Service Product Design and Consumer Refund Policies

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Customers often exhibit considerable uncertainty in their service valuations. In response, firms may tailor their products and allow service cancellations. We consider the joint product customization and refund policy decisions of a monopolistic firm selling to a heterogeneous customer population with imperfect signals on their valuations. Our results shed light on how customers' valuation uncertainty, characterized by the valuation heterogeneity and signal quality, drives the interaction between product line and refund policy designs. In particular, when the valuation heterogeneity is high, the firm may choose to offer a single quality level with full refund, leading to a *variety reduction* in the product line. In contrast, when the valuation heterogeneity is low, the firm will always offer a full product line without any refund. At moderate valuation heterogeneity, both qualities and refunds are subject to more customization, and partial refund can be optimal when the signal quality is high, even though our setup does not involve aggregate demand uncertainty, capacity limitations, competition, or channel conflicts. Interestingly, despite its appeal, generous refund terms do not increase aggregate customer surplus. Furthermore, the firm may not have incentives to reduce customers' valuation uncertainty even if doing so is costless. We verify the robustness of our results and discuss their practical implications.

Key words: product line design; cancellations; return policy; partial refunds; consumer uncertainty; services; quality

This version: October 8, 2019

1. Introduction

Customers often exhibit considerable uncertainty in their service valuations.[1] In response, firms may tailor their products and allow service cancellation alongside some refunds. A prime example is the airline industry. At Air Canada, for instance, the economy-class customers are offered different peripheral services, in addition to a seat on the airplane, at different rates and sometimes drastically different refund terms. As illustrated in Figure 1, an Economy Standard ticket is non-refundable while Economy Latitude is fully refundable at Air Canada. An Economy Latitude ticket also enjoys other benefits including free same-day change, complementary checked baggages,

[1] We use "product" and "service" interchangeably throughout the paper.

and priority check-in, all of which are not available for Economy Standard. Therefore, Air Canada *simultaneously* customizes its products and varies its refund terms for its economy class tickets.

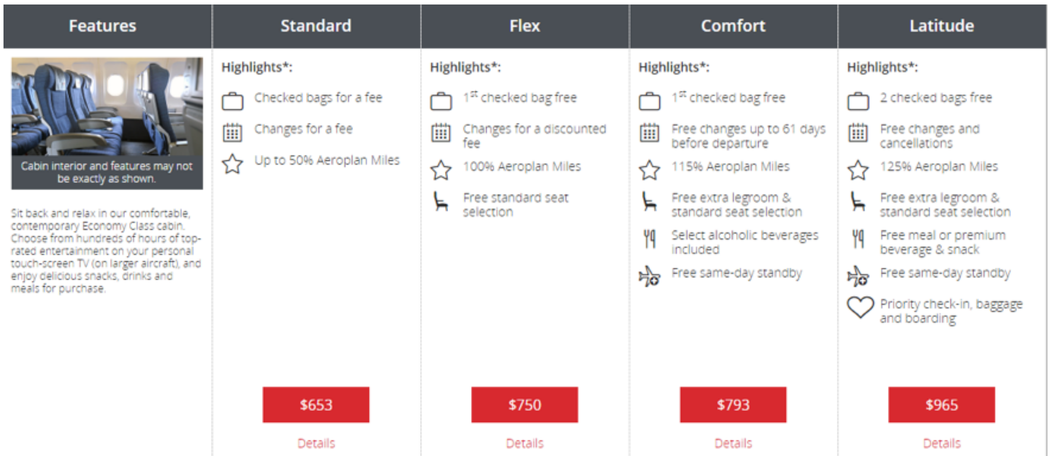

Figure 1 Economy class offerings for a flight from Denver to Montreal at Air Canada.

Product customization and refund terms can vary dramatically across industries, or even within the same firm. Indeed, for Air Canada, the product options and refund terms for business class tickets are substantially different from the economy class — only a single, premium service level is offered for the business class, along with various refund-pricing options. Unlike the airline industry, however, theme parks such as Disney World offer numerous admission options and in-park services, but all are non-transferable and non-refundable.

The joint product customization and refund design are often observed when there is a temporal separation between purchase and consumption. At the time of purchase, customers have valuation uncertainty and can only make product choices based upon their perceived valuations. Once the uncertainty is resolved, customers may exercise the return (or equivalently, cancel the service) depending upon their revealed valuations. Therefore, even though both product customization and refund policies are levers for customer discrimination, the former acts *ex ante*, while the latter acts *ex post*.

Product design and consumer refunds have each been studied extensively in the marketing and operations literature. Much of this literature, however, considers refund policies under a *given* product line, and often assumes that a single product or quality level is offered. For the research on product line design, the main focus has been on quality customization and has not incorporated refund policies. In this paper, we aim to bridge the gap by incorporating refund terms as an important dimension of product design. Several research questions naturally arise:

1. How do product customization and customer refunds interact with each other?

2. How does customers' valuation uncertainty drive a firm's product customization and refund terms?
3. When should firms reduce customers' valuation uncertainty via information provision?

1.1 Summary of Main Results

We consider the following setup. A monopolistic firm serves a customer population with uncertain valuations. Customers are either high-type or low-type, valuing product quality differently. Customers do not observe their types before the purchase but receive an imperfect signal indicating their types. The firm configures its product qualities, prices, and refund policies. Consumers self-select the product to purchase based on the signal they receive. After the purchase, customers discover their true types and decide whether to return the product and obtain a refund.

In this setup, we explicitly model customers' valuation uncertainty in two dimensions. The first dimension, *valuation heterogeneity*, measures the discrepancy between the high vs. low valuation types. It concerns the heterogeneity among customers' *true* valuations that are only revealed *ex post*. The second dimension, *imperfect signal*, determines the customers' *ex ante* perception of their valuation types. The quality of the signal captures the inter-temporal difference between customers' *perceived* and *true* valuations. We show that both dimensions are important drivers of our results.

We fully characterize the firm's optimal product line with an endogenous refund policy, which allows us to investigate the interaction between product customization and refund policies. As a baseline, we show that when refunds are not allowed, it is optimal to offer quality-differentiated products to different customer segments. However, when refunds are allowed, the firm's product line is subject to change except when customers' valuation heterogeneity is low. As valuation heterogeneity increases, the quality difference within the optimal product line shrinks while the refunds become more generous. In particular, when the valuation heterogeneity is high, it can be optimal for the seller to offer a single product with full refund to all customers, leading to a *variety reduction* effect. In this way, the firm can engage all the high-type customers even when their valuation perception is uncertain *ex ante*, which cannot be achieved without *ex post* refunds. Therefore, the flexibility to vary refund policies allows the firm to work with a narrower span of quality levels while earning higher profit.

In answering the second research question, we summarize the optimal product line design as follows:

1. When the customers' valuation heterogeneity is high, it is optimal for the firm to offer fully-refundable premium-quality products to all customers.

2. When customers' valuation heterogeneity is moderate and the signal quality is low, it is optimal for the firm to offer non-refundable good-quality service to customers who are optimistic about their valuations, and fully-refundable premium-quality service to pessimistic customers.
3. When customers' valuation heterogeneity is moderate and the signal quality is high, it is optimal for the firm to offer a premium product with a partial refund to the optimistic customers and a non-refundable low-quality product to the pessimistic customers.
4. When customers' valuation heterogeneity is low, it is optimal for the firm to adopt a no-refund policy while providing high- and low-quality products to the optimistic and pessimistic customers, respectively.

These findings suggest that the optimal product line design is jointly determined by both dimensions of the valuation uncertainty. Specifically, the product line is mainly driven by the *valuation heterogeneity* when it is extremely high or low (Results 1 and 4); otherwise, it critically depends upon the *signal quality* (Results 2 and 3). These results seem to match many observations in the marketplace. For instance, business-class airfare can be rather standardized and is often fully refundable. This may be attributed to the significant valuation heterogeneity for business trips, which is either mission-critical or has almost no value depending on the business needs (Result 1). On the contrary, visitors to Disney World theme parks are only offered non-refundable tickets since they all place a reasonably similar value for such an experience, implying low valuation heterogeneity (Results 4). Results 2 and 3 emphasize the role of the signal the customers receive. Notably, in Result 3, partial refunds can emerge as an optimal policy, even though our setup does not involve aggregate demand uncertainty, capacity limitations, competition, or channel conflicts — which are common causes identified in the existing literature for partial refunds. Instead, partial refunds in our paper result from moderate valuation heterogeneity and high signal quality, which make it more efficient to jointly customize quality *ex ante* and tailor refunds *ex post*.

For the last research question, we find that reducing customers' valuation uncertainty does not always benefit the firm. That is, a firm may not be interested in improving the signal quality or letting customers have more accurate valuation *ex ante*, even if it can be done at little or no cost. Indeed, when the valuation heterogeneity is high, it is advisable for the firm not to take any action at all. When the valuation heterogeneity is moderate, the firm should weaken the signal quality if it is low, or strengthen it if it is high. When the valuation heterogeneity is low, the firm would weaken any existing signal. In general, information provision is preferable only when customers have higher valuation heterogeneity.

We consider two extensions of our main model. In the first one, we allow the cost of cancelled product to be only partially recoverable. In the second one, we allow customers to upgrade if they turn out to be high type. We show that our key insights are robust in both extensions.

The rest of the paper is organized as follows. Section 2 surveys the most recent and relevant literature. Section 3 presents the model preliminaries and analyzes a benchmark case when refunds are not allowed. Section 4 analyzes the optimal product line design with and without standardization and discuss the implications on customer welfare. Section 5 investigates the firm's incentive to improve the signal quality. Section 6 considers two model extensions and verifies the robustness of our main results. Section 7 concludes. All technical proofs are relegated to the Appendix.

2. Literature

Our research intersects with two streams of literature: consumer refund policy and product line design.

There is an extensive literature on consumer refund policies, covering the various reasons that may trigger consumer returns such as opportunism (Chu et al. 1998, Shang et al. 2017), product quality (Moorthy and Srinivasan 1995, Hsiao and Chen 2012), product mismatch (Davis et al. 1995, Guo 2009, Shulman et al. 2010), alternate options (Xie and Gerstner 2007), unattended service (Gallego et al. 2015), customer no-shows (Ringbom and Shy 2004, Ringbom and Shy 2008) and valuation uncertainty (Che 1996, Courty and Li 2000, Liu and Xiao 2008, Su 2009, Chen 2011, Shulman et al. 2011, Akcay et al. 2013, Akan et al. 2015, Shulman et al. 2015), just to list a few. Furthermore, consumer refund policies have been studied in both manufacturing and service settings. Most of the aforementioned papers focus on the manufacturing setting, with a few dedicated to service settings such as Courty and Li (2000), Xie and Gerstner (2007), Liu and Xiao (2008), Ringbom and Shy (2008), Guo (2009), and Akan et al. (2015). Due to the volume of papers involving consumer refund policies, we do not attempt to conduct a comprehensive literature review. Rather, we will only discuss those papers that are most relevant to ours, namely, papers that study the service settings where returns are triggered by consumers' valuation uncertainty, and those that offer insights on partial refunds.

In the service context, customer refunds are usually triggered by valuation uncertainty, as cancellation normally takes place before a service is experienced. Courty and Li (2000) introduce a price-refund menu as a way for the seller to screen customers of different valuation distributions. Akan et al. (2015) extend the problem by allowing customers to learn their valuations at different times so that the seller can screen on both the amount and timing of the refund. Liu and Xiao (2008) suggest that either the service or the refund should be offered on a more restricted basis when there is a capacity constraint; in addition, they study the impact of capacity rationing on the price-refund menu design. Chen (2011) considers a capacity-constrained seller facing both aggregate demand uncertainty and heterogeneous customer valuations, and develops an optimal selling scheme that induces truthful revelation. Even though we do not consider the capacity issue in the current paper, our results corroborate the idea that refund may not be offered to all customers due to quality customization. Our work is also related to the recent literature on the impact of information provision regarding uncertain valuations. Shulman et al. (2015) find that information provision may increase the cancellation rate, particularly when the pre-purchase valuation is moderate with regard to the price. By configuring the optimal quality-price-refund menu, our study further shows that information provision may harm the firm's profit, depending on the level of valuation uncertainty.

