$, we know that $\mathscr{K}^*$ is the CPNE for any subset of players S also. Therefore $\mathscr{K}^*$ is self-enforcing.

Second, we can show that $\mathscr{K}^*$ that strictly dominates any other strategy $\{P_1', ..., P_a', K_1' \cup Q_1', ..., K_b' \cup Q_b', K_{b+1}', ..., K_l'\}$ where $a>0$. Comparing the expected profit, it is sufficient to prove that $V_{n^*}(n^*F)\pi(l) \geq V_{\min_{t=1,...,a;i=1,...,b}\{p_t, n_i+q_i\}}(\min_{t=1,...,a;i=1,...,b}\{p_t, n_i+q_i\}F)\pi(l+a)$ or equivalently,

$$\frac{\pi(l)}{\pi(l+a)} \geq \frac{V_{\min_{t=1,...,a;i=1,...,b}\{p_t,n_i+q_i\}}(\min_{t=1,...,a;i=1,...,b}\{p_t,n_i+q_i\}F)}{V_{n^*}(n^*F)} \tag{A15}$$

By Corollary 1, $\pi(l+1) \geq \pi(l+a)$ hence $\dfrac{\pi(l)}{\pi(l+a)} \geq U(l)$. Together with (A13),

$$\frac{\pi(l)}{\pi(l+a)} \geq \frac{V_1(F)}{V_{n^*}(n^*F)} \geq \frac{V_{\min_{t=1,...,a;i=1,...,b}\{p_t,n_i+q_i\}}(\min_{t=1,...,a;i=1,...,b}\{p_t,n_i+q_i\}F)}{V_{n^*}(n^*F)}.$$

Hence (A15) holds. Also, $\mathscr{K}^*$ dominates other structures with $a=0$ due to part (iii) of Definition 3.

Finally, we prove that $\mathscr{K}^*$ cannot be a CPNE if the reverse of (A13) holds. Consider the players that join K_1 under efficient assignment. Let S be this set of players, then $n^*(\mathscr{K}^*_{-S}, y-|S|) = n^*(\mathscr{K},y)$. The reverse of (A13) then implies the reverse of (A14). Applying Proposition A1 (i), it suggests that S is not playing its CPNE strategy in $\mathscr{K}^*$. Thus $\mathscr{K}^*$ is not self-enforcing and cannot be CPNE.

(ii). By (i), when the reverse of (A13) holds, there should be $a \geq 1$. Let 1 out of the Y player form a new coalition and the revised structure is $\mathscr{K}' = \mathscr{I}_1 \cup \mathscr{K}$. By (i) the CPNE satisfies $a=1$ if and only if $U(l+1) \geq \dfrac{V_1(F)}{V_{n^*(\mathscr{K}',y-1)}(n^*(\mathscr{K}',y-1)F)}$, and $a \geq 2$ if the reverse holds. The same argument carries for $a<n$.

(iii). The result is implied by the proof of (ii). □

Proof of Proposition 2 For a structure $\mathscr{S} = \{S_1, ..., S_m\}$ to be CPNE, the strategy of each single player s should satisfy Lemma A5 given the rest $n-1$ players stay with the same strategy in $\mathscr{S}$. Therefore there must be

$$|S_i - S_j| \leq 1, \quad \forall i,j \in \{1,...,m\} \tag{A16}$$

Otherwise, suppose $S_i - S_j \geq 2$ for some $i \neq j$. Then for an arbitrary supplier in S_j its strategy violates (i) in Lemma A5 thus does not constitute a one-player CPNE. Also, given the number of coalitions m, there is only one *ordered* structure that satisfies (A16), with $|S_m| = \lceil n/m \rceil$ and $\lfloor n/m \rfloor \leq |S_1| \leq \lceil n/m \rceil$. Denote such structure as $\mathscr{S}_m$. Then the CPNE, if *ordered*, should be belong to the set $\{\mathscr{S}_m : 1 \leq m \leq n\}$. □

Proof of Theorem 1 We can identify the conditions for various types of coalition being CPNE with the aids of Proposition A1. Consider $\mathscr{K} = \mathscr{I}_1$ and $y = n-1$. Then $l = 1$, $n^*(\mathscr{K}, y) = n$, and $n^*(\mathscr{I}_i \cup \mathscr{K}, y-i) = \lceil n/(i+1) \rceil$. Proposition A1 (i) suggests that grand coalition among n players is CPNE if $U(1) \geq \frac{V_1(F)}{V_n(nF)}$. The conditions for independent coalition and $\mathscr{S}_m$ be CPNE follow immediately after Proposition A1 (ii) and (iii), respectively. $\square$

Proof of Proposition 3.

(i) By Assumption 1 (b), $V_k(kF)/V_1(F)$ is a unimodal in F. Thus for any $\frac{U(m)}{\lceil \frac{n}{m} \rceil}$, either that $\frac{U(m)}{\lceil \frac{n}{m} \rceil} < \frac{V_1(F)}{V_{\lceil \frac{n}{m} \rceil}(\lceil \frac{n}{m} \rceil F)}$ for all $F > \mu$, or there exists $\mu < F' < F''$ such that $\frac{U(m)}{\lceil \frac{n}{m} \rceil} \geq \frac{V_1(F)}{V_{\lceil \frac{n}{m} \rceil}(\lceil \frac{n}{m} \rceil F)}$ when $F \in [F', F'']$. Denote such interval $[F', F'']$ as $\mathscr{F}_m$. Suppose $F \notin \cup_{k=1}^{m-1} \mathscr{F}_k$. Then according to Theorem 1, m coalitions will be formed if $F \in \mathscr{F}_m$, and more than m coalitions will be formed otherwise, e.g., $F > F''$. Thus larger alliances are more likely to be formed with moderately small F.

(ii) By Assumption 1 (a), $\frac{\partial V_k(kF)}{\partial k} < 0$. Therefore, $\frac{V_1(F)}{V_k(kF)}$ increases with k. Followed by Theorem 1, large alliances is more likely to be formed with small n. $\square$

Proof of Proposition 4. (i). It is sufficient to show that $U(m)$ increases with the pass-through rates identified in Table A2. For linear-power demand, the pass-through rate is $\frac{\theta}{\theta+1}$. We only need to show that $U(m)$ is increasing in θ. For complementary suppliers,

$$U^C(m) = (\frac{\theta+m+1}{\theta+m})^{\theta+1} = \left[(1+\frac{1}{\theta+m})^{\theta+m}\right]^{\frac{\theta+1}{\theta+m}}$$

Since both $(1+\frac{1}{\theta+m})^{\theta+m}$ and $\frac{\theta+1}{\theta+m}$ increase with θ, $U^C(m)$ should be increasing in θ. For substitutable suppliers,

$$U^S(m) \sim (\frac{\theta+1/(m+1)}{\theta+1/m})^{\theta+1} = \left[1+\frac{1}{m(m+1)\theta+m}\right]^{-\theta-1} = \left[\left(1+\frac{1}{m(m+1)\theta+m}\right)^{m(m+1)\theta+m}\right]^{\frac{-\theta-1}{m(m+1)\theta+m}}$$

As $m > 1$, both $\left(1+\frac{1}{m(m+1)\theta+m}\right)^{m(m+1)\theta+m}$ and $\frac{-\theta-1}{m(m+1)\theta+m}$ increase with θ. Therefore $U^S(m)$ also increases with θ. The proof with iso-elastic demand is similar.

(ii). Since the nature of the suppliers only affect the first component of RASF, it is sufficient to show that $U(m)$ for substitutable suppliers are overall smaller than those for complementary suppliers. First consider exponential demand. By Table A4, $\ln U^C(m) = 1$ and $\ln U^S(m) = 2\ln\left(1+\frac{1}{m}\right) - \frac{1}{m} + \frac{1}{m+1} = 2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{km^k} - \frac{1}{m} + \frac{1}{m+1} = \frac{1}{m} + \frac{1}{m+1} + 2\sum_{k=2}^{\infty}\frac{(-1)^{k+1}}{km^k}$. Since $\sum_{k=2}^{\infty}\frac{(-1)^{k+1}}{km^k} \leq 0$ and for $m \geq 2$ there is $1 > \frac{1}{m} + \frac{1}{m+1}$, we have $\ln U^S(m) \leq \ln U^C(m)$ for $m > 1$. It can also be verified that $\ln U^S(1) = .89 \leq \ln U^C(1) = 1$. Therefore the statement holds for exponential demand.