Another research question that has attracted considerable attention in the literature is when full or partial refunds should be offered. Davis et al (1995) use a stylized model to demonstrate the conditions under which a full refund should be offered. Ringbom and Shy (2004) propose the use of common prices but different partial refunds in order to discriminate among customers of different no-show rates. Xie and Gerstner (2007) show that in the presence of finite capacity, a seller can benefit from a partial refund for service cancellation as freed capacity can be used to serve other consumers. Ringbom and Shy (2008) study collusive pricing and refund policies in the service industry. They show that monopolistic and collusive service providers offer full refund, while competitive service providers offer partial refunds. Guo (2009) studies the rationale behind offering partial refunds in a competitive setting and identifies capacity scarcity as a key driver for partial refunds that are adopted in equilibrium. Su (2009) studies consumer refund policies in a supply chain setting. He shows that it is optimal to offer partial refunds, which are driven by aggregate demand uncertainty. Shulman et al. (2010) also demonstrate the optimality of partial refunds in a supply chain. Their focus is whether the manufacturer or the retailer should process product returns. Shulman et al. (2011) further show that partial refunds can be more frugal in a competitive than monopolistic environment. Hsiao and Chen (2012) illustrate that there are conditions under which refund can exceed the full price. Tran et al. (2018) study return policy design in a supply chain setting and find partial refund can be introduced to decrease profit variability. Partial refunds also constitute an important element of our result. We show that partial refunds can emerge as an optimal policy, even when there are no capacity limitations, aggregate demand uncertainty, risk aversion, competition, or channel conflicts, and the key drivers lie in quality customization and valuation uncertainty.

With respect to the stream of research that considers product line design, the existing literature has examined this issue under various context, including market segmentation (see Moorthy 1984 and references), distribution channel (e.g., Villas-Boas 1998, Liu and Cui 2010), advertising (e.g., Villas-Boas 2004), consumer evaluation and research costs (e.g., Villas-Boas 2009, Kuksov and Villas-Boas 2010), preference structure (e.g., Orhun 2009, Kim et al. 2013), just to list a few. We, however, focus on how the product line design can be affected by customers' valuation uncertainty. In this regard, this paper is most relevant to those considered in Guo and Zhang (2012) and Xiong and Chen (2014). Both papers consider a mixture of high- and low-type customers, and study how to design the best product offerings for each type. Specifically, Guo and Zhang (2012) study the impact of consumer deliberation on optimal product design. Xiong and Chen (2014) allow consumers to choose a standard product before learning about their types, or pay for a learning fee and choose afterwards. However, neither of the two papers considers the option of refunds after customers discover their true types.

3. Model Preliminaries

A service provider (henceforth referred to as the "firm," with a masculine pronoun) serves a market with total demand normalized to 1. The firm can serve the market with one or more products. Each product is defined by a triplet (q, p, β) with quality q, price p, and the refund rate $\beta \in [0,1]$. Products can differ from each other on all or a subset of these attributes. For example, two products can share the same quality but have different prices or refund rates. The cost of serving one unit of product at quality level q is $q^2/2$. If the product is returned, no cost will be incurred and an amount βp will be refunded to the customer. This setup is meant to resonate with service settings, where cancelled service does not incur any materialized cost. However, such a setup may not be suitable for physical goods where product cost is incurred prior to purchase and returns may involve non-trivial costs such as shipping and repackaging. Nevertheless, as will be discussed later in §6.1, allowing only part of the product cost $q^2/2$ be recoverable does not impose structural changes to our main results.

3.1 Valuation Uncertainties

Each customer (henceforth referred to by a feminine pronoun) has a quality valuation θ and derives a product valuation θq for a product with quality q (Mussa and Rosen 1978). The parameter θ is uncertain for each customer, who can be "high-type" with quality valuation θ_H, or "low-type" with quality valuation θ_L. Specifically,

$$\theta = \begin{cases} \theta_H, & \text{with probability } \alpha, \\ \theta_L, & \text{with probability } 1-\alpha, \end{cases}$$

where $0 \leq \alpha \leq 1$ and $\theta_L \leq \theta_H$. In order to simplify our notation, we use $\bar{y}$ to denote $1-y$ for any $y \in [0,1]$ in the rest of the paper. The distribution of θ is known to both the firm and the customers, and the ratio θ_H/θ_L, henceforth referred to as *valuation heterogeneity*, is an important characteristic of customers' valuation uncertainty.

Customers do not observe their valuations prior to purchase. Instead, each customer receives a private signal, "Good" or "Bad," indicating her true type. Subsequently, we refer to these customers as the "good-signal" or "bad-signal" customers, respectively. The quality of the signal can be measured by the probability of getting the right signal conditional on one's true type:

$$P\{\text{Good} \,|\, \text{High-type}\} = P\{\text{Bad} \,|\, \text{Low-type}\} = \rho,$$

where we assume $\rho \in (1/2, 1]$; thus the signals are informative. As one's valuation type is only revealed after the purchase, the *signal quality* ρ reflects the disparity between the perceived and true valuations, representing another important characteristic of customers' valuation uncertainty.

The signal and valuation types jointly divide customers into four categories based on their perceived valuation *ex ante* and true valuation *ex post*, as illustrated in Table 1.

Table 1 Customer categorization based on *ex ante* and *ex post* types.

		Prior to Purchase	
		Good Signal	Bad Signal
Post Purchase	High Valuation	$\rho\alpha$	$\bar{\rho}\alpha$
	Low Valuation	$\bar{\rho}\bar{\alpha}$	$\rho\bar{\alpha}$

Prior to purchase, there are two segments of customers — a fraction $\rho_G = \rho\alpha + \bar{\rho}\bar{\alpha}$ are good-signal customers and the rest, a fraction of $\rho_B = \bar{\rho}_G = \rho\bar{\alpha} + \bar{\rho}\alpha$, are bad-signal customers. Each segment of customers can apply Bayes' Rule with the knowledge of valuation distribution to estimate their true types. For instance, a good-signal customer is of high-type with probability α_G and a bad-signal customer is of high-type with probability α_B, where

$$\alpha_G = \frac{\rho\alpha}{\rho\alpha + \bar{\rho}\bar{\alpha}}, \quad \alpha_B = \frac{\bar{\rho}\alpha}{\bar{\rho}\alpha + \rho\bar{\alpha}}.$$

The expected valuations for good- and bad-signal customers are $\theta_G = \alpha_G\theta_H + (1-\alpha_G)\theta_L$ and $\theta_B = \alpha_B\theta_H + (1-\alpha_B)\theta_L$, respectively. Observe that, since $\rho \in (1/2, 1]$, a good signal implies a higher chance of being a high-type customer, as well as higher expected quality valuations; i.e., $\alpha_G > \alpha > \alpha_B$ and $\theta_G > \theta_B$.

After the purchase, valuation uncertainty is resolved and customers learn their true types. The customers again fall into two segments – a fraction α are high-type and the rest $\bar{\alpha}$ are low-type. Customers make return decisions based on their true valuation types.

3.2 Customer Choice and Market Outcomes

Given that the signals are private for individual customers, the firm faces an adverse selection problem. The firm needs to design a product line with at most two products, product "G", (q_G, p_G, β_G), and product "B", (q_B, p_B, β_B), catering to the good- and bad-signal customers, respectively. When $(q_G, p_G, \beta_G) = (q_B, p_B, \beta_B)$, the two products are identical and only one product is offered to all customers. The firm can also choose not to serve the good- or bad-signal customers; in our analysis, this can be done by assigning them a dummy product $(0, 0, 1)$.

Let u_{si} denote the expected surplus for customers receiving signal s and purchasing product i, where $s, i \in \{G, B\}$. To induce the good-signal customers to choose product "G" and bad-signal customers to choose product "B", the firm needs to ensure that both individual rationality (IR) and incentive compatibility (IC) constraints are satisfied. That is,

$$
\begin{aligned}
& u_{GG} \geq 0, && (IR_G) \\
& u_{BB} \geq 0, && (IR_B) \\
& u_{GG} \geq u_{GB}, && (IC_G) \\
& u_{BB} \geq u_{BG}. && (IC_B)
\end{aligned}
$$

The following Lemma is a result of the (IR) constraints, noting that a customer will return the product if and only if the potential refund exceeds the true product valuation.

LEMMA 1. *Only the low-type customers may exercise the refunds.*

Therefore, the expected surplus for customers receiving signal s and purchasing product i is

$$u_{si} = \alpha_s(\theta_H q_i - p_i) + \bar{\alpha}_s \max\{\theta_L q_i - p_i, -\bar{\beta}_i p_i\} = \alpha_s \theta_H q_i + \bar{\alpha}_s \max\{\theta_L q_i, \beta_i p_i\} - p_i, \tag{1}$$

and a customer who has purchased product $i \in \{G, B\}$ will return the product if and only if (i) she turns out to be of low-type ($\theta = \theta_L$), and (ii) the refund exceeds her product valuation ($\theta_L q_i \leq \beta_i p_i$). In other words, a customer will keep the product with quality q_i should her valuation turns out to be high, or return the product and obtain a refund of $\beta_i p_i$ if her valuation turns out to be low. In any event, the quality q_i and refund $\beta_i p_i$ jointly determine the customer's reward after the uncertainty resolves. It then follows that there are four possible market outcomes with regard to customer refunds:

RR: all low-type customers will claim the refund;

NR: only bad-signal (purchasing product "B") low-type customers will claim the refund;

RN: only good-signal (purchasing product "G") low-type customers will claim the refund;

NN: no customer will claim the refund.

In order to design the optimal product line, the firm needs to investigate the optimal product line design under each market outcome, and compare the profits across the four candidate solutions.

3.3 Benchmark: Product Line Design without Refund

We first analyze the benchmark case where no refund is allowed. It also corresponds to the product design under the market outcome **NN**. Before proceeding, we introduce the following notations and relations that will be used in the remainder of the paper:

$$\begin{aligned} \phi(\lambda,\alpha_1,\alpha_2) &= \frac{\lambda\alpha_2-\alpha_1}{\bar{\alpha}_1-\lambda\bar{\alpha}_2}, \qquad \forall\, 0\le\alpha_1<\alpha_2\le 1, 0\le\lambda\le 1,\\ \alpha_0 &= \frac{\alpha_B-\rho\alpha}{\bar{\rho}_G}\le\alpha_B\le\alpha\le\alpha_G,\\ \theta_0 &= \max\{\alpha_0\theta_H+\bar{\alpha}_0\theta_L,0\}\le\theta_B\le\theta_G\le\theta_H,\\ \phi_0 &= -\frac{\alpha_0}{1-\alpha_0}. \end{aligned}$$

Whenever comparison arises, we refer to θ_0 as "inferior", θ_G as "moderate", and θ_H as "high."

When no refund is allowed, i.e., $\beta_G=\beta_B=0$, the firm solves the following optimization problem:

$$\max_{q_G,q_B,p_G,p_B\ge 0}\ \rho_G(p_G-q_G^2/2)+\bar{\rho}_G(p_B-q_B^2/2)$$
$$\text{s.t. } (IR_G),(IR_B),(IC_G),(IC_B).$$

Note that this formulation allows both single- and dual-quality design, as well as partial or full market coverage. The results are summarized in the following Lemma.

Lemma 2 **(Optimal Product Line Design without Refund)**. *When no refund is offered,*

(i) the optimal product line is

$$\begin{aligned}(q_G^{NN},p_G^{NN}) &= (\theta_G,\theta_G(\theta_G-\theta_0)+\theta_0\theta_B),\\ (q_B^{NN},p_B^{NN}) &= (\theta_0,\theta_0\theta_B);\end{aligned}$$

(ii) When $\frac{\theta_L}{\theta_H}\ge\phi_0$*, the firm is better off with dual-quality design and full market coverage* ($\theta_G>\theta_0>0$)*; otherwise, the firm is better off with single-quality design and partial market coverage* ($\theta_G>\theta_0=0$)*.*

Lemma 2 shows that each customer segment will be assigned a distinct quality level. Good-signal customers will be served with moderate quality ($q_G^N = \theta_G$), and bad-signal customers will be served with inferior quality ($q_B^N = \theta_0$) or unserved when $\theta_0 = 0$. When some threshold condition is satisfied, the entire market will be covered at two distinct quality levels; otherwise, the market will be partially covered by one product. Specifically, dual quality design is optimal for any valuation heterogeneity (θ_L/θ_H) when $\alpha_0 \geq 0$, or equivalently, $\alpha \geq \frac{\rho^2+\rho-1}{\rho(2\rho-1)}$. As will be shown in the next section, this may not be true when refund is allowed.

4. Optimal Product Line Design

We now analyze the optimal product line design where refunds are allowed. The firm needs to design two products, (q_G, p_G, β_G) and (q_B, p_B, β_B), targeting good- and bad-signal customers, respectively. As discussed in Section 3.2, in order to find the optimal product line design, the firm compares the optimal product lines under each possible market outcome. As an example, the optimal design with the market outcome **RR** can be identified by solving:

$$\max_{q_G,q_B,p_G,p_B,\beta_G,\beta_B\geq 0} \Pi^{RR} = \rho_G\left(p_G - \bar{\alpha}_G\beta_G p_G - \frac{\alpha_G q_G^2}{2}\right) + \bar{\rho}_G\left(p_B - \bar{\alpha}_B\beta_B p_B - \frac{\alpha_B q_B^2}{2}\right)$$

$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0, & (IR_G) \\ u_{BB} & \geq & 0, & (IR_B) \\ u_{GG} & \geq & u_{GB}, & (IC_G) \\ u_{BB} & \geq & u_{BG}, & (IC_B) \\ \beta_G p_G & \geq & \theta_L q_G, & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B, & (RR_B) \\ \beta_G & \leq & 1, & \\ \beta_B & \leq & 1. & \end{array}$$

The firm's profit in the objective function includes initial revenues, net of refunds issued to customers, minus the service cost for customers who choose to keep the product. The (IR) and (IC) constraints ensure that the products are targeting the right set of customers, and the (RR) constraints ensure the **RR** market outcome — that all low-valuation customers, whether purchasing product G or B, will exercise the refund.

The solution under the market outcome **NN** was characterized in Lemma 2. The problem formulations for market outcomes **NR** and **RN** can be written in a similar fashion as that of **RR** above; we relegate the details to Appendix A. Comparing the profits across all market outcomes gives the results in Table 2. The key findings of the optimal design are summarized in the following theorem.

THEOREM 1 **(Optimal Product Line Design)**. *When refunds are allowed, there exists* $\bar{\phi}^* \geq \underline{\phi}^* \geq 0$ *and* $\frac{1}{2} \leq \hat{\rho} \leq 1$*, such that*

Table 2 Optimal Product Line

Valuation Heterogeneity / Signal Quality	$High: \theta_L/\theta_H \leq \underline{\phi}^*$	Moderate: $\underline{\phi}^* < \theta_L/\theta_H \leq \bar{\phi}^*$ — $Low: \rho \leq \hat{\rho}$	Moderate: $\underline{\phi}^* < \theta_L/\theta_H \leq \bar{\phi}^*$ — High: $\rho > \hat{\rho}$	$Low: \theta_L/\theta_H > \bar{\phi}^*$
Quality (q_G^*, q_B^*)	(θ_H, θ_H)	(θ_G, θ_H)	(θ_H, θ_0)	(θ_G, θ_0)
Price (p_G^*, p_B^*)	$(\alpha_G\theta_H^2 \sim \theta_H^2, \theta_H^2)$	(θ_G^2, θ_H^2)	$(\theta_G(\theta_H - \theta_0) + \theta_0\theta_B \sim \theta_H(\theta_H - \theta_0) + \theta_0\theta_B, \theta_0\theta_B)$	$(\theta_G(\theta_G - \theta_0) + \theta_0\theta_B, \theta_0\theta_B)$
Refund Rate (β_G^*, β_B^*)	$(\frac{1}{\bar{\alpha}_G} - \frac{\alpha_G}{\bar{\alpha}_G}\frac{\theta_H^2}{p_G^*}, 1)$	$(0, 1)$	$(\frac{1}{\bar{\alpha}_G} - \frac{\alpha_G\theta_H^2 - (\theta_G - \theta_B)\theta_0}{\bar{\alpha}_G p_G^*}, 0)$	$(0, 0)$
Market Outcome	**RR**	**NR**	**RN**	**NN**

In the table above, $\underline{\phi}^* = \min\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\}$ and $\bar{\phi}^* = \max\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\}$.

(i) *if $\frac{\theta_L}{\theta_H} \leq \underline{\phi}^*$, the market outcome is **RR**, and the firm may offer a single, high quality, fully refundable product ($q_G = q_B = \theta_H$, $\beta_G = \beta_B = 1$) to all customers;*

(ii) *if $\underline{\phi}^* \leq \frac{\theta_L}{\theta_H} \leq \bar{\phi}^*$ and $\rho \leq \hat{\rho}$, the market outcome is **NR**, and the firm should offer moderate quality product to good-signal customers ($q_G = \theta_G$) without any refund ($\beta_G = 0$), and high quality product to bad-signal customers ($q_B = \theta_H$) with full refund ($\beta_B = 1$);*

(iii) *if $\underline{\phi}^* \leq \frac{\theta_L}{\theta_H} \leq \bar{\phi}^*$ and $\rho > \hat{\rho}$, the market outcome is **RN**, and the firm should offer high quality product to good-signal customers ($q_G = \theta_H$) with partial refund ($0 < \beta_G < 1$) and inferior quality product to bad-signal customers ($q_B = \theta_0$) without any refund ($\beta_B = 0$);*

(iv) *if $\frac{\theta_L}{\theta_H} \geq \bar{\phi}^*$, the market outcome is **NN**, and the firm should offer moderate quality product to good-signal customers ($q_G = \theta_G$) and inferior quality product to bad-signal customers ($q_B = \theta_0$) without any refund ($\beta_G = \beta_B = 0$).*

The results clearly demonstrate how the optimal product line varies with respect to the valuation uncertainty. To begin with, we observe that when the valuation heterogeneity is high ($\theta_L/\theta_H \leq \underline{\phi}^*$), so the valuation uncertainty is high *ex post*, the firm should defer all customer discrimination *ex post* accordingly. That is, the firm can rely upon a single, high quality ($q_G^* = q_B^* = \theta_H$) product that is fully refundable ($\beta_G^* = \beta_B^* = 1$), to serve both good- and bad-signal customers. Even though there is room for refund customization (product G tolerates some price-refund trade off as shown in Table 2), only one quality level will be offered to all customers. This is in stark contrast to the case when refund is not allowed, where the firm may adopt a dual-quality design ($q_G^{NN} = \theta_G > q_B^{NN} = \theta_0 > 0$) under certain conditions (e.g., when $\alpha_0 \geq 0$). In fact, it can be shown that there always exists situations where the optimal design with refund entails a single high-quality level, while the optimal design with no refund calls for two distinct quality levels.

COROLLARY 1 **(Variety Reduction)**. *There are always $\phi_0 \le \underline{\phi}^*$ and customer refund induces a variety reduction in product line design when $v_L/v_H \in (\phi_0, \underline{\phi}^*]$.*

Recall from Lemma 2 and Theorem 1 that ϕ_0 is the threshold above which the no-refund design uses dual quality, and $\underline{\phi}^*$ is the threshold below which the customized design offers a single quality. As shown in the proof of Corollary 1, there is a non-trivial region within which customized design offers less quality levels than the benchmark no-refund design. We refer to this effect as *variety reduction.* This variety reduction effect is akin to the seminal result of Moorthy (1984), who shows that customers' incentive compatibility constraints sometimes cause the firm to combine products for different customer segments. The single-quality design entails extracting value from the high-type customers only. Since low-type customers do not value the product much, serving them will cannibalize the revenue from high-type customers. The strategy of targeting high-type customers only, however, cannot be operationalized without refund; since customers do not know their type at the time of purchase, type-based screening at the time of purchase is infeasible. Therefore, without refund, the firm is heavily dependent upon imperfect, signal-based screening tools *ex ante*, such as quality customization, for customer discrimination. With refund, the firm can perform more effective, type-based screening *ex post*, by offering premium quality service at high-type customers' maximum willingness to pay.

When the valuation heterogeneity is rather low ($\theta_L/\theta_H > \bar{\phi}^*$), hence the valuation uncertainty is low *ex post*, offering refunds will encourage returns from some customers and leaving these customers unserved, potentially hurting the firm's revenue. Thus the firm will discriminate customers *ex ante* via quality customization, and will not offer any refund. Coupled with the no refund policy, the firm offers the lowest quality among all scenarios, i.e., θ_G for good-signal customers and θ_0 for bad-signal customers, since customers only pay for the expected quality and do not have a recourse when their valuations turn out to be low.

Finally, when valuation heterogeneity is moderate ($\underline{\phi}^* < \theta_L/\theta_H \le \bar{\phi}^*$), the optimal product line depends on the signal quality, namely the *ex ante* valuation uncertainty. In this case, both refund and quality are customized, and refund is only allowed for one product. When the signal is weak ($\rho \le \hat{\rho}$), the *ex ante* valuation uncertainty is high. The firm should then tone down *ex ante* quality customization and put more weight on *ex post* refund customization. As shown in the third column in Table 2, customers are offer good and premium quality products, whereas one is non-refundable and the other is fully refundable. When the signal is strong ($\rho > \hat{\rho}$) and the *ex ante* valuation uncertainty is low, however, the firm should shift more weight to *ex ante* quality customization. Indeed this is the scenario where product qualities exhibit the largest gap, θ_H vs θ_0, and partial refund may arise as a result of reduced discrimination *ex post*. We summarize the effects of valuation uncertainty in Corollary 2.

Table 3 Customization Strategy

Valuation Uncertainty		**Product Line Customization**	
ex post	*ex ante*	*ex post*	*ex ante*
High		Full	None
Moderate	High	Full or None	Low
Moderate	Low	Partial or None	High
Low		None	Moderate

Corollary 2 **(Effects of Valution Uncertainty)**. *(i) The firm offers higher quality levels and more liberal refund policies as the valuation heterogeneity increases;*

(ii) Under moderate valuation heterogeneity, the bad-signal customers are offered a high quality product and more generous refund than good-signal customers when the signal is weak; the opposite is true when the signal is strong.

Note that in Corollary 2(ii) when the valuation heterogeneity is moderate and signal is strong, it is valuable for the firm to fine tune the refund terms and offer a partial refund ($0 < \beta_G^* < 1$). Specifically, such fine tuning of refund terms does not add value in the scenario when the valuation heterogeneity is moderate but signal is weak ($\rho \leq \hat{\rho}$). Hence partial refunds reflect the adjustment in *ex post* customer discrimination in response to a more accurate signal. This finding is summarized in Corollary 3.

Corollary 3 **(Partial Refunds)**. *Partial refunds ($0 < \beta < 1$) may arise when the valuation heterogeneity is moderate and the signal is strong ($\rho > \hat{\rho}$).*

Note that the threshold signal quality $\hat{\rho}$ changes with α. The next proposition characterizes the values of $\hat{\rho}$ for different α values.

Proposition 1 **(Threshold Signal Quality)**. *The threshold signal quality $\hat{\rho}$ varies with α. In particular, we have (i) $\hat{\rho} = 0.5$ for any $\alpha \in (0, \frac{1}{9}]$; (ii) $0.5 < \hat{\rho} \leq 1$ when $\alpha \in (\frac{1}{9}, \frac{1}{2})$; (iii) $\hat{\rho} = 1$ for any $\alpha \in [0.5, 1]$.*

Together, Theorem 1 and Proposition 1 suggest that whether partial refund should be offered is influenced by both valuation uncertainty and market composition. In a market dominated by low-type customers ($\alpha < \frac{1}{9}$), offering a full refund policy can undermine the profitability of the firm.

The firm should use partial refunds at all times ($\hat{\rho} = 0.5$). As the fraction of low-type customers drops (α increases), the firm can afford to offer more liberal refund policies coupled with higher quality when the signal is less accurate ($\rho < \hat{\rho} \in (0.5, 1)$). When the majority of customers are high-type customers ($\alpha \geq 0.5$), the threshold quality $\hat{\rho} = 1$, and the firm can restrict to full-or-no refunds without regard to the signal quality.

4.1 Numerical Examples

We conduct a numerical study to illustrate the impact of valuation uncertainties and market composition on the optimal product line. The results are shown in Figure 2. The vertical axis corresponds to market composition (α), and the horizontal axis represents signal strength (ρ). We consider several different valuation heterogeneity corresponding to different values of θ_L/θ_H.