For linear-power demand, it can be verified that $U^C(m) = U^S(m)$ when $\theta = 1$. To show that $U^C \geq U^S$ for general θ, it is then suffice to prove that $\partial U^C(m)/\partial\theta \geq \partial U^S(m)/\partial\theta$ for all $\theta \geq 1$. Note that $\partial \ln U^C(m)/\partial\theta = \ln\left(\frac{\theta+m+1}{\theta+m}\right)^{\theta+1} - \frac{\theta+1}{(\theta+m)(\theta+m+1)} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k(\theta+m)^k} - \frac{\theta+1}{(\theta+m)(\theta+m+1)} =$

$\sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{k(\theta+m)^k}+\frac{m-\frac{\theta+m+1}{2(\theta+m)}}{(\theta+m)(\theta+m+1)}$ and $\partial\ln U^S(m)/\partial\theta=\ln\left(\frac{\theta+1/(m+1)}{\theta+1/m}\right)^{\theta+1}+\frac{\theta+1}{(m\theta+1)[(m+1)\theta+1]}=$ $\sum_{k=1}^{\infty}\frac{-1}{k[m(m+1)\theta+m+1]^k}+\frac{\theta+1}{(m\theta+1)[(m+1)\theta+1]}=\sum_{k=2}^{\infty}\frac{-1}{k[m(m+1)\theta+m+1]^k}+\frac{1-1/(m+1)}{(m\theta+1)[(m+1)\theta+1]}$. Obviously for $\theta\geq 0$ and $m\geq 1$, there are $\sum_{k=3}^{\infty}\frac{(-1)^{k+1}}{k(\theta+m)^k}\geq 0\geq\sum_{k=2}^{\infty}\frac{-1}{k[m(m+1)\theta+m+1]^k}$ and $\frac{m-\frac{\theta+m+1}{2(\theta+m)}}{(\theta+m)(\theta+m+1)}\geq\frac{1-1/(m+1)}{(m\theta+1)[(m+1)\theta+1]}$. Thus $\partial\ln U^C(m)/\partial\theta\geq\partial\ln U^S(m)/\partial\theta$ for any $\theta>0$. Given $U^C=U^S$ at $\theta=1$, this proves that $U^C(m)\geq\leq U^S(m)$ for all $\theta\geq\leq 1$.

The case with iso-elastic demand can be proved in a a similar fashion. $\square$

Proof of Corollary 5 The main statement follows Theorem 1 and Proposition 3 (i). To show that independent structure will be formed for substitutable suppliers when $F\to\infty$, we first note from Table A4 that $\partial U(m)/\partial m\leq 0$ for substitutable suppliers, and

LEMMA A4. *For all demand functions $D(p)$, $3>U(1)\geq 2>U(2)\geq ...\geq U(m)\geq 1$.*

Proof. $U(m)\geq 1$ is true by the definition of $U(m)$. Further, as $U(m)$ decreases with m, it is suffice to show that $3>U(1)\geq 2>U(2)$. For linear-power demand $D(p)=(a-bp)^{\theta}$, we have $U(1)=4\left(\frac{1/2+\theta}{1+\theta}\right)^{\theta+1}$ and $U(2)=\frac{9}{4}\left(\frac{1/3+\theta}{1/2+\theta}\right)^{\theta+1}$. Obviously both $U(1)$ and $U(2)$ increase with θ. Therefore $U(1)\geq 4\left(\frac{1/2+\theta}{1+\theta}\right)^{\theta+1}\Big|_{\theta=0}=2$. On the other hand, $U(1)=4\left(1-\frac{1/2}{1+\theta}\right)^{1+\theta}$ and $U(2)=\frac{9}{4}\left(1-\frac{1/6}{1/2+\theta}\right)^{1/2+\theta}\left(1-\frac{1/6}{1/2+\theta}\right)^{1/2}$. Thus $\lim_{\theta\to\infty}U(1)=4e^{-1/2}=2.43<3$ and $\lim_{\theta\to\infty}U(2)=\frac{9}{4}e^{-1/6}=1.9<2$. For exponential demand $D(p)=ae^{-bp}$, it can be numerically verified that $3>U(1)=2.43>2>U(2)=1.90$. For iso-elastic demand $D(p)=ap^{-b}$, we have $U(1)=4\left(\frac{b-1}{b-1/2}\right)^{b-1}$ and $U(2)=\frac{9}{4}\left(\frac{b-1/2}{b-1/3}\right)^{b-1}$. Both $U(1)$ and $U(2)$ decrease with b. As there must be $b>n$ and $n\geq 2$, b must be at least 3. Thus $U(1)\leq 4\left(\frac{b-1}{b-1/2}\right)^{b-1}\Big|_{b=3}=\frac{64}{25}<3$ and $U(2)\leq\frac{9}{4}\left(\frac{b-1/2}{b-1/3}\right)^{b-1}\Big|_{b=3}=\frac{2025}{1024}<2$. Finally, $U(1)=4\left(1-\frac{1/2}{b-1/2}\right)^{b-1/2}\left(1-\frac{1/2}{b-1/2}\right)^{-1/2}$. Therefore $\lim_{b\to\infty}U(1)=4e^{-1/2}=2.43>2$. We can see that for all $D(p)$ there is $3>U(1)\geq 2>U(2)$. This proves the lemma. $\square$

We next identify CPNE of the substitutable suppliers using Lemma A4 and Theorem 1. When $n=2$, Lemma A4 suggests $U(1)\geq 2=\lim_{F\to\infty}V_1(F)/V_2(2F)$. Therefore $RASF_1\geq 0$ and grand coalition is the CPNE. When $n>2$, the RASFs can be approximated by $RASF_m\to U(m)-\lceil\frac{n}{m}\rceil\lim_{F\to\infty}V_1(F)/V_{\lceil\frac{n}{m}\rceil}(\lceil\frac{n}{m}\rceil F)=U(m)-\lceil\frac{n}{m}\rceil$. Then $RASF_1=U(1)-n<0$. For any $2\leq m\leq n-1$, $\lceil\frac{n}{m}\rceil\geq 2>U(m)$ hence $RASF_m<0$. Therefore independent structure is the CPNE. $\square$

2. Asymmetric Suppliers

DEFINITION 4. A coalition structured $\mathscr{K}=\{K_1,K_2,...,K_l\}$ is *F-ordered* if $V_{n_i+1}(F+F_{K_i})\geq V_{n_j+1}(F+F_{K_j})$ for any $i<j$.

Single-player game: For an arbitrary player with reserve fund F, suppose the rest of the $n-1$ players form F-ordered coalitions $\{K_1, K_2, ..., K_l\}$ where $n_i = |K_i|$ for $i = 1, ..., l$. The player has $l+1$ strategies: join coalition K_i (if K_i also accepts it) for $i \in \{1, 2, ..., l\}$, or be an independent player. The expected profit for the single player is $G_{n_i+1}(F + F_{K_i}) \frac{F}{F + F_{K_i}} \pi(l)$ and $G_1(F)\pi(l+1)$ respectively. For each member in coalition K_i with reserve fund level L, the expected profit is $G_{n_i+1}(F + F_{K_i}) \frac{L}{F + F_{K_i}} \pi(l)$ and $G_{n_i}(F_{K_i}) \frac{L}{F_{K_i}} \pi(l+1)$ respectively. CPNE are the ones that maximizes the player's expected profit:

Lemma A5. *The CPNE for the game with single-player s with reserve fund level F is:*
(i) $\{K_1 \cup \{s\}, K_2, ..., K_l\}$, that is, join K_1, if $U(l) \geq \frac{V_1(F)}{V_{n_1+1}(F + F_{K_1})}$,
(ii) $\{\{s\}, K_1, K_2, ..., K_l\}$, that is, be independent, if $U(l) \leq \frac{V_1(F)}{V_{n_1+1}(F + F_{K_1})}$
where $\{K_1, K_2, ..., K_l\}$ is an F-ordered structure among the rest of the players $N - \{s\}$.

***y*-player game.** Consider a set of suppliers S with size y and the other $n-y$ players N/S forming coalitions $\mathscr{K} = \{K_1, ..., K_l\}$. Similar to that in the symmetric case, strategies of the y players will be in the form of

$$\{P_1, ..., P_a, K_1 \cup Q_1, K_2 \cup Q_2, ..., K_l \cup Q_l\} \tag{A17}$$

where $a, l \geq 0$, $|P_i| = p_i > 0$, for $i = 1, ..., a$, $|Q_j| = q_j \geq 0$ for $j = 1, ..., l$ and $\sum_{i=1}^{a} p_i + \sum_{j=1}^{l} q_j = y$. If all members in S join the coalitions in $\mathscr{K}$ without forming any additional coalition ($a = 0$), the strategy can be expressed by $\mathscr{Q} = \{Q_1, ..., Q_l\}$, where $Q_j \cap Q_p = \emptyset$ for any $j \neq p$ and $\bigcup_{i=1}^{l} Q_i = S$. The coalition structure will be revised to $\mathscr{K}' = \{K_1', ..., K_l'\} = \{K_1 \cup Q_1, ..., K_l \cup Q_l\}$. We can show the following:

Definition 5. Supplier coalitions $\{K_1, ..., K_l\}$ are *V-similar* if $V_{n_i}(F_{K_i}) \geq V_{n_j+1}(F_{K_j} + F_s)$ $\forall i, j \in \{1, 2, ..., l\}$ and $s \in K_i$.

Lemma A6. *Given players N/S form coalitions $\mathscr{K} = \{K_1, ..., K_l\}$ and an $a = 0$ strategy $\mathscr{Q} = \{Q_1, ..., Q_l\}$ for players in S.*

(i) $\mathscr{Q}$ is self-enforcing only if $U(l) \geq \max_{1 \leq i \leq l, s \in Q_i} \{\frac{V_1(F_s)}{V_{n_i'}(F_{K_i'})}, \}$ and $\{K_1', ..., K_l'\}$ are V-similar, where $\mathscr{K}' = \{K_1 \cup Q_1, ..., K_l \cup Q_l\}$;

(ii) $\mathscr{Q}$ is a CPNE if the conditions in (i) hold and for any other strategy $\mathscr{Q}' = \{Q_1', ..., Q_l'\}$ that also satisfies the conditions in (i), $\mathscr{Q}$ is not strongly dominated by $\mathscr{Q}'$;

(iii) if the conditions in (i) hold, any strategy as in (A17) *with $a > 0$ is not a CPNE.*

Proof.

(i) A self-enforcing strategy should form the NE for any single-player game. By Lemma A5 the two conditions are necessary to hold.

(ii) The statement obviously holds for $y = 1$. Suppose it also holds for any set with size $y \leq Y - 1$. Now consider the set S with size Y. For any $S' \subset S$, denote $\mathscr{Q}_{S'}$ the strategy for S', $\mathscr{Q}_{-S'}$ the strategy for S/S' and $\mathscr{K}_{-S'}'$ the coalition structure for the $n - |S'|$ suppliers N/S' in $\mathscr{K}'$. Then by definition, $\mathscr{Q}$ is self-enforcing if for all $S' \subset S$, $\mathscr{Q}_{S'}$ is the CPNE given suppliers in N/S' act according to $\mathscr{K}_{-S'}'$. Note that the lemma holds for $|S'| < Y$. Applying these to all $S' \subset S$, we can see that $\mathscr{Q}$ is self-enforcing under the stated conditions.

Next, we need to verify that $\mathscr{Q}$ is not be strongly dominated by another self-enforcing strategy. First, we can show that any self-enforcing strategy $\{P_1,...,P_a,Q_1,...,Q_l\}$, where $\bigcup_{i=1}^{a} P_i \bigcup_{j=1}^{l} Q_j = S$ and $a \neq 0$, is dominated by a self-enforcing strategy $\mathscr{Q}' = \{Q_1',...,Q_l'\}$, where $\bigcup_{j=1}^{l} Q_j' = S$. The former results in coalition structure $\{P_1,...,P_a,K_1\cup Q_1,...,K_l\cup Q_l\}$ and the later forms coalitions $\{K_1\cup Q_1',...,K_l\cup Q_l'\}$. It is sufficient to show that

$$\begin{aligned}&\pi(l)L \min_{i:K_i\cup Q_i' \text{ contains at least one low-endowed supplier}} V_{n_i+q_i'}(F_{K_i\cup Q_i'}) \\ \geq\ &\pi(l+a)L \max_{P_j,K_i\cup Q_i \text{ contains at least one low-endowed supplier}} \{V_{p_j}(F_{P_j}),V_{n_i+q_i}(F_{K_i\cup Q_i})\}\end{aligned} \quad \text{(A18)}$$

and

$$\begin{aligned}&\pi(l)H \min_{i:K_i\cup Q_i' \text{ contains at least one high-endowed supplier}} V_{n_i+q_i'}(F_{K_i\cup Q_i'}) \\ \geq\ &\pi(l+a)H \max_{P_j,K_i\cup Q_i \text{ contains at least one high-endowed supplier}} \{V_{p_j}(F_{P_j}),V_{n_i+q_i}(F_{K_i\cup Q_i})\}\end{aligned} \quad \text{(A19)}$$

Note that $\{K_1\cup Q_1',...,K_l\cup Q_l'\}$ needs to satisfy the CPNE condition for any single supplier. Then for any $K_i\cup Q_i'$ containing a low-endowed supplier, there should be $U(l) \geq \dfrac{V_1(L)}{V_{n_i+q_i'}(F_{K_i\cup Q_i'})}$. Therefore,

$$U(l) \geq \frac{V_1(L)}{\min_{i:K_i\cup Q_i' \text{ contains at least one low-endowed supplier}} V_{n_i+q_i'}(F_{K_i\cup Q_i'})}.$$

Since $\pi(l)/\pi(l+a) \geq U(l)$, (A18) is immediately implied by the above and Assumption 1 (a). Similar argument applies to any high-endowed supplier, which will derive (A19).

In addition, as $\mathscr{Q} = \{Q_1,...,Q_l\}$ is not dominated by any other self-enforcing strategy $\mathscr{Q}' = \{Q_1',...,Q_l'\}$, $\mathscr{Q}$ is a CPNE under the stated conditions. The statement also holds true for $y = Y$.

(iii) is proved in the course of (ii) (a). □

Proposition A2. *Consider a set of players S of size $|S|$=y, and the rest of the $n-y$ players forming ordered coalitions $\mathscr{K}_{N/S} = \{K_1,K_2,...,K_l\}$. There exists a set of functions $T_1,T_2,...,T_y$ such that*

(i) the CPNE satisfies $a=0$ if and only if $U(l) \geq T_1(\mathscr{K}_{N/S},S)$

(ii) the CPNE satisfies $0<a<y$, if and only if $U(l+i) \leq T_{i+1}(\mathscr{K}_{N/S},S)$ for $0 \leq i \leq a-1$ and $U(l+a) \geq T_{a+1}(\mathscr{K}_{N/S},S)$.

(iii) the CPNE is $a=y$ if and only if $U(l+i) \leq T_{i+1}(\mathscr{K}_{N/S},S)$, for $0 \leq i \leq y-1$.

Proof. (i) The CPNE satisfies $a=0$ if some strategy $\mathscr{Q} = \{Q_1,...,Q_l\}$ where $\bigcup_{i=1}^{l} Q_i = S$ is CPNE. By Lemma A6, it requires that

$$U(l) \geq= \min_{\mathscr{Q}} \{ \max_{1\leq i\leq l, s\in Q_i} \{ \frac{V_1(F_s)}{V_{n_i'}(F_{K_i'})} \} : \bigcup_{j=1}^{l} Q_j = S \text{ and } \{K_1',...,K_l'\} \text{ are V-similar}\} \quad \text{(A20)}$$

The RHS of (A20) defines $T_1(\mathscr{K}_{N/S},S)$.