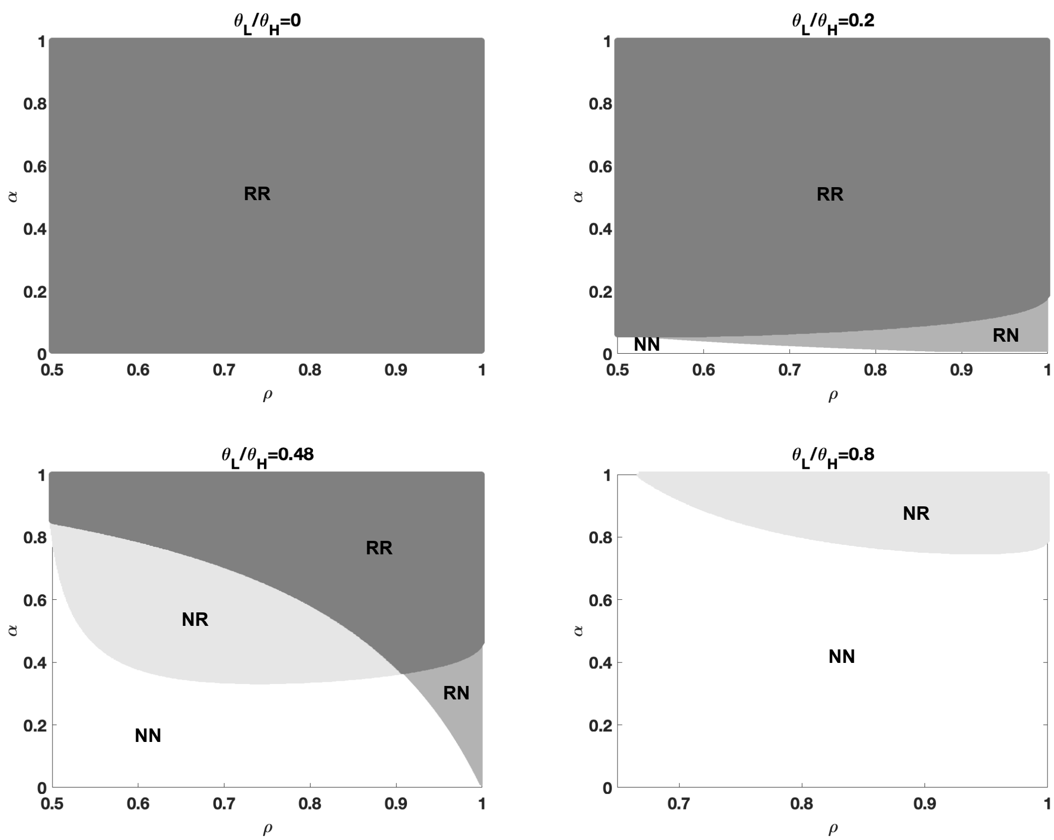

Figure 2 Optimal market outcome (**RR**: all low-type customers will return, **RN**: only good-signal-low-type customers will return, **NR**: only bad-signal-low-type customers will return, **NN**: no customer will return) based upon valuation uncertainties (θ_L/θ_H: valuation heterogeneity; ρ: signal quality) and market composition (α: fraction of high-type customers).

The top left graph describes the scenario where the low-valuation customer will derive zero utility ($\theta_L/\theta_H = 0$); therefore, the valuation heterogeneity is high. In this scenario, the firm will make refunds available to all customers (i.e., the market outcome is **RR**), regardless of the market composition (α) or signal quality (ρ).

As the valuation heterogeneity decreases (θ_L/θ_H increases), it is more likely that the refund will be restricted to one of the products, but **RR** is still optimal as long as the fraction of high-valuation customers (α) and the valuation heterogeneity are reasonably high. In other words, the boundary conditions for the **RR** outcome ($\underline{\phi}^*$) is characterized by both the fraction α and signal quality ρ. For example, to ensure that $\theta_L/\theta_H = 0.2$ falls below $\underline{\phi}^*$, the fraction α should exceed 0.07 for the null signal ($\rho = 0.5$) and 0.18 for near-perfect signal (ρ approaching 1). For a relatively low heterogeneity level $\theta_L/\theta_H = 0.48$, α should be over 0.9 (for the null signal) or 0.42 (for the near-perfect signal) for the market outcome to be **RR**.

When the valuation heterogeneity drops below a certain threshold, e.g., $\theta_L/\theta_H > 0.5$, **RR** is never optimal, and **NN** becomes more dominant. In all four graphs, the market outcome **NN** is optimal under conditions that are opposite to those inductive to **RR** — when the fraction of high-valuation customers (α) as well as the valuation heterogeneity are both low. Specifically, when θ_L/θ_H approaches 1, **NN** will always be adopted.

The bottom left graph ($\theta_L/\theta_H = 0.48$) illustrates the ranges for **RN** and **NR** when the valuation heterogeneity is moderate ($\underline{\phi}^* \le \theta_L/\theta_H \le \bar{\phi}^*$). In general, **RN** occupies the southeast region while **NR** occupies the northwest. Consistent with our analytical results, refund will only be exercised by the good-signal customers (**RN**) if the signal is strong but high-valuation customers are rare, and only by bad-signal customers (**NR**) when the signal is weak but high-valuation customers are plentiful. The actual market outcome depends upon the valuation heterogeneity (θ_L/θ_H). For example, consider the case with $\alpha = 0.7$ and $\rho = 0.6$, the optimal product line would yield the market outcome **RR** $\rightarrow$ **NR** $\rightarrow$ **NN** as θ_L/θ_H increases from 0 to 1; but with $\alpha = 0.3$ and a more accurate signal $\rho = 0.95$, the market outcome would vary from **RR** $\rightarrow$ **RN** $\rightarrow$ **NN** as θ_L/θ_H increases from 0 to 1.

We also wish to comment on the instance where the signal is perfect and customers know their true types before purchase, i.e., $\rho = 1$. It can be verified that market outcomes shown on the right edge of each graph in Figure 2 will yield the same profit for the firm as **NN**. Thus, the firm can simply use the product line developed in Lemma 2 corresponding to the market outcome **NN**.

4.2 Standardization

We have now fully characterized the optimal product line. Next, we consider product line standardization where either a standard quality or a common refund is offered to all customers. Standardization can simplify the product line, and can be useful when the cost of introducing additional products or managing a complicated refund policy is high. We also find that sometimes it can be optimal to customize either the product quality or the refund terms, but not both, even if the firm possesses such capability.

Proposition 2 **(Optimality of Standarization)**.

(i) A standard refund maximizes the firm's expected profit when the signal is strong and the high-type customers does not form the majority;

(ii) A standard quality maximizes the firm's expected profit when the valuation heterogeneity is high.

When the valuation heterogeneity is low or high ($\theta_L/\theta_H \geq \bar{\phi}^*$ or $\theta_L/\theta_H \leq \underline{\phi}^*$), it can be derived from Theorem 1 that a common refund rate ($\beta_G = \beta_B$) is optimal. When the valuation heterogeneity is moderate ($\underline{\phi}^* \leq \theta_L/\theta_H \leq \bar{\phi}^*$) and the signal is strong ($\rho \geq \hat{\rho}$), the optimal design involves an **RN** market outcome with partial refund to the good-signal customers. In this scenario, we are able to show that the optimality would sustain even if the same partial refund is extended to the entire population, as the bad-signal customers would prefer not to exercise such refund and the market outcome remains to be **RN**; see details in Appendix C. Thus a standard refund is optimal when the signal quality ρ is over the threshold $\hat{\rho}$. In addition, given that $\hat{\rho} = 1$ when $\alpha > 0.5$ as shown in Proposition 1, the optimality of standard refunds requires the high-type customers not forming the majority of the market ($\alpha < 0.5$).

Propositionpp(ii) follows immediately from Theorem 1. As shown in Theorem 1, offering a single quality ($q_G = q_B$) is optimal if and only if the valuation heterogeneity is high ($\theta_L/\theta_H \leq \underline{\phi}^*$). Otherwise there is always a need for quality customization. The driving force of quality standardization was discussed after Corollary 1.

In addition, we investigate the optimal design when the firm is required to offer a single quality level across all customers, but may customize refunds; see details in Appendix B. It turns out that the firm would always apply highly differentiated refund terms (none or full) to effectively discriminate the customers *ex post*, due to the fact that offering a standard quality makes it impossible to discriminate customers *ex ante*. Consequently, partial refund will not be used together with offering a standard quality.

4.3 Consumer Welfare

How does customer welfare vary across customers? Would customers be better off under customized refund rates than a standard refund rate? We summarize customers' surplus in Tables 4 and 5. Overall, customers fall into four categories depending on the signals they receive and valuation types, denoted by **GH**, **GL**, **BH** and **BL**, where the first letter represents the signal ("Good" or "Bad") and the second reflects the valuation type ("High" or "Low"). The surplus for each customer category is calculated under all possible market outcomes induced by the optimal design with a standard (Proposition 6), as well as customized (Theorem 1) refund policy.

Table 4 Consumer welfare under *standard* refund policy

Customer Type	Full Refund	Partial Refund	No Refund
GH	0	$\theta_H(\theta_H-\theta_G)+\theta_0(\theta_G-\theta_B)$	$\theta_G(\theta_H-\theta_G)+\theta_0(\theta_G-\theta_B)$
GL	0	$-\theta_H(\theta_G-\theta_L)+\theta_0(\theta_G-\theta_B)<0$	$-\theta_G(\theta_G-\theta_L)+\theta_0(\theta_G-\theta_B)<0$
BH	0	$\theta_0(\theta_H-\theta_B)$	$\theta_0(\theta_H-\theta_B)$
BL	0	$-\theta_0(\theta_B-\theta_L)<0$	$-\theta_0(\theta_B-\theta_L)<0$

Table 5 Consumer welfare under *customized* refund policy (when not coinciding with the standard policy)

Customer Type	Full Refund	Customized Refund	No Refund
GH	0	$\theta_G(\theta_H-\theta_G)$	$\theta_G(\theta_H-\theta_G)+\theta_0(\theta_G-\theta_B)$
GL	0	$-\theta_G(\theta_G-\theta_L)<0$	$-\theta_G(\theta_G-\theta_L)+\theta_0(\theta_G-\theta_B)<0$
BH	0	0	$\theta_0(\theta_H-\theta_B)$
BL	0	0	$-\theta_0(\theta_B-\theta_L)<0$

In general, customers who are offered a full refund will always end up with a zero surplus. Other than that, high-valuation customers receive a positive surplus, while low-valuation customers receive a negative surplus. *Ex ante*, good-signal customers expect a positive surplus $\theta_0(\theta_G-\theta_B)$, while bad-signal customers expect a zero surplus under both standard and customized refund policies. This suggests that customizing the refund policy does not necessarily change customers' *ex ante* expected surplus.

Note that the optimal product lines under the standard and customized refund may coincide with each other, e.g., when the conditions in Proposition 4.2(ii) hold. To make the comparison more meaningful, we consider scenarios where the policies are distinct under the standard and customized refund, which occurs under moderate valuation heterogeneity, i.e., $\theta_L/\theta_H\in[\underline{\phi}^*,\bar{\phi}^*]$. In this region, a standard refund will allow *no* refund while the customized refunds will offer *distinct* refunds. From Tables 4 and 5, high-type customers are always better off under the standard refund, while low-type customers are better off under the customized refund. These findings are summarized in the following proposition:

Proposition 3. *(i) The refund policy does not change the expected customer surplus based on signals;*

(ii) High-type customers receives more (resp. less) surplus under the standard (resp. customized) refund policy, while the reverse applies to low-type customers;

(iii) The standard refund increases the variance in customer surplus, while the customized refund reduces the variance in the customer surplus.

5. Reducing Customer Uncertainty via Information Provision

In our model, one source of valuation uncertainty originates from the imperfect signal customers receive. Refunds can be viewed as an *ex post* information provision device that allows customers to adjust their purchase decisions. Should customers have better knowledge of their types, however, the firm may serve them differently. We therefore would like to investigate whether the firm may benefit from any *ex ante* information provision by improving the signal strength, which is the focus of this section. The findings are summarized as follows:

Proposition 4 **(Information Provision)**.

(i) When the valuation heterogeneity is high, the firm has no incentive to improve the signal quality;

(ii) When the valuation heterogeneity is low, the firm has incentive to weaken the signal;

(iii) When the valuation heterogeneity is moderate, the firm may have an incentive to strengthen the signal when it is high, and weaken the signal when it is low.

To illustrate the findings, we conduct a numerical study under a similar set of parameters as in Figure 2. The firm's optimal profit as a function of the signal strength ρ is plotted in Figure 3.

When the valuation heterogeneity is high as in the top left graph (θ_L/θ_H=0), the firm can solely rely upon refund to extract all surplus from the high-type customers. Therefore, improving signal adds little value to the firm.

When valuation heterogeneity is low, e.g., when $\theta_L/\theta_H = 0.8$, as Theorem 1 (iv) suggests, the firm inclines to use quality customization than information provision. Therefore, the firm does not gain much from information provision. In fact, the firm may wish to weaken the signal to make the customers less informed *ex ante*.