(ii). If $U(l) \leq T_1(\mathscr{K},S)$, then $a \geq 1$. That is, the players S will form at least one additional coalition of their own. With a slight abuse of notation, consider adding a void empty coalition K_0 to the structure $\mathscr{K}$. That is, $\mathscr{K} = \{K_0,K_1,K_2,...,K_l\}$. Then Lemma A6 still applies. Therefore the CPNE satisfies $a=1$ if

$$U(l) \geq= \min_{\mathscr{Q}} \{ \max_{0\leq i\leq l, s\in Q_i} \{ \frac{V_1(F_s)}{V_{n_i'}(F_{K_i'})} \} :$$

$$\bigcup_{j=0}^{l} Q_j = S \text{ and } \{K_0^{'}, ..., K_l^{'}\} \text{ are of similar sizes}\} \tag{A21}$$

Then the RHS defines $T_2(\mathscr{K}_{N/S}, S)$. Similar to the proof of Lemma A6, we can prove that the CPNE satisfies $a = 1$ when $U(l+0) \leq T_1$ and $U(l+1) \geq T_2$. The proof for other values of a follows in a similar fashion. Specifically, let $\mathscr{K} = \{K_{-a+1}, ..., K_0, K_1, ..., K_l\}$ where K_i's are empty sets for $i \leq 0$. Then

$$T_{a+1}(\mathscr{K}_{N/S}, S) = \min_{\mathscr{Q}} \{ \max_{-a+1 \leq i \leq l, s \in Q_i} \{ \frac{V_1(F_s)}{V_{n_i^{'}}(F_{K_i^{'}})} \} : \bigcup_{j=-a+1}^{l} Q_j = S \text{ and } \{K_{-a+1}^{'}, ..., K_l^{'}\} \text{ are of similar sizes}\}$$

(iii). Followed by (i) and (ii), the CPNE must be $a = |S| = y$ under this condition. □

Proof of Theorem 2. We prove the theorem under the aids of Proposition A2. Let S be the set of an arbitrary $n-1$ suppliers, and $N/S = \{s\}$. Then $\mathscr{K}_{N/S} = \{\{s\}\}$ consists of a collation of an independent supplier s hence $l = 1$. According to Proposition A2 (i),

$$T_1(\{\{s\}\}, N/\{s\}) = \max_{b \in N/\{s\}} \frac{V_1(F_b)}{V_n(F_N)}$$

Grand coalition is the unique CPNE i.i.f. $U(1) \geq T_1(\{\{s\}\}, N/\{s\})$ for any $s \in N$. Since N contains both high- and low-endowed suppliers, we need

$$U(1) \geq \max\{\frac{V_1(L)}{V_n(F_N)}, \frac{V_1(H)}{V_n(F_N)}\}.$$

This proves (i).

Following the same steps, we can identify that two coalitions will be formed if $U(1) \leq \max\{\frac{V_1(L)}{V_n(F_N)}, \frac{V_1(H)}{V_n(F_N)}\}$ and for any $s \in N$ there is $U(2) \geq T_2(\{\{s\}\}, N/\{s\}) = \min\{\max_{b_0 \in Q_0, b_1 \in Q_1} \{ \frac{V_1(F_{b_0})}{V_{|Q_0|}(F_{Q_0})} \frac{V_1(F_{b_1})}{V_{|Q_1|+1}(F_{Q_1} + F_s)} \}\}$ for all $Q_0 \bigcup Q_1 = N/\{s\}$ and $\{Q_0, Q_1 \bigcup \{s\}\}$ are V-similar. Overall, we need $U(2) \geq \min\{\max_{1 \leq i \leq 2, b \in S_i} \{ \frac{V_1(F_b)}{V_{|S_i|}(F_{S_i})} :$ $\mathscr{Q}$ is a 2-partition of S and $\{Q_i\}_1^2$ are V-similar$\}$. The same can be proved for other m's. □

Proof of Proposition 6. Consider $n_H H + n_L L = n_H H^{'} + n_L L^{'}$ and $H/L > H^{'}/L^{'}$. There should be $L < L^{'} < H^{'} < H$. By Theorem 2 (i), grand coalition is more likely to be stable under $(L^{'}, H^{'})$ if $T_1 \geq T_1^{'}$. Note that $V_1^{'}(F) = \frac{Fg(F) - G(F)}{F^2}$ and $\frac{\partial}{\partial F}[Fg(F) - G(F)] = g^{'}(F) < 0$ for $F \geq \mathbf{E}[\xi_i]$. Thus $V_1(F)$ is concave on $[\mathbf{E}[\xi_i], \infty)$. It can be verified that for exponential distribution, there will always be $V_1^{'}(F) \leq 0$. Thus $T_1 = \max\{V_1(L)/V_n(F_N), V_1(H)/V_n(F_N)\} > \max\{V_1(L^{'})/V_n(F_N), V_1(H^{'})/V_n(F_N)\} = T_1^{'}$, and grand coalition is more likely to be stable in the latter setting. For normal and Erlang distribution, suppose $V_1^{'}(F) = 0$ at $F = \hat{F}$. Then $T_1 > T_1^{'}$ when $H > L \geq \hat{F}$. The same will also hold when $L < H \leq \hat{F}$. Thus the statement holds true when reserve fund levels are all above or below some threshold. □

Proof of Proposition 7. As $n_H/n_L > n_H^{'}/n_L^{'}$, $n_H H + n_L L = n_H^{'} H^{'} + n_L^{'} L^{'}$ and $H/L = H^{'}/L^{'}$, there should be $L < L^{'}$ and $H < H^{'}$. Followed by the same argument as in the proof of Proposition 6, $V_1(F)$ is concave on $[\mathbf{E}[\xi_i], \infty)$. Also, $V_1^{'}(F) \leq 0$ under exponential distribution. Thus $T_1 = \max\{V_1(L)/V_n(F_N), V_1(H)/V_n(F_N)\} = V_1(L)/V_n(F_N) > V_1(L^{'})/V_n(F_N) = \max\{V_1(L^{'})/V_n(F_N), V_1(H^{'})/V_n(F_N)\} = T_1^{'}$. By Theorem 2 (i), it is more likely for $T_1^{'}$ to exceed $U(1)$, which implies the stability of grand coalition. For normal and Erlang distribution, suppose $V_1^{'}(F) = 0$ at $F = \hat{F}$. Then $T_1 \geq T_1^{'}$ as long as $H > L \geq \hat{F}$. □

Robustness Analysis

Equilibrium Investment Decisions

LEMMA A7. $\dfrac{V_1(L)}{V_k(L+(k-1)F)}$ *decreases in* L*, when* ξ_i *follows i.i.d. exponential distribution.*

Proof.

$$\frac{\partial}{\partial L}\left[\frac{V_1(L)}{V_k(L+(k-1)F)}\right] = \frac{\partial}{\partial L}\left[\frac{G_1(L)}{G_k(L+(k-1)F)}\frac{L+(k-1)F}{L}\right]$$
$$= -\frac{G_1}{G_k}\frac{(k-1)F}{L^2} + (\frac{g_1}{G_k} - g_k\frac{G_1}{G_k^2})(1+\frac{(k-1)F}{L})$$
$$= \frac{G_1}{G_k}\left[-\frac{(k-1)F}{L^2} + (\frac{g_1}{G_1} - \frac{g_k}{G_k})(1+\frac{(k-1)F}{L})\right]$$

To prove that $\frac{\partial}{\partial L}\left[\frac{V_1(L)}{V_k(L+(k-1)F)}\right] < 0$ one needs to show that $-\frac{(k-1)F}{L^2} + (\frac{g_1}{G_1} - \frac{g_k}{G_k})(1+\frac{(k-1)F}{L}) < 0$ or equivalently

$$\frac{g_1(L)}{G_1(L)} - \frac{g_k(L+(k-1)F)}{G_k(L+(k-1)F)} \le \frac{1}{L} - \frac{1}{L+(k-1)F}.$$

By (A3),

$$\frac{g_k(x)}{G_k(x)} = \frac{\frac{1}{\theta}(\frac{x}{\theta})^{k-1}/(k-1)!}{\sum_{i=k}^{\infty}\frac{(x/\theta)^i}{i!}} = \frac{1}{\theta(k-1)!}\frac{1}{\sum_{i=k}^{\infty}\frac{(x/\theta)^{i+1-k}}{i!}} = \frac{1}{(k-1)!}\frac{1}{\sum_{i=k}^{\infty}\frac{x^{i+1-k}}{i!\theta^{i-k}}}$$

increases in k. Thus $\dfrac{g_k(x)}{G_k(x)} \ge \dfrac{g_1(x)}{G_1(x)} = \dfrac{\frac{1}{\theta}}{e^{\frac{x}{\theta}}-1}$.

Also, it can be verified that $\dfrac{1}{e^x-1} - \dfrac{1}{x}$ increases in x. Hence $\frac{g_1(x)}{G_1(x)} - \frac{1}{x} = \frac{\frac{1}{\theta}}{e^{\frac{x}{\theta}}-1} - \frac{1}{x}$ increases in x. Therefore,

$$\frac{g_1(L)}{G_1(L)} - \frac{g_k(L+(k-1)F)}{G_k(L+(k-1)F)} \le \frac{g_1(L)}{G_1(L)} - \frac{g_1(L+(k-1)F)}{G_1(L+(k-1)F)} \le \frac{1}{L} - \frac{1}{L+(k-1)F}.$$

□

Given reserve fund level F. Suppose that m coalitions will be formed if $F_j = F$ for $j = 1, 2, ..., n$.

LEMMA A8. *There exist* $l_{m+k} \le ... \le l_{m+1} \le l_m \le F$ *and* $F \le H_m \le H_{m+1} \le ... \le H_{m+t}$ *for some* $k, t \ge 0$*, such that*

(i) m *coalitions will be formed if there are* $n-1$ *suppliers with reserve fund* F *and one supplier* i *with reserve fund* $L_m \le F_i \le F$ *or* $F \le F_i \le H_m$*;*

(ii) $m+l$ *coalitions will be formed if there are* $n-1$ *suppliers with reserve fund* F *and one supplier* i *with reserve fund* $L_{m+l} \le F_i \le L_{m+l-1}$ *or* $H_{m+l-i} \le F_i \le H_{m+l}$*.*

(ii) $m+k$ *(resp.* $m+t$*) coalitions will be formed if there are* $n-1$ *suppliers with reserve fund* F *and one supplier* i *with reserve fund* $F_i \le L_{m+l}$ *(resp.* $F_i \ge H_{m+l}$*).*

Proof. Without loss of generality, consider one supplier with reserve fund level L and $n-1$ supplier with reserve fund level H. V-similar coalition structure can only be (1) $\lfloor n/m \rfloor$ high-endowed suppliers, (2) $\lfloor n/m \rfloor$ high-endowed suppliers and 1 low-endowed suppliers, (3) $\lceil n/m \rceil$ high-endowed suppliers. Let $k = \lfloor n/m \rfloor$. Then by Theorem 2 and Lemma A7, $T_m = \max\{\dfrac{V_1(L)}{V_{k+1}(L+kH)}, \dfrac{V_1(H)}{V_{k+1}((k+1)H)}, \dfrac{V_1(H)}{V_k(kH)}\} = \dfrac{V_1(L)}{V_{k+1}(L+kH)}$ is decreasing in L. In other words, larger L yields smaller T_m thus larger alliances will be formed (smaller m^*).

Similarly, for one supplier with reserve fund level H and $n-1$ supplier with reserve fund level L, the V-similar coalition structure can only be (1) k low-endowed suppliers, (2) $k+1$ low-endowed suppliers, and (3) k' low-endowed suppliers and 1 high-endowed suppliers, where $k' \le k$. Thus $T_m = \max\{\frac{V_1(L)}{V_{k'+1}(H+k'L)}, \frac{V_1(L)}{V_{k+1}((k+1)L)}, \frac{V_1(L)}{V_k(kL)}\} = \max\{\frac{V_1(L)}{V_{k'+1}(H+k'L)}, \frac{V_1(L)}{V_{k+1}((k+1)L)}\}$ increases in H. That is, larger H yields larger T_m thus smaller alliances will be formed (larger m^*).

These prove the three points all together. $\square$

Proof of Proposition 8. Consider the reserve fund F such that m coalitions will be formed if $F_j = F$ for $j = 1,2,...,n$. For any supplier i, assume that all the other $n-1$ suppliers invest at F. Then if the supplier i also invests at F, the expected profit is $\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)\frac{F}{\frac{n}{m}F} - vF$. If it invests at $L < F$ such that $m+l$ coalitions, where $0 \le l \le n-m$, will be formed, then the expected profit is $\pi(m+l)G_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F + L)\frac{L}{(\frac{n}{m+l}-1)F+L} - vL$. Supplier i then prefers investing at F than $L_{m+l} \le L < L_{m+l-1}$ if

$$\begin{aligned} v &\le \frac{\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)\frac{F}{\frac{n}{m}F} - \pi(m+l)G_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L)\frac{L}{(\frac{n}{m+l}-1)F+L}}{F-L} \\ &= \pi(m)V_{\frac{n}{m}}(\frac{n}{m}F) - \frac{L}{F-L}\left[\pi(m+l)V_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L) - \pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)\right] \end{aligned} \tag{A22}$$

and prefers investing at F than $L_m \le L < F$ if

$$\begin{aligned} v &\le \frac{\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)\frac{F}{\frac{n}{m}F} - \pi(m)G_{\frac{n}{m}}((\frac{n}{m}-1)F+L)\frac{L}{(\frac{n}{m}-1)F+L}}{F-L} \\ &= \pi(m)V_{\frac{n}{m}}(\frac{n}{m}F) - \frac{L}{F-L}\left[\pi(m)V_{\frac{n}{m}}((\frac{n}{m}-1)F+L) - \pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)\right] \end{aligned} \tag{A23}$$

Note that for the second component in (A23) there is

$$\begin{aligned} &\frac{L}{F-L}\left[\pi(m)V_{\frac{n}{m}}((\frac{n}{m}-1)F+L) - \pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)\right] = \pi(m)\frac{L}{F-L}\left[\frac{G_{\frac{n}{m}}((\frac{n}{m}-1)F+L)}{(\frac{n}{m}-1)F+L} - \frac{G_{\frac{n}{m}}(\frac{n}{m}F)}{\frac{n}{m}F}\right] \\ &= \pi(m)\frac{L}{F-L}\left[\frac{\frac{n}{m}FG_{\frac{n}{m}}((\frac{n}{m}-1)F+L) - [(\frac{n}{m}-1)F+L]G_{\frac{n}{m}}(\frac{n}{m}F)}{[(\frac{n}{m}-1)F+L]\frac{n}{m}F}\right] \\ &= \pi(m)\frac{m}{nF}\frac{L}{(\frac{n}{m}-1)F+L}\left(\frac{n}{m}F\frac{(G_{\frac{n}{m}}((\frac{n}{m}-1)F+L) - G_{\frac{n}{m}}(\frac{n}{m}F)}{F-L} + G_{\frac{n}{m}}(\frac{n}{m}F)\right) \end{aligned}$$

which increases in L hence maximized at $L = F$.

For the second component in (A22) we can show the following:

(a) $\frac{L}{F-L}\left[\pi(m+l)V_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L) - \pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)\right]$ increases in L, as

$$\begin{aligned} &\frac{L}{F-L}\left[\pi(m+l)V_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L) - \pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)\right] \\ &= \frac{L}{F-L}\left[\frac{\pi(m+l)G_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L)}{(\frac{n}{m+l}-1)F+L} - \frac{\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)}{\frac{n}{m}F}\right] \\ &= \frac{L}{F-L}\frac{\frac{n}{m}F\pi(m+l)G_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L) - [(\frac{n}{m+l}-1)F+L]\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)}{[(\frac{n}{m+l}-1)F+L](\frac{n}{m})F} \end{aligned}$$

$$= \frac{m}{nF}\frac{L}{(\frac{n}{m+l}-1)F+L}\left(\frac{\frac{n}{m}\pi(m+l)G_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L)-\frac{n}{m+l}\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)}{F-L}F+\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)\right)$$

which apparently increases in L hence maximized at $L=L_{m+l-1}$.