When the valuation heterogeneity is moderate, the benefit of *ex ante* information provision can go both ways. On one hand, strengthening the signal may reduce the firm's profit, particularly when the signal is weak to begin with, e.g., when $\rho < 0.7$, θ_L/θ_H=0.48 and $\alpha = 0.7$. Accordingly to the discussion under Theorem 1, this is when quality customization dominates refund. Following the same line of reasoning as the above case, information provision is not rewarding to the firm.

On the other hand, signal improvement can benefit the firm when it is already high, e.g., when $\theta_L/\theta_H = 0.48$ or 0.2 and $\alpha < 0.5$. This is the case where firms put more weight on *ex post* information provision than *ex ante* quality customization. Therefore, the firm may benefit from signal improvement as a form of information provision.

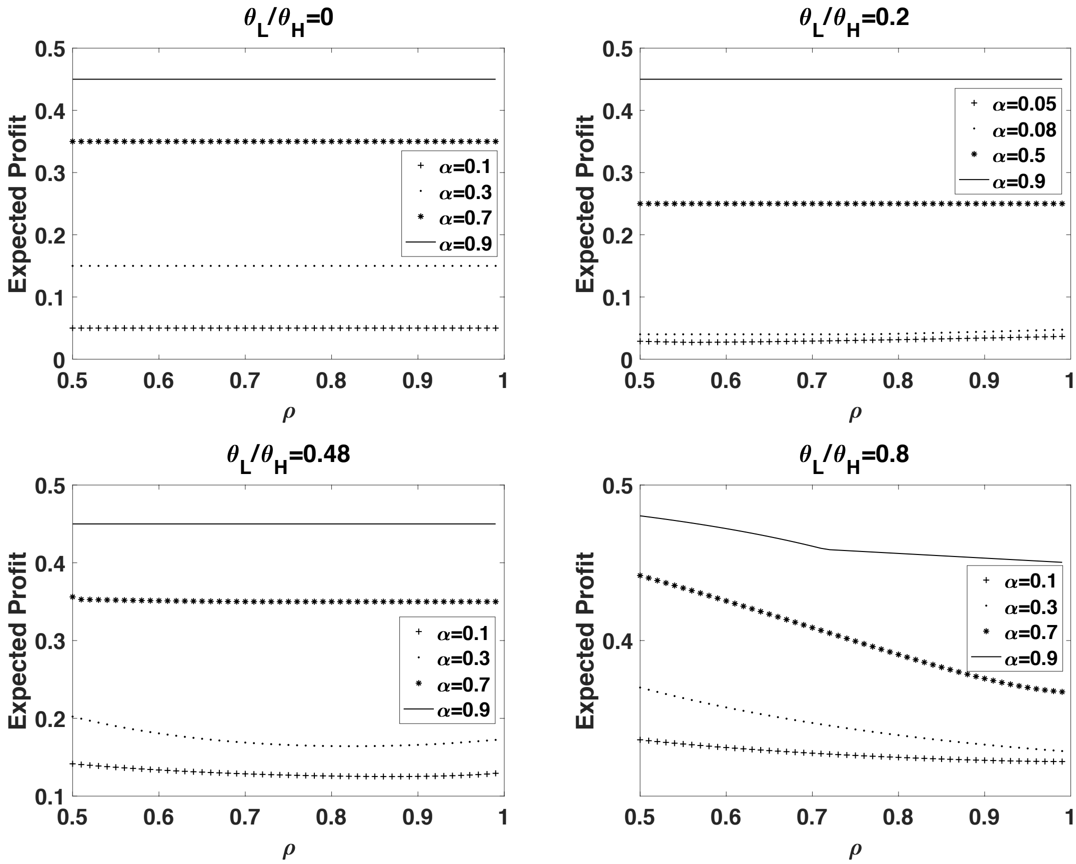

Figure 3 The optimal expected profit as the customer valuation, composition and signal quality vary.

Table 6 Firm's Information Provision Strategy

	High Valuation Heterogeneity	**Moderate** Valuation Heterogeneity	**Low** Valuation Heterogeneity
ex ante	No action	Strengthen strong signal; Weaken ambiguous signal	Weaken all signals
ex post	Full refunds	Customized refunds	No refund

We summarize the managerial implications of the above findings in Table 6. When the valuation heterogeneity is high, the firm can solely rely on *ex post* information provision devices such as customer refunds. When the valuation heterogeneity is low, the firm may abandon *ex post* information provision and proactively engage in activities that weaken customers' signals *ex ante*, e.g., providing complicated catalogues or overwhelming advertisements, adding to the difficulty for customers to evaluate their valuations. When the valuation heterogeneity is moderate, the firm may adopt both *ex post* and *ex ante* information provision devices. Specifically, the firm may wish to magnify the current signal – that is, strengthen the signal if it is already strong, e.g., via customer consultation or product review, or weaken the signal if it is noisy. We should point out that the above analysis did not consider the cost of information provision. In practice, the firm may need to do a benefit/cost analysis to determine whether any *ex ante* information provision activity is worthwhile to pursue.

6. Extensions

In this section, we verify the robustness of the results by exploring two extensions. §6.1 studies whether the insights will hold when returned products incur some cost. §6.2 considers the possibility of service upgrades.

6.1 Partially Recoverable Cost

In our analysis so far, we assume that a returned product or service cancellation would not incur any quality-related cost. This assumption is reasonable as long as no cost is incurred until service delivery. However, it is still valuable to examine whether similar results hold in a more general setting. In this section, we verify the robustness of our results by considering the scenario where only part of the quality-related cost is recoverable for a returned product.

Assume that in the event of a return, a unit of product at quality level q will cost the firm $\lambda q^2/2$, where $\lambda \in [0,1]$. Our analysis so far corresponds to the special case $\lambda = 0$. In other words, λ reflects the portion of the product cost that is *unrecoverable* upon service cancellation. The introduction of λ has little impact on the (IC) and (IR) constraints, but calls for revising objective functions for market outcomes involving customer returns, i.e., **RR**, **RN** and **NR**.

For example, the optimal design under market outcome **RR** is given by

$$\max_{q_G,q_B,p_G,p_B,\beta_G,\beta_B\geq 0} \Pi^{RR} = \rho_G\left(p_G - \bar{\alpha}_G\beta_G p_G - \frac{\alpha_G q_G^2}{2} - \bar{\alpha}_G\lambda\frac{q_G^2}{2}\right) + \bar{\rho}_G\left(p_B - \bar{\alpha}_B\beta_B p_B - \frac{\alpha_B q_B^2}{2} - \bar{\alpha}_B\lambda\frac{q_B^2}{2}\right)$$

$$\text{s.t.} \qquad (IR_G),(IR_B),(IC_G),(IC_B),(RR_G),(RR_B).$$

Following a similar analysis as in §4, we derive the optimal design in the following theorem.

Theorem 2 **(Optimal Product Line Design under Partially Recoverable Product Cost)**. *Under partially recoverable product cost where $0 \leq \lambda \leq 1$,*

*(i) if $\frac{\theta_L}{\theta_H} \leq \min\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}},\alpha_G,1),\phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}},\alpha_0,1)\right\}$, the market outcome is **RR**, and the firm may offer fully refundable products at close quality levels ($q_G = \frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H$, $q_B = \frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H$, $\beta_G = \beta_B = 1$);*

*(ii) if $\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}},\alpha_G,1) \leq \frac{\theta_L}{\theta_H} \leq \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}},\alpha_0,1)$, the market outcome is **NR**, and the firm should offer moderate quality product to good-signal customers ($q_G = \theta_G$) without any refund ($\beta_G = 0$), and high quality product to bad-signal customers ($q_B = \frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H$) with full refund ($\beta_B = 1$);*

(iii) *if* $\phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}},\alpha_0,1) \le \frac{\theta_L}{\theta_H} \le \phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}},\alpha_G,1)$*, the market outcome is* ***RN****, and the firm should offer high quality product to good-signal customers (*$q_G = \frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H$*) with partial refund (*$0 < \beta_G < 1$*) and inferior quality product to bad-signal customers (*$q_B = \theta_0$*) without any refund (*$\beta_B = 0$*);*

(iv) *if* $\frac{\theta_L}{\theta_H} \ge \max\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}},\alpha_G,1), \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}},\alpha_0,1)\right\}$*, the market outcome is* ***NN****, and the firm should offer moderate quality product to good-signal customers (*$q_G = \theta_G$*) and inferior quality product to bad-signal customers (*$q_B = \theta_0$*) without any refund (*$\beta_G = \beta_B = 0$*).*

Theorem 2 shows that the structure of the results does not change when part or all of the product cost is unrecoverable. At $\lambda = 0.2$, Figure 5 demonstrates strong similarity to Figure 2 where $\lambda = 0$. It also shows that as the valuation heterogeneity increases (θ_L/θ_H decreases), the firm relies more upon refund than quality customization to maximize its profit. Specifically, the quality difference between the two products shrinks when the valuation heterogeneity is high. The *variety reduction* effect identified in Corollary 1 continues to hold if we define variety as the span, rather than the number, of quality levels.

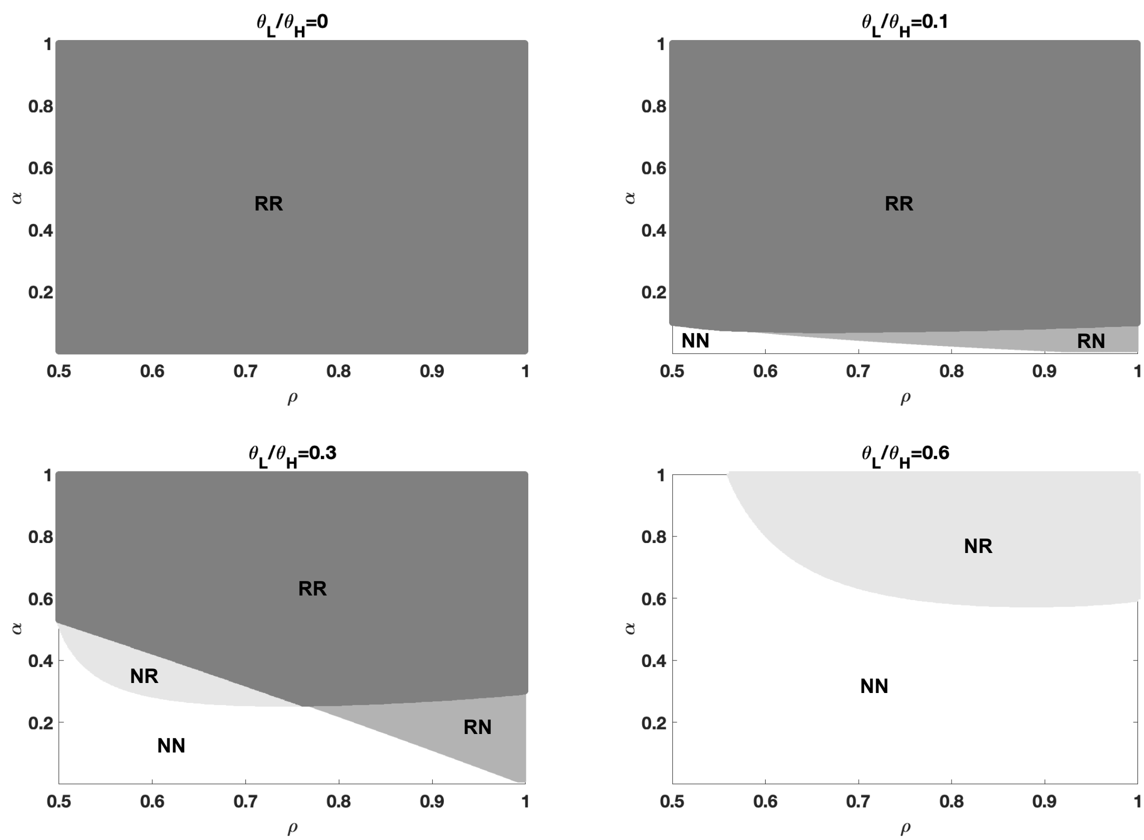

Figure 4 Optimal market outcome with partially recoverable product cost ($\lambda = 0.2$)

The effect of signal strength is also similar to what was identified in Theorem 1. Although doing the same analytical characterization on signal quality ρ is challenging for general λ, Figure 5 confirms that **NR** dominates when the signal is weak (small ρ), and **RN** dominates when the signal is strong (large ρ).