(b) By Theorem 2, at $L=L_{m+l-1}$,

$$\pi(m+l-1)V_{\frac{n}{m+l-1}}((\frac{n}{m+l-1}-1)F+L)\geq\pi(m+l)V_1(L)\geq\pi(m+l)V_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L)$$

Therefore, the upper bound for v is minimized at $L=F$ and supplier i would not invest at any $L<F$ if any only if

$$v\leq\frac{\partial\left\{\pi(m)G_{\frac{n}{m}}((\frac{n}{m}-1)F+L)\frac{L}{(\frac{n}{m}-1)F+L}\right\}}{\partial L}|_{L\to F}=\pi(m)\left[g_{\frac{n}{m}}(\frac{n}{m}F)\frac{m}{n}+V_{\frac{n}{m}}(\frac{n}{m}F)\frac{n-m}{n}\right]$$

Followed by a similar argument, if supplier i invests at $H>F$ such that $m+l$ coalitions, where $0\leq l\leq n-m$, will be formed, then the expected profit is $\pi(m+l)G_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+L)\frac{H}{(\frac{n}{m+l}-1)F+H}-vH$. Supplier i then prefers investing at F than $H_{m+l}\geq H>H_{m+l-1}$ if

$$\begin{aligned}v\geq&\frac{\pi(m+l)G_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+H)\frac{H}{(\frac{n}{m+l}-1)F+H}-\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)\frac{F}{\frac{n}{m}F}}{H-F}\\&=\pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)-\frac{H}{H-F}\left[\pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)-\pi(m+l)V_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+H)\right]\end{aligned}\tag{A24}$$

and prefers investing at F than $H_m\geq H>F$ if

$$\begin{aligned}v\geq&\frac{\pi(m)G_{\frac{n}{m}}((\frac{n}{m}-1)F+H)\frac{H}{(\frac{n}{m}-1)F+H}-\pi(m)G_{\frac{n}{m}}(\frac{n}{m}F)\frac{F}{\frac{n}{m}F}}{H-F}\\&=\pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)-\frac{H}{H-F}\left[\pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)-\pi(m)V_{\frac{n}{m}}((\frac{n}{m}-1)F+H)\right]\end{aligned}\tag{A25}$$

For the second component in (A25) there is

$$\begin{aligned}&\frac{H}{H-F}\left[\pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)-\pi(m)V_{\frac{n}{m}}((\frac{n}{m}-1)F+H)\right]=\pi(m)\frac{H}{H-F}\left[\frac{G_{\frac{n}{m}}(\frac{n}{m}F)}{\frac{n}{m}F}-\frac{G_{\frac{n}{m}}((\frac{n}{m}-1)F+H)}{(\frac{n}{m}-1)F+H}\right]\\&=\pi(m)\frac{H}{H-F}\frac{[(\frac{n}{m}-1)F+H]G_{\frac{n}{m}}(\frac{n}{m}F)-\frac{n}{m}FG_{\frac{n}{m}}((\frac{n}{m}-1)F+H)}{[(\frac{n}{m}-1)F+H]\frac{m}{n}F}\\&=\pi(m)\frac{m}{nF}\frac{H}{(\frac{n}{m}-1)F+H}\left[G_{\frac{n}{m}}(\frac{n}{m}F)-\frac{n}{m}F\frac{G_{\frac{n}{m}}((\frac{n}{m}-1)F+H)-G_{\frac{n}{m}}(\frac{n}{m}F)}{H-F}\right]\end{aligned}$$

which is minimized at $H=F$.

In a similar fashion as the L case, we can show that for the second component in (A24) with given l, $\frac{H}{H-F}\left[\pi(m)V_{\frac{n}{m}}(\frac{n}{m}F)-\pi(m+l)V_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+H)\right]$ decreases in H, and for boundary $H=H_{m+l-1}$, there is

$$\pi(m+l-1)V_{\frac{n}{m+l-1}}((\frac{n}{m+l-1}-1)F+H)\geq\pi(m+l)V_1(F)\geq\pi(m+l)V_{\frac{n}{m+l}}((\frac{n}{m+l}-1)F+H).$$

Hence the lower bound of v is maximized at $H=F$. That is, supplier i would not invest at any $H>F$ if any only if

$$v \geq \frac{\partial\left\{\pi(m)G_{\frac{n}{m}}((\frac{n}{m}-1)F+H)\frac{H}{(\frac{n}{m}-1)F+H}\right\}}{\partial H}|_{H\to F} = \pi(m)\left[g_{\frac{n}{m}}(\frac{n}{m}F)\frac{m}{n}+V_{\frac{n}{m}}(\frac{n}{m}F)\frac{n-m}{n}\right]$$

Therefore, for supplier i to invest in F also, the investment cost should equal

$$v=\pi(m)\left[g_{\frac{n}{m}}(\frac{n}{m}F)\frac{m}{n}+V_{\frac{n}{m}}(\frac{n}{m}F)\frac{n-m}{n}\right].$$

By Theorem 1, m is determined by $m^*(F)$. Hence $\pi(m^*)\left[g_{\frac{n}{m^*}}(\frac{n}{m^*}F)\frac{m^*}{n}+V_{\frac{n}{m^*}}(\frac{n}{m^*}F)\frac{n-m^*}{n}\right]$ is a piece-wise decreasing, left-continuous function of F. Thus for any given cost v there exists a unique reserve fund level F that the suppliers can be stable at. In particular, if $v=\pi(m^*)\left[g_{\frac{n}{m}}(\frac{n}{m^*}F)\frac{m^*}{n}+V_{\frac{n}{m^*}}(\frac{n}{m^*}F)\frac{n-m^*}{n}\right]$ for some F, then m^* coalitions will be formed. $\square$

Allocation Rules.

Proposition A3. *Consider a set of players S of size $|S|=y$, and the rest of the $n-y$ players forming ordered coalitions $\mathscr{K}_{N/S}=\{K_1,K_2,...,K_l\}$. There exists a set of functions $T_0,T_1,...,T_{y-1}$ such that*

(i) $a=0$ is CPNE when $U(l)\geq T_0(\mathscr{K}_{N/S},S)$

(ii) $0<a<y$ is CPNE when $U(l+i)\leq T_i(\mathscr{K}_{N/S},S)$ for $0\leq i\leq a-1$ and $U(l+a)\geq T_a(\mathscr{K}_{N/S},S)$

(iii) $a=y$ is CPNE when $U(l+i)\leq T_i(\mathscr{K}_{N/S},S)$, for $0\leq i\leq y-1$.

Proof. For ease of exposition, denote K_i^u the powered-total of all elements in set K_i. That is, if K_i contains n_{Li} low-endowed supplier and n_{Hi} high-endowed supplier, then $K_i^u=n_{Li}L^u+n_{Hi}H^u$.

For single-player game $S=\{s\}$, the threshold T_0 can be characterized by

$$T_0(\mathscr{K},\{s\}) = \begin{cases} \max_i\{\dfrac{G_1(L)}{L^u}\dfrac{K_i^u+L^u}{G_{|K_i|+1}(K_i^1+L)}\} & \text{if } s \text{ is a low-endowed supplier} \\ \max_i\{\dfrac{G_1(H)}{H^u}\dfrac{K_i^u+H^u}{G_{|K_i|+1}(K_i^1+H)}\} & \text{if } s \text{ is a high-endowed supplier} \end{cases}$$

Obviously the proposition holds for $y=1$.

Further, suppose the proposition holds for all $y<Y$. Now consider a set S of size Y and the other players N/S forming coalitions $\mathscr{K}=\{K_1,...,K_l\}$. In this scenario, redefine

$$V_k(F_{K_i})=\frac{G_k(F_{K_i})}{K_i^u}.$$

Then, similar to the proof of Proposition A2, $a=0$ is self-enforcing if

$$U(l)\geq=\min_{\mathscr{Q}}\{\max_{1\leq i\leq l,s\in K_j'/K_j}\{\frac{V_1(s)}{V_{n_i'}(F_{K_i'})}\}:\bigcup_{j=1}^{l}K_j'/K_j=S \text{ and } \{K_1',...,K_l'\} \text{ are V-similar}\} \tag{A26}$$

and $T_0(\mathscr{K},S)$ follows the RHS of (A26). We can also prove that there is no other self-enforcing strategy $a>0$ in which every supplier in S is strictly better off under the condition of (A26). First consider $u>1$, and any low-endowed suppliers, which belongs to set K under strategy $a=0$ and S under $a>0$. Then its expected profit under $a=0$ is $\pi(l)G_{|K|+1}(K^1+L)\dfrac{L^u}{K^u+L^u}\geq\pi(l+1)G_1(L)$ — the inequality follows the self-enforcing condition of any single player. The supplier's expected profit under $a>0$ is given by $\pi(l+a)G_{|S|+1}(S^1+L)\dfrac{L^u}{S^u+L^u}$. By Assumption 1 (a), it can be verified that $G_1(L)\geq G_{|S|+1}(S^1+L)\dfrac{L^u}{S^u+L^u}$ for $u\geq 1$. Also there is $\pi(l+1)\geq\pi(l+a)$. So $\pi(l)G_{|K|+1}(K^1+L)\dfrac{L^u}{K^u+L^u}\geq\pi(l+1)G_1(L)\geq\pi(l+a)G_{|S|+1}(S^1+L)\dfrac{L^u}{S^u+L^u}$. The low-endowed suppliers are no better off under any $a>0$ strategy. Similar can be proved that when $u<1$, $G_1(H)\geq G_{|S|+1}(S^1+H)\dfrac{H}{S^1+H}\geq G_{|S|+1}(S^1+H)\dfrac{H^u}{S^u+H^u}$. Thus high-endowed suppliers are no better off under any $a>0$ strategy. Therefore $a=0$ is a CPNE. The same argument carries for any $a\leq y-1$, and the form of T_a follows (A22). Thus the proposition also holds for $y=Y$. $\square$

Proof of Proposition 9.

(i) Implied by Proposition A3 (i), the proof is similar to Theorem 2 (i).

(ii) $\max\{\dfrac{G_1(L)}{L^u},\dfrac{G_1(H)}{H^u}\}\dfrac{n_LL^u+n_HH^u}{G_n(F_N)}=\max\{\dfrac{G_1(L)[n_L+n_H(H/L)^u]}{G_n(F_N)},\dfrac{G_1(H)[n_H(L/H)^u+n_H]}{G_n(F_N)}\}$. Note that $\dfrac{G_1(L)[n_L+n_H(H/L)^u]}{G_n(F_N)}$ increases in u, $\dfrac{G_1(H)[n_H(L/H)^u+n_H]}{G_n(F_N)}$ decreases in u, and $\dfrac{G_1(L)[n_L+n_H(H/L)^u]}{G_n(F_N)}|_{u=0}<\dfrac{G_1(H)[n_H(L/H)^u+n_H]}{G_n(F_N)}|_{u=0}$. Therefore there exists a unique $u_f=\dfrac{\ln G_1(L)/G_1(H)}{\ln L/H}$ at which $\dfrac{G_1(L)[n_L+n_H(H/L)^{u_f}]}{G_n(F_N)}=\dfrac{G_1(H)[n_H(L/H)^{u_f}+n_H]}{G_n(F_N)}$. Specifically, $\max\{\dfrac{G_1(L)}{L^u},\dfrac{G_1(H)}{H^u}\}\dfrac{n_LL^u+n_HH^u}{G_n(F_N)}$ takes the value $\dfrac{G_1(H)[n_H(L/H)^u+n_H]}{G_n(F_N)}$ when $0<u\leq u_f$ and $\dfrac{G_1(L)[n_L+n_H(H/L)^u]}{G_n(F_N)}$ when $u_f\leq u$, and is minimized at $u=u_f$. By (i), grand coalition is more likely to be achieved as u approaches u_f. $\square$

Default Premium on Wholesale Prices.

• For n symmetric suppliers, we can show that Lemma A5 holds true if δ is less than certain threshold.

Consider assembly system with iso-elastic demand, i.e., $D=ap^{-b}$. Other problems can be proved in a similar manner. By Proposition 1, the expected profit for any alliance given coalition structure $\mathscr{S}$ is $\pi(\mathscr{S})=\dfrac{a}{b}(1-1/b)^b(\dfrac{\widetilde{C}}{1-m/b})^{1-b}$, where m is the number of coalitions, and $\tilde{C}=nc+\sum_1^m\bar{G}_{|Sk|}(F_{S_k})\delta_{S_k}=nc+\sum_1^m\bar{G}_{|Sk|}(F_{S_k})|S_k|\delta$. The expected profit for a supplier s in alliance k is then

$$\Pi_s(\mathscr{S})=\pi(\mathscr{S})G_{|S_k|}(F_{S_k})\frac{F}{F_{S_k}}=\frac{a}{b}(1-1/b)^b(\frac{\widetilde{C}}{1-m/b})^{1-b}V_{|S_k|}(F_{S_k})F.$$

For any **single supplier** s, suppose the rest suppliers form ordered coalitions $\mathscr{K}=\{K_1,...,K_l\}$ where $n_1\leq n_2\leq...\leq n_l$, and denote the new structure if supplier i joins coalition t as $\mathscr{K}^t=\{K_i,...,K_t\cup\{s\},...,K_l\}$. Then supplier s will decides between being in dependent, i.e., $K^0=\{\{s\},K_1,...,K_l\}$, and joining coalition t^* where

$$t^*=\arg\max_{1\leq t\leq l}\{\Pi_s(\mathscr{K}^t)\}.$$

LEMMA A9. *There exists a $\delta_0>0$ such that $\Pi_s(\mathscr{K}^t)$ decreases with t and $t^*=1$ when $\delta\leq\delta_0$.*

Proof. We prove the statement for iso-elastic demand $D=ap^{-b}$. Other kinds of demand can be proved in a similar fashion. If supplier s join alliance t, then the expected profit is

$$\frac{aF}{b}(1-1/b)^b(1-l/b)^{b-1}\left[C+\delta\sum_1^l\bar{G}_{|S_k|}(F_{S_k})n_k-\delta\bar{G}_{|S_t|}(F_{S_t})n_t+\delta\bar{G}_{|S_t+1|}(F_{S_t}+F)(n_t+1)\right]^{1-b}V_{|S_k|+1}(F_{S_k}+F).$$

To prove the lemma is equivalent to showing that the following decreases with $\hat{n}$:

$$\left[C+\delta\sum_{1}^{l}\bar{G}_{|S_k|}(F_{S_k})n_k-\delta\bar{G}_{\hat{n}}(\hat{n}F)\hat{n}+\delta\bar{G}_{\hat{n}+1}((\hat{n}+1)F)(\hat{n}+1)\right]^{1-b}V_{\hat{n}+1}((\hat{n}+1)F)$$