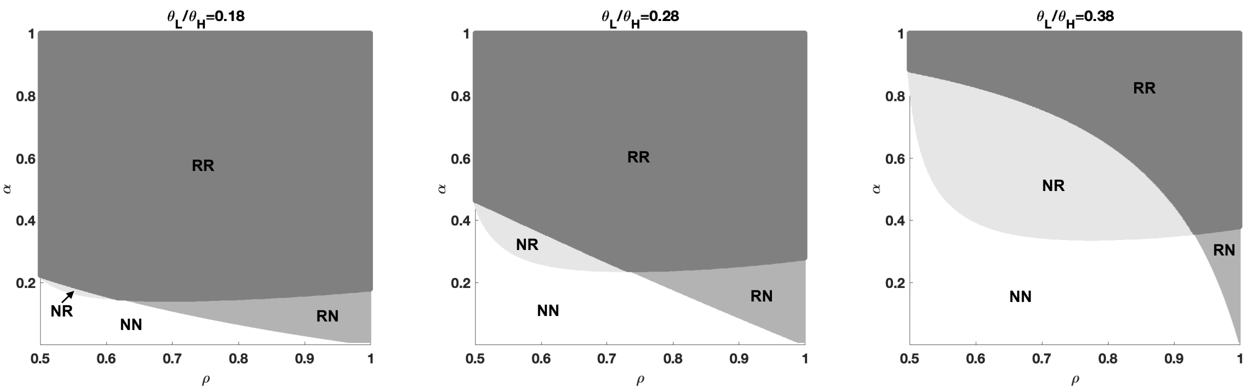

Figure 5 Optimal market outcome with partially recoverable product cost ($\lambda = 0.2$) under moderate *aggregate* uncertainty.

Corollary 4 below provides additional insights on how the cost of refunds may affect the optimal product line design:

Corollary 4. *The firm tends to offer more quality customization and less generous refunds as the unrecoverable portion of the product cost, λ, increases.*

Corollary 4 identifies the non-recoverable cost as a factor that affects the trade off between *ex ante* (quality customization) and *ex post* (customer refunds) customer discrimination. As a result, a firm should carefully examine the nature of its customer base as well as its cost structure, when deciding on his quality and refund terms. That is, in service domains where cancellation yields little quality-related sunk cost, a firm should focus more on refund design and less on quality customization; for industries where returns may cause substantial, irreversible quality-related cost, e.g., tailored service, make-to-order products, the firm should offer more differentiated quality levels and be less generous on refund terms.

6.2 Service Upgrade

Refund policy gives low-type customers the recourse to withdraw from current service after learning about their true valuation. A natural question to ask is whether similar recourse should be extended to high-type customers as well, and to what extent it may impact the refund policy design.

In this subsection, we explore the possibility that customers may upgrade the service after discovering their true valuations. That is, a customer can switch to an alternate product *ex post*, and pay the difference in prices plus certain upgrade fee (f), if the quality of the alternate product is higher. Intuitively, the scope of upgrade should be limited to high-type customers only; otherwise, all customers will upgrade *ex post* and no one would rationally purchase the low quality product to begin with, making the problem trivial to study.

The expected surplus for customers receiving signal s and purchasing product i is

$$u_{si} = \begin{cases} \alpha_s(\theta_H q_i - p_i) + \bar{\alpha}_s \max\{\theta_L q_i - p_i, -\bar{\beta}_i p_i\}, & \text{when } q_i > q_j \\ \alpha_s \max\{\theta_H q_i - p_i - f, \theta_H q_j - p_j\} + \bar{\alpha}_s \max\{\theta_L q_i - p_i, -\bar{\beta}_i p_i\}, & \text{when } q_i \leq q_j \end{cases} \tag{2}$$

where $\{s, i, j\} \in \{G, B\}$ and $i \neq j$. Depending on which product has higher quality, the upgrade decision will apply one way or the other; when $q_G \geq q_B$, high-type customers who have purchased product "B" will *upgrade* to product "G" and those who have purchased product "G" can only *stay* with the same product, and *vice versa.*

Similar to the discussion in §3.2, there are still four market outcomes with regard to product returns. Together with possible upgrades, there are eight scenarios to consider. We use superscript "U" to denote upgrading outcome. For example, $\mathbf{RN}^U$ means that bad-signal customers who purchased a non-refundable product will upgrade.

We summarize our findings as follows:

Theorem 3. *There exists a $\phi^U \geq 0$, such that*

(i) when $\frac{\theta_L}{\theta_H} \leq \phi^U$, $\boldsymbol{RN}^U$ is the optimal design, where $q_G = \theta_H, q_B = 0$, $\beta_G \leq 1, \beta_B = 0$;

(ii) when $\frac{\theta_L}{\theta_H} > \phi^U$, $\boldsymbol{N}^U\boldsymbol{N}$ is the optimal design, where $q_G = \theta_L, \theta_H > q_B > \theta_L$, $\beta_G = \beta_B = 0$.

Furthermore, in both cases, the upgrade fee f can take any value within the range $[0, \bar{\alpha}_G(\theta_H - \theta_L)|q_G - q_B|]$, and the price p_i as well as the refund rate β_i of the higher quality product linearly decrease in f.

Theorem 3 suggests that when valuation heterogeneity is high ($\frac{\theta_L}{\theta_H} \leq \phi^U$), the firm should offer a single, high quality, refundable product to good-signal customers. The upgrade fee f can take any value within a range as long as the price p_G and the refund rate β_G are adjusted accordingly; the higher f is, the lower p_G and β_G are. Specifically, if the firm makes upgrade free ($f = 0$), product G becomes fully refundable ($\beta_G = 1$) and the firm charges the maximum willingness to pay θ_H^2 for product G. Bad-signal customers are always awarded a zero-quality product at zero price. Effectively, this allows bad-signal customers to buy *ex post* should their valuations turn out to be high. [2] This essentially yields the same revenue and market outcome as in **RR** without service upgrade option. Thus the variety reduction effect characterized in Corollary 1 sustains even in the

[2] In fact, all customers are indifferent between the two products — regardless of their signal, one may either purchase product "G" up front, or buy nothing (product "B") and upgrade to "G" later.

presence of service upgrade. This also conforms to airline practice for business-class tickets, where the refund policy and purchase timing are quite flexible and service quality is consistently high.

When valuation heterogeneity is relatively low ($\frac{\theta_L}{\theta_H} > \phi^U$), no refund is allowed; a low-quality product is offered to good-signal customers and a good-quality product is offered to bad-signal customers, and the former will upgrade to the good-quality product if their valuations turn out to be high. [3] The market outcome is the same as in **NN** without service upgrade option. However, the customers are getting a *reversed* signal-based quality assignment, in that good-signal customers will obtain a lower quality product than bad-signal customers due to the availability of service upgrade. This allows the firm to extract all surplus from customers regardless of their signal types, which cannot be obtained without service upgrade (e.g., good-signal customers receive positive surplus under **NN** according to Lemma 2). This is consistent with the practice of some service providers, such as theme parks and concerts, who are more likely to allow service upgrades than refunds.

In general, service upgrade suppresses refund customization, and only market outcomes resembling **RR** and **NN** without service upgrade may appear. Therefore, it is still beneficial for the firm to adopt refunds under some conditions.

7. Concluding Remarks

In this paper, we investigate the optimal product line design with endogenous consumer refund policies. In our model, returns occur due to customer valuation uncertainty, reflected by the heterogeneity in their true valuations revealed *ex post*, and imperfect signal that determines their perceived valuations *ex ante*. We study how the valuation uncertainty may affect the quality and refund options available to customers. In general, higher quality and more liberal refund terms are offered with higher valuation heterogeneity. Specifically, when the valuation heterogeneity is high, the firm can safely rely upon a single-quality design with full refunds, whereas when the valuation heterogeneity is low, a dual-quality design with no refund. When the valuation heterogeneity is moderate, the firm should customize both quality and refunds, and it is the signal quality that determines which customer segment will get the refunds. We also study the effect of either quality or refund standardization, and whether the firm may have incentive to reduce valuation uncertainty via *ex ante* information provision. Our main results are summarized in Table 7. The findings are consistent with many observations in practice, and offer some general guidelines for managers to create the best service offerings for their target market.

[3] In this instance, good-signal customers are indifferent between the two products while bad-signal customers strictly prefer product "B."

Table 7 Implications of the Optimal Product Line Design

Characteristics of Valuation Uncertainty	**Optimal Product Line** (G for good-signal customers and B for bad-signal customers)	**Allow Standard-** Quality	Refund	**Information Provision Strategy**
High valuation heterogeneity	G: high quality, partial/full refund B: high quality, full refund	Yes	Yes	No Action
Moderate valuation heterogeneity **Low** signal quality	G: good quality, no refund B: high quality, full refund	No	No	Weaken Signal
Moderate valuation heterogeneity **High** signal quality	G: high quality, partial refund B: low quality, no refund	No	Yes	Strengthen Signal
Low valuation heterogeneity	G: good quality, no refund B: low quality, no refund	No	Yes	Weaken Signal

There are several meaningful dimensions in which the paper can be extended. First, we focused on quality-related cost and have ignored other related costs. It would be valuable to examine the impact of the fixed salvage value and transaction costs (e.g., Economides 1999) on the optimal product line and refund policies.

Second, given the perishability of service capacity, it is promising to consider the problem under a multi-period capacitated setting and study the extent to which the perishability and capacity availability may affect the optimal product line. Although the capacity issue has not been the central focus for most marketing papers, its relevance has been shown to be critical in some recent literature (Xie and Gerstner 2007, Liu and Xiao 2008, Guo 2009).

Third, there are different ways to specify valuation uncertainty and how this uncertainty resolves over time. For example, valuation uncertainty for an air ticket can be caused by whether the trip will be made or preferences for different departure times, resulting in quite different valuation heterogeneity *ex post*. It would be interesting to investigate the implications of such differences in the future.

Finally, while our results are derived under a monopolistic setting, it would also be interesting to investigate them in a competitive environment. The effect of competition has been studied in several recent papers (e.g., Guo 2009, Shulman et al. 2011). However, to our knowledge, competitive product line design with consumer refunds has not been considered so far. This is another fruitful avenue for future research.

Acknowledgments

This research is partially supported by the Natural Sciences and Engineering Research Council of Canada (RGPIN-2011-402324) and the Concordia University Research Chair program.

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Online Appendix: Service Product Design and Consumer Refund Policies

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October 8, 2019

We use the following lemmas throughout our proofs. The lemmas can be shown after some algebra. We state them without proof.

LEMMA A1. *Let* $0 \le \alpha_1 < \alpha_2 \le 1$ *and* $0 \le \lambda \le 1$. *The following statements hold:*

(i) $\frac{\mathsf{E}_{\alpha_1}[\theta]}{\mathsf{E}_{\alpha_2}[\theta]} \ge (\le)\lambda$ *is equivalent to*

$$\frac{\theta_L}{\theta_H} \ge (\le)\phi(\lambda, \alpha_1, \alpha_2) = \frac{\lambda\alpha_2 - \alpha_1}{\bar{\alpha}_1 - \lambda\bar{\alpha}_2};$$

(ii) $\phi(\lambda, \alpha_1, \alpha_2)$ *decreases with* α_1, *and increases with* α_2 *and* λ.

(iii) $\phi(\sqrt{\alpha_1}, \alpha_1, 1) = \frac{\sqrt{\alpha_1}}{1+\sqrt{\alpha_1}}$ *increases with* α_1.

(iv) $\phi(\sqrt{\frac{\alpha_1}{\alpha_2}}, \alpha_1, \alpha_2) = \frac{\sqrt{\alpha_1}(\sqrt{\alpha_2}-\sqrt{\alpha_1})}{1+\sqrt{\alpha_1/\alpha_2}+\sqrt{\alpha_1}(\sqrt{\alpha_2}-\sqrt{\alpha_1})} \le \phi(\sqrt{\alpha_1}, \alpha_1, 1)$ *for* $0 \le \alpha_1 < \alpha_2 \le 1$.