A negative FOC of the above requires that

$$\delta\Lambda_1(\hat{n})\leq\Lambda_2(\hat{n})$$

where

$$\begin{aligned}\Lambda_1(\hat{n}) &= (1-b)V_{\hat{n}+1}((\hat{n}+1)F)(G_{\hat{n}}(\hat{n}F)-G_{\hat{n}+1}((\hat{n}+1)F)+G'_{\hat{n}}(\hat{n}F)\hat{n}-G'_{\hat{n}+1}((\hat{n}+1)F)(\hat{n}+1)\\ &\quad +V'_{\hat{n}+1}((\hat{n}+1)F)(\sum_{1}^{l}\bar{G}_{|S_k|}(F_{S_k})n_k-\bar{G}_{\hat{n}}(\hat{n}F)\hat{n}+\bar{G}_{\hat{n}+1}((\hat{n}+1)F)(\hat{n}+1))\\ \Lambda_2(\hat{n}) &= -V'_{\hat{n}+1}((\hat{n}+1)F)C\end{aligned}$$

and $V'_{\hat{n}+1}((\hat{n}+1)F)=V_{\hat{n}+1}((\hat{n}+1)F)-V_{\hat{n}}(\hat{n}F)$, $G'_{\hat{n}+1}((\hat{n}+1)F)=G_{\hat{n}+1}((\hat{n}+1)F)-G_{\hat{n}}(\hat{n}F)$. Therefore the FOC is negative if $\delta\leq\delta_0=\min_{\hat{n}}\{\Lambda_2(\hat{n})/\Lambda_1(\hat{n}):\Lambda_1(\hat{n})>0\}$. $\square$

When δ is less than δ_0, the structure of Lemma A5 holds and
(i) $\{K_1\cup\{s\},K_2,...,K_l\}$, that is, join K_1, if $\dfrac{\pi(\mathscr{K}^1)}{\pi(\mathscr{K}^0)}\geq\dfrac{V_1(F)}{V_{n_1+1}((n_1+1)F)}$
(ii) $\{\{s\},K_1,K_2,...,K_l\}$, that is, be independent, if $\dfrac{\pi(\mathscr{K}^1)}{\pi(\mathscr{K}^0)}\leq\dfrac{V_1(F)}{V_{n_1+1}((n_1+1)F)}$
where ordered structure $\{K_1,K_2,...,K_l\}$ represents the strategy for the rest of the players $N-\{s\}$.

For y**-player games**, given that the rest of the $n-y$ players form ordered coalitions $\mathscr{K}=\{K_1,K_2,...,K_l\}$, denote $\mathscr{K}\vee y$ as the *water-filling* structure $\mathscr{K}^*(\mathscr{K},Y)$. Then similarly for Proposition A1,
(i) the CPNE satisfies $a=0$ if and only if $\dfrac{\pi((\mathscr{K}\vee y-1)^1)}{\pi((\mathscr{K}\vee y-1)^0)}\geq\dfrac{V_1(F)}{V_{n^*}(n^*F)}$, where $n^*=n^*(\mathscr{K},y)$
(ii) the CPNE satisfies $0<a<y$, if and only if

$$\begin{aligned}\frac{\pi((\mathscr{K}\vee y-1)^1)}{\pi((\mathscr{K}\vee y-1)^0)} &\leq \frac{V_1(F)}{V_{n^*}(n^*F)},\ \text{where } n^*=n^*(\mathscr{K},y)\\ \frac{\pi((\mathscr{I}_i\cup\mathscr{K}\vee y-1-i)^1)}{\pi((\mathscr{I}_i\cup\mathscr{K}\vee y-1-i)^0)} &\leq \frac{V_1(F)}{V_{n^*}(n^*F)},\ \text{for } 1\leq i\leq a-1,\ \text{where } n^*=n^*(\mathscr{I}_i\cup\mathscr{K},y-i)\\ \frac{\pi((\mathscr{I}_a\cup\mathscr{K}\vee y-1-a)^1)}{\pi((\mathscr{I}_a\cup\mathscr{K}\vee y-1-a)^0)} &\geq \frac{V_1(F)}{V_{n^*}(n^*F)},\ \text{where } n^*=n^*(\mathscr{I}_a\cup\mathscr{K},y-a)\end{aligned}$$

(iii) the CPNE is $a=y$, if and only if

$$\begin{aligned}\frac{\pi((\mathscr{K}\vee y-1)^1)}{\pi((\mathscr{K}\vee y-1)^0)} &\leq \frac{V_1(F)}{V_{n^*}(n^*F)},\ \text{where } n^*=n^*(\mathscr{K},y)\\ \frac{\pi((\mathscr{I}_i\cup\mathscr{K}\vee y-1-i)^1)}{\pi((\mathscr{I}_i\cup\mathscr{K}\vee y-1-i)^0)} &\leq \frac{V_1(F)}{V_{n^*}(n^*F)},\ \text{for } 1\leq i\leq y-1,\ \text{where } n^*=n^*(\mathscr{I}_i\cup\mathscr{K},y-i)\end{aligned}$$

where $\mathscr{I}_k$ is the coalition structure with k independent suppliers, i.e., $\mathscr{I}_k=\{P_1,...,P_k\}$ where $|P_i|=1$ for $i=1,...,k$.

Finally, let $\mathscr{S}(m,N)$ be an m-partition of N such that $-1\leq|S_i|-|S_j|\leq1$ for any $i\neq j$ and I_k represent a set of k suppliers. Then the **CPNE** can be characterized similar to Theorem 1:
(i) form grand coalition ($m^*=1$) if $\dfrac{\pi(N)}{\pi(\{N-I_1\}^0)}\geq\dfrac{V_1(F)}{V_n(nF)}$;

(ii) form independent structure ($m^*=n$) if $\dfrac{\pi(\mathscr{S}^1(m,N-I_1))}{\pi(\mathscr{S}^0(m,N-I_1))} < \dfrac{V_1(F)}{V_{\lceil n/m\rceil}(\lceil n/m\rceil F)}$ for any $1 \le m \le n-1$;

(iii) form m^* coalitions where $1<m^*<n$, if $\dfrac{\pi(\mathscr{S}^1(m,N-I_1))}{\pi(\mathscr{S}^0(m,N-I_1))} < \dfrac{V_1(F)}{V_{\lceil n/m\rceil}(\lceil n/m\rceil F)}$ for any $1 \le m \le m^*-1$, and $\dfrac{\pi(\mathscr{S}^1(m^*,N-I_1))}{\pi(\mathscr{S}^0(m^*,N-I_1))} \ge \dfrac{V_1(F)}{V_{\lceil n/m^*\rceil}(\lceil n/m^*\rceil F)}$.

This leads to the use of refined *Risk Adjusted Stability Factor* as follows

(i) grand coalition is stable if $RASF_1 = \dfrac{\pi(N)}{\pi(\{N/\{s\}\}^0)} - \dfrac{V_1(F)}{V_n(nF)} \ge 0$;

(ii) independent structure is stable if $RASF_m = \dfrac{\pi(\mathscr{S}^0(m,N/\{s\}))}{\pi(\mathscr{S}^1(m,N/\{s\}))} - \dfrac{V_1(F)}{V_{\lceil n/m\rceil}(\lceil n/m\rceil F)} < 0$ for any $1 \le m \le n-1$;

(iii) m^* coalitions, where $1<m^*<n$, will be formed if $RASF_m = \dfrac{\pi(\mathscr{S}^0(m,N/\{s\}))}{\pi(\mathscr{S}^1(m,N/\{s\}))} - \dfrac{V_1(F)}{V_{\lceil n/m\rceil}(\lceil n/m\rceil F)} < 0$ for any $1 \le m \le m^*-1$, and $RASF_{m^*} = \dfrac{\pi(\mathscr{S}^0(m^*,N/\{s\}))}{\pi(\mathscr{S}^1(m^*,N/\{s\}))} - \dfrac{V_1(F)}{V_{\lceil n/m^*\rceil}(\lceil n/m^*\rceil F)} \ge 0$,

where $\mathscr{S}(m,N)$ is the m-partition of N such that $-1 \le |S_i| - |S_j| \le 1$ for any $i \ne j$ and I_k represent a set of k suppliers, and for any single supplier s and ordered coalition structure $\mathscr{K} = \{K_1,...,K_l\}$, $\mathscr{K}^t = \{K_i,...,K_t \cup \{s\},...,K_l\}$ and $\mathscr{K}^0 = \{\{s\},K_1,...,K_l\}$.

• When δ is great than the threshold, Lemma A9 may not hold. In face, the SOC of the expected profit is negative. Then instead of always joining the smallest coalition, there might exist a “preferred” coalition size n^* that a supplier would join the alliance whose member is most close to n^*, whereas $n^*=1$ in the special case when δ is less than the threshold. This generally implies that there will be more incentive for suppliers to join form alliances.

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