LEMMA A2. *For* $\rho \ge (\frac{1}{2}, 1]$, *we have*

$$\alpha_B \le \alpha \le \alpha_G, \tag{A1}$$

$$\bar{\alpha}_G \le \bar{\alpha} \le \bar{\alpha}_B. \tag{A2}$$

Appendix A: Proof of Results in the Main Text

Proof of Lemma 1

Since a customer will return the product if and only if the potential refund exceeds the product valuation, the expected utility for customer receiving signal s and purchasing product i is $u_{si} = \alpha_s \max\{\theta_H q_i - p_i, -\bar{\beta}_i p_i\} + \bar{\alpha}_s \max\{\theta_L q_i - p_i, -\bar{\beta}_i p_i\}$. If product i is to be returned by a high-type customer, then it will be returned by both types of customers due to $\theta_L q_i - p_i \le \theta_H q_i - p_i \le -\bar{\beta}_i p_i$. It will yield $u_{si} = -\bar{\beta}_i p_i \le 0$ thus violating the (IR) constraint. Therefore, for both products, only the low-type customers *may* exercise the refund, i.e., $\max\{\theta_H q_i - p_i, -\bar{\beta}_i p_i\} = \theta_H q_i - p_i$ for $i \in \{G, B\}$. ∎

Proof of Lemma 2

(i) First take note that $\theta_G q_G - p_G \geq \theta_G q_B - p_B \geq \theta_B q_B - p_B \geq (\theta_B q_G - p_G)^+$. Since the objective function is strictly increasing in p_G, at optimality, we must have

$$\theta_G q_G^{NN} - p_G^{NN} = \theta_G q_B^{NN} - p_B^{NN}, \tag{A3}$$

which suggests that $p_G^{NN} - p_B^{NN} = \theta_G(q_G^{NN} - q_B^{NN})$. Furthermore, the constraint $\theta_B q_B - p_B \geq \theta_B q_G - p_G$ suggests that $p_G^{NN} - p_B^{NN} \geq \theta_B(q_G^{NN} - q_B^{NN})$. Since $\theta_G \geq \theta_B$, at optimality $p_G^{NN} \geq p_B^{NN}$ and $q_G^{NN} \geq q_B^{NN}$. Thus, we can simultaneously increase p_G and p_B to maintain (A3). It follows that at optimality

$$\theta_B^N q_B^N - p_B^{NN} = 0. \tag{A4}$$

The objective function of the optimization problem can be rewritten as

$$\max_{q_G, q_B \geq 0} \rho_G(\theta_G q_G - \theta_G q_B + \theta_B q_B - q_G^2/2) + \bar{\rho}_G(\theta_B q_B - q_B^2/2).$$

The optimal quality levels are then

$$q_G^{NN} = \theta_G, \;\; q_B^{NN} = \max\{\theta_G - \frac{\theta_G - \theta_B}{\bar{\rho}_G}, 0\} = \theta_0. \tag{A5}$$

The respective optimal prices are given by

$$\begin{aligned} p_B^{NN} &= \theta_B q_B^{NN} = \theta_G\theta_B - \frac{(\theta_G - \theta_B)\theta_B}{\bar{\rho}_G} = \theta_0\theta_B, \\ p_G^{NN} &= p_B^{NN} + \theta_G(q_G^{NN} - q_B^{NN}) = \theta_G\theta_B + \frac{(\theta_G - \theta_B)^2}{\bar{\rho}_G} = \theta_G(\theta_G - \theta_0) + \theta_0\theta_B. \end{aligned}$$

(ii) The market is fully covered by two distinct products if and only if $\theta_0 = \alpha_0\theta_H + \bar{\alpha}_0\theta_L > 0$. The condition is equivalent to $\frac{\theta_L}{\theta_H} > -\frac{\alpha_0}{1-\alpha_0}$. ■

Proof of Theorem 1

We first establish the $\underline{\phi}^*$ and $\bar{\phi}^*$ by characterizing the conditions under which each market outcome is optimal, and then proof the existence of $\hat{\rho} \in [0.5, 1]$.

To begin with, consider the optimal design under each market outcome:

• **RR.** When both the good- and bad-signal customers exercise the refund, the problem for the firm can be formulated as follows:

$$\max_{p_G, p_B, \beta_G, \beta_B} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta_G p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2)$$

$$\text{s.t.} \quad \begin{array}{llll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & u_{GB} & (IC_G) \\ u_{BB} & \geq & u_{BG} & (IC_B) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_G & \leq & 1 & \\ \beta_B & \leq & 1 & \end{array}$$

where $U_{GG} = -p_G + \bar{\alpha}_G \beta_G p_G + \alpha_G \theta_H q_G$, $U_{BB} = -p_B + \bar{\alpha}_B \beta_B p_B + \alpha_B \theta_H q_B$, $U_{GB} = U_{GG} + (1 - \bar{\alpha}_G \beta_G) p_G - (1 - \bar{\alpha}_G \beta_B) p_B - \alpha_G \theta_H (q_G - q_B)$, and $U_{BG} = U_{BB} - (1 - \bar{\alpha}_B \beta_G) p_G + (1 - \bar{\alpha}_B \beta_B) p_B + \alpha_B \theta_H (q_G - q_B)$. The (IC) constraints essentially require that

$$\frac{1 - \bar{\alpha}_G \beta_G}{\alpha_G} p_G - \frac{1 - \bar{\alpha}_G \beta_B}{\alpha_G} p_B \leq \theta_H (q_G - q_B) \leq \frac{1 - \bar{\alpha}_B \beta_G}{\alpha_B} p_G - \frac{1 - \bar{\alpha}_B \beta_B}{\alpha_B} p_B.$$

By (A1), the above implies that $p_G \leq p_B$. In addition, as high-type customers are always better off than low-type customers under the same product, i.e., $U_{GB} \geq U_{BB}$ and $U_{GG} \geq U_{BG}$, there should be $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$.

Without affecting the optimal solution, the objective function can be re-written as $\min_{p_G, p_B, \beta} \rho_G U_{GG} + \bar{\rho}_G U_{BB}$. Hence the optimal solution should be binding at (IR_B) and (IC_G). The problem can be simplified to:

$$\max_{p_G, p_B, \beta_G, \beta_B} \Pi^{RR} = \rho_G (p_G - \bar{\alpha}_G \beta_G p_G - \alpha_G q_G^2/2) + \bar{\rho}_G (p_B - \bar{\alpha}_B \beta_B p_B - \alpha_B q_B^2/2)$$

$$\text{s.t.} \quad \begin{array}{llll} p_B - \bar{\alpha}_B \beta_B p_B & = & \alpha_B \theta_H q_B & (IR_B) \\ p_G - \bar{\alpha}_G \beta_G p_G & = & \alpha_G \theta_H q_G - \alpha_G \theta_H q_B + p_B - \bar{\alpha}_G \beta_B p_B & (IC_G) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_G & \leq & 1 & \\ \beta_B & \leq & 1 & \end{array}$$

In maximizing the objective function, we need to maximize the RHS of (IC_G), which equals to $\alpha_G \theta_H q_G - \alpha_G \theta_H q_B + \alpha_B \theta_H q_B + (\alpha_G - \alpha_B) \beta_B p_B$. By (IR_B), there should be $\beta_B^* = 1$ and $p_B^* = \theta_H q_B$. Overall,

$$p_G^* \in [\alpha_G, 1] \theta_H q_G, \ \beta_G^* = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G}{\bar{\alpha}_G} \frac{\theta_H q_G}{p_G}, \ p_B^* = \theta_H q_B, \ \beta_B^* = 1 \quad \text{(A6)}$$

The expected profit is: $\Pi^* = \rho_G (\alpha_G \theta_H q_G - \alpha_G q_G^2/2) + \bar{\rho}_G (\alpha_B \theta_H q_B - \alpha_B q_B^2/2)$. Therefore, the optimal quality is

$$q_G^{RR} = \theta_H, \quad q_B^{RR} = \theta_H, \quad p_G^{RR} \in [\alpha_G, 1] \theta_H^2, \quad p_B^{RR} = \theta_H^2, \quad \beta_G^{RR} = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G \theta_H}{\bar{\alpha}_G} \frac{q_G^{RR}}{p_G^{RR}}, \quad \beta_B^{RR} = 1,$$

$$\Pi^{RR} = \rho_G \frac{\alpha_G \theta_H^2}{2} + \bar{\rho}_G \frac{\alpha_B \theta_H^2}{2}.$$

- **NR.** When only the bad-signal customer will exercise the refund, we can let $\beta_G = 0$ without loss of generality and the problem can be formulated by:

$$\max_{p_G, p_B, \beta_B} \Pi^{NR} = \rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B \beta_B p_B - \alpha_B q_B^2/2)$$
$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & u_{GB} & (IC_G) \\ u_{BB} & \geq & u_{BG} & (IC_B) \\ \beta_B p_B & \geq & \theta_L q_B & (NR_B) \\ \beta_B & \leq & 1 & \end{array}$$

where $U_{GG} = -p_G + \theta_G q_G$, $U_{BB} = -p_B + \bar{\alpha}_B \beta_B p_B + \alpha_B \theta_H q_B$, $U_{GB} = -p_B + \bar{\alpha}_G \beta_B p_B + \alpha_G \theta_H q_B$, and $U_{BG} = -p_G + \theta_B q_G$. Followed by a similar analysis as in RR, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$. Thus, both (IR_B) and (IC_G) will be binding. The problem can be simplified to

$$\max_{p_G, p_B, \beta_B} \Pi^{NR} = \rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B \beta_B p_B - \alpha_B q_B^2/2)$$
$$\text{s.t.} \quad \begin{array}{lcll} p_B - \bar{\alpha}_B \beta_B p_B & = & \alpha_B \theta_H q_B & (IR_B) \\ p_G & = & \theta_G q_G + p_B - \alpha_G \theta_H q_B - \bar{\alpha}_G \beta_B p_B & (IC_G) \\ \beta_B p_B & \geq & \theta_L q_B & (NR_B) \\ \beta_B & \leq & 1 & \end{array}$$

The optimum can be obtained as follows:

$$p_G^* = \theta_G q_G, \;\; \beta_G^* = 0, \;\; p_B^* = \theta_H q_B, \;\; \beta_B^* = 1. \tag{A7}$$

and the expected profit is $\Pi^* = \rho_G(\theta_G q_G - q_G^2/2) + \bar{\rho}_G(\alpha_B \theta_H q_B - \alpha_B q_B^2/2)$. Therefore, the optimal quality is

$$q_G^{NR} = \theta_G, \quad q_B^{NR} = \theta_H, \quad p_G^{NR} = \theta_G^2, \quad p_B^{NR} = \theta_H^2, \quad \beta_G^{NR} = 0, \quad \beta_B^{NR} = 1,$$
$$\Pi^{NR} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\alpha_B \theta_H^2}{2}.$$

- **RN.** When the refund only goes to good-signal customers, we can let $\beta_B = 0$ without loss of generality and the following problem needs to be solved:

$$\max_{p_G, p_B, \beta_G} \Pi^{RN} = \rho_G(p_G - \bar{\alpha}_G \beta_G p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - q_B^2/2)$$
$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & u_{GB} & (IC_G) \\ u_{BB} & \geq & u_{BG} & (IC_B) \\ \beta_G p_G & \geq & \theta_L q_G & (RN_G) \\ \beta_G & \leq & 1 & \end{array}$$

where $U_{GG} = -p_G + \alpha_G\theta_H q_G + \bar{\alpha}_G\beta_G p_G$, $U_{BB} = -p_B + \theta_B q_B$, $U_{GB} = -p_B + \theta_G q_B$, and $U_{BG} = -p_G + \alpha_B\theta_H q_G + \bar{\alpha}_B\beta_G p_G$. Followed by a similar argument as other scenarios, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$, both (IR_B) and (IC_G) are binding:

$$\begin{array}{llll} \max\limits_{p_G,p_B,\beta_G} \Pi^{RN} & = & \rho_G(p_G - \bar{\alpha}_G\beta_G p_G - \alpha_G q_G^2/2 + \bar{\rho}_G(p_B - q_B^2/2) & \\ \text{s.t.} \quad p_B & = & \theta_B q_B & (IR_B) \\ \quad p_G & = & \alpha_G\theta_H q_G + \bar{\alpha}_G\beta_G p_G + p_B - \theta_G q_B & (IC_G) \\ \quad \beta_G p_G & \geq & \theta_L q_G & (RN_G) \\ \quad \beta_G & \leq & 1 & \end{array}$$

It can be verified that the optimal menu would be of the following form:

$$\begin{array}{l} p_G^* \in [\theta_G q_G - (\theta_G - \theta_B)q_B, \theta_B q_B + \theta_H(q_G - q_B)],\ \beta_G^* = \dfrac{1}{\bar{\alpha}_G} - \dfrac{\alpha_G\theta_H q_G - (\theta_G - \theta_B)q_B}{\bar{\alpha}_G p_G^*} \\ p_B^* = \theta_B q_B,\ \beta_B^* = 0. \end{array}$$

The optimal expected profit is then $\Pi^* = \rho_G[\alpha_G\theta_H q_G - (\theta_G - \theta_B)q_B - \alpha_G q_G^2/2] + \bar{\rho}_G(\theta_B q_B - q_B^2/2)$. Thus,

$$\begin{array}{l} q_G^{RN} = \theta_H, \quad q_B^{RN} = \theta_0, \quad p_G^{RN} \in \theta_0\theta_B + [\theta_G, \theta_H](\theta_H - \theta_0), \quad p_B^{RN} = \theta_0\theta_B, \\ \beta_G^{RN} = \dfrac{1}{\bar{\alpha}_G} - \dfrac{\alpha_G\theta_H^2 - (\theta_G - \theta_B)\theta_0}{\bar{\alpha}_G p_G^{RN}}, \quad \beta_B^{RN} = 0, \\ \Pi^{RN} = \rho_G \dfrac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \dfrac{\theta_0^2}{2}. \end{array}$$

where $\theta_0 = \max\{\alpha_0\theta_H + (1-\alpha_0)\theta_L, 0\}$ and $\alpha_0 = \frac{\alpha_B - \rho\alpha}{\bar{\rho}_G} \leq \alpha_B \leq \alpha_G$.

- **NN.** When no customer will exercise the refund, analysis can be retrieved to Lemma 2 and the optimal profit is

$$\Pi^{NN} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}.$$

Comparing the optimal profits across four kinds of products as follows, yields the optimal product design:

$$\begin{aligned} \Pi^{RR} &= \rho_G \frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \frac{\alpha_B\theta_H^2}{2} \\ \Pi^{NR} &= \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\alpha_B\theta_H^2}{2} \\ \Pi^{RN} &= \rho_G \frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2} \\ \Pi^{NN} &= \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2} \end{aligned}$$

By Lemma A1 and noting that $\phi(\sqrt{\alpha_B}, \alpha_0, 1) = \frac{\sqrt{\alpha_B} - \alpha_0}{1-\alpha_0} \geq \frac{-\alpha_0}{1-\alpha_0}$, it can be summarized that:

NN is optimal product if $\frac{\theta_L}{\theta_H} \geq \max\left\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\right\}$.

NR is optimal if $\phi(\sqrt{\alpha_G},\alpha_G,1)\le\frac{\theta_L}{\theta_H}\le\phi(\sqrt{\alpha_B},\alpha_0,1)$.

RN is optimal if $\phi(\sqrt{\alpha_B},\alpha_0,1)\le\frac{\theta_L}{\theta_H}\le\phi(\sqrt{\alpha_G},\alpha_G,1)$.

RR is optimal if $\frac{\theta_L}{\theta_H}\le\min\left\{\phi(\sqrt{\alpha_G},\alpha_G,1),\phi(\sqrt{\alpha_B},\alpha_0,1)\right\}$.

Therefore, $\underline{\phi}^*=\min\left\{\phi(\sqrt{\alpha_G},\alpha_G,1),\phi(\sqrt{\alpha_B},\alpha_0,1)\right\}$ and $\bar{\phi}^*=\max\left\{\phi(\sqrt{\alpha_G},\alpha_G,1),\phi(\sqrt{\alpha_B},\alpha_0,1)\right\}$.

We next show that there exists a $\hat{\rho}\in[0.5,1]$ such that $\phi(\sqrt{\alpha_B},\alpha_0,1)\le\phi(\sqrt{\alpha_G},\alpha_G,1)$ when $\rho\le\hat{\rho}$ and $\phi(\sqrt{\alpha_B},\alpha_0,1)\ge\phi(\sqrt{\alpha_G},\alpha_G,1)$ when $\rho>\hat{\rho}$. By definition, $\phi(\sqrt{\alpha_G},\alpha_G,1)-\phi(\sqrt{\alpha_B},\alpha_0,1)=\frac{\sqrt{\alpha_G}-\alpha_G}{1-\alpha_G}-\frac{\sqrt{\alpha_B}-\alpha_0}{1-\alpha_0}=\frac{1-\sqrt{\alpha_B}}{1-\alpha_0}-\frac{1-\sqrt{\alpha_G}}{1-\alpha_G}$. Therefore, $\phi(\sqrt{\alpha_B},\alpha_0,1)\le\phi(\sqrt{\alpha_G},\alpha_G,1)$ is equivalent to $\frac{1-\sqrt{\alpha_B}}{1-\alpha_0}\ge\frac{1-\sqrt{\alpha_G}}{1-\alpha_G}=\frac{1}{1+\sqrt{\alpha_G}}$, and after some algebra, the condition is reduced to

$$S(\rho,\alpha)=-\sqrt{\alpha_B\alpha_G}-\sqrt{\alpha_B}+\sqrt{\alpha_G}+\alpha_0\ge 0. \tag{A8}$$

- When $\alpha\in[0,0.5]$, there are $\bar{\alpha}\ge\alpha$ and $\bar{\rho}_G\ge\rho_G$. At $\rho=0.5$, we have $\alpha_B=\alpha_G=\alpha_0=\alpha$ and $S(0.5,\alpha)=0$. We next prove that S is convex in ρ, i.e.,

$$\frac{\partial^2 S}{\partial\rho^2}=-\frac{\partial^2\sqrt{\alpha_B\alpha_G}}{\partial\rho^2}-\frac{\partial^2\sqrt{\alpha_B}}{\partial\rho^2}+\frac{\partial^2\sqrt{\alpha_G}}{\partial\rho^2}+\frac{\partial^2\alpha_0}{\partial\rho^2}\ge 0 \tag{A9}$$

by showing that (a) $\frac{\partial^2\alpha_0}{\partial\rho^2}\ge 0$; (b) $\frac{\partial^2\sqrt{\alpha_G}}{\partial\rho^2}-\frac{\partial^2\sqrt{\alpha_B}}{\partial\rho^2}\ge 0$; and (c) $\frac{\partial^2\sqrt{\alpha_G\alpha_B}}{\partial\rho^2}\le 0$.

As a preliminary, it can be verified that

$$\begin{aligned}&\frac{\partial\alpha_G}{\partial\rho}=\frac{\alpha\bar{\alpha}}{\rho_G^2},\ \frac{\partial\alpha_B}{\partial\rho}=-\frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2},\ \frac{\partial\alpha_0}{\partial\rho}=\frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2}-\frac{2\alpha\bar{\alpha}}{\bar{\rho}_G^3}\\&\frac{\partial^2\alpha_G}{\partial\rho^2}=\frac{2\alpha\bar{\alpha}(\bar{\alpha}-\alpha)}{\rho_G^3},\ \frac{\partial^2\alpha_B}{\partial\rho^2}=\frac{2\alpha\bar{\alpha}(\bar{\alpha}-\alpha)}{\bar{\rho}_G^3},\ \frac{\partial^2\alpha_0}{\partial\rho^2}=-\frac{2\alpha\bar{\alpha}(\bar{\alpha}-\alpha)}{\bar{\rho}_G^3}+\frac{6\alpha\bar{\alpha}(\bar{\alpha}-\alpha)}{\bar{\rho}_G^4}\end{aligned} \tag{A10}$$

(a) apparently holds by the derivatives above.

To prove (b), it can be obtained that $\frac{\partial^2\sqrt{\alpha_G}}{\partial\rho^2}-\frac{\partial^2\sqrt{\alpha_B}}{\partial\rho^2}=\frac{1}{\rho^{3/2}\alpha^{1/2}\rho_G^{5/2}}[2(\bar{\alpha}-\alpha)\rho-\bar{\alpha}]-\frac{1}{\bar{\rho}^{3/2}\alpha^{1/2}\bar{\rho}_G^{5/2}}[2(\bar{\alpha}-\alpha)\bar{\rho}-\bar{\alpha}]$. Note that $2(\bar{\alpha}-\alpha)\bar{\rho}-\bar{\alpha}\le 0$. When $2(\bar{\alpha}-\alpha)\rho-\bar{\alpha}\ge 0$, apparently there is $\frac{\partial^2\sqrt{\alpha_G}}{\partial\rho^2}-\frac{\partial^2\sqrt{\alpha_B}}{\partial\rho^2}\ge 0$. Otherwise, suppose $2(\bar{\alpha}-\alpha)\rho-\bar{\alpha}\le 0$. Consider a function $T(y)=\frac{1}{y^{3/2}(y\alpha+(1-y)\bar{\alpha})^{5/2}}[2(\bar{\alpha}-\alpha)y-\bar{\alpha}]$. After some algebra, we have $\frac{\partial T(y)}{\partial y}=\frac{3\bar{\alpha}-6(\bar{\alpha}-\alpha)y}{y^{5/2}(y\alpha+(1-y)\bar{\alpha})^{5/2}}\ge 0$ for $y\in[0,\frac{\bar{\alpha}}{2(\bar{\alpha}-\alpha)}]$. Therefore, $T(y)$ increases on the interval satisfying $2(\bar{\alpha}-\alpha)y-\bar{\alpha}\le 0$, which includes $[\bar{\rho},\rho]$. Hence there is also $\frac{\partial^2\sqrt{\alpha_G}}{\partial\rho^2}-\frac{\partial^2\sqrt{\alpha_B}}{\partial\rho^2}=\alpha^{-1/2}[T(\rho)-T(\bar{\rho})]\ge 0$.

For (c), after some algebra, the second derivative of $\sqrt{\alpha_G\alpha_B}$ can be reduced to $\frac{\partial^2\sqrt{\alpha_G\alpha_B}}{\partial\rho^2}=\frac{\alpha^2\bar{\alpha}^2}{4(\alpha_B\alpha_G)^{3/2}}[-(\frac{\alpha_G}{\bar{\rho}_G^2}+\frac{\alpha_B}{\rho_G^2})^2+4\alpha_B\alpha_G\frac{\bar{\alpha}-\alpha}{\alpha\bar{\alpha}}(\frac{\alpha_G}{\bar{\rho}_G^3}+\frac{\alpha_B}{\rho_G^3})]$. Note that $\frac{\bar{\alpha}-\alpha}{\alpha\bar{\alpha}}(\frac{\alpha_G}{\bar{\rho}_G^3}+\frac{\alpha_B}{\rho_G^3})\rho_G^2\bar{\rho}_G^2=\frac{\bar{\alpha}-\alpha}{\alpha\bar{\alpha}}(\frac{\alpha_G\rho_G^2}{\bar{\rho}_G}+\frac{\alpha_B\bar{\rho}_G^2}{\rho_G})=\frac{\bar{\alpha}-\alpha}{\bar{\alpha}}(\frac{\rho\rho_G}{\bar{\rho}_G}+\frac{\bar{\rho}\bar{\rho}_G}{\rho_G})$. Since $\rho_G\le\bar{\rho}_G$ and $\frac{\rho}{\bar{\rho}_G}\ge\frac{\bar{\rho}}{\rho_G}$, there is $\frac{\rho\rho_G}{\bar{\rho}_G}+\frac{\bar{\rho}\bar{\rho}_G}{\rho_G}\le\frac{\rho\bar{\rho}_G}{\bar{\rho}_G}+\frac{\bar{\rho}\rho_G}{\rho_G}=1$. Thus $\frac{\bar{\alpha}-\alpha}{\alpha\bar{\alpha}}(\frac{\alpha_G}{\bar{\rho}_G^3}+\frac{\alpha_B}{\rho_G^3})\rho_G^2\bar{\rho}_G^2=\frac{\bar{\alpha}-\alpha}{\bar{\alpha}}(\frac{\rho\rho_G}{\bar{\rho}_G}+\frac{\bar{\rho}\bar{\rho}_G}{\rho_G})\le\frac{\bar{\alpha}-\alpha}{\bar{\alpha}}\le 1$. Therefore, $\frac{\partial^2\sqrt{\alpha_G\alpha_B}}{\partial\rho^2}\le\frac{\alpha^2\bar{\alpha}^2}{4(\alpha_B\alpha_G)^{3/2}}[-(\frac{\alpha_G}{\bar{\rho}_G^2}+\frac{\alpha_B}{\rho_G^2})^2+4\frac{\alpha_B\alpha_G}{\rho_G^2\bar{\rho}_G^2}]\le 0$.

Therefore, when $\alpha\in[0,0.5]$, $S(\rho,\alpha)$ is convex for ρ on $[0.5,1]$ with one root being $\rho=0.5$. Denote the other root as ρ_0. Then $\hat{\rho}=\min\{\max\{0.5,\rho_0\},1\}$.

- When $\alpha \in [0.5, 1]$, there are $\bar{\alpha} \leq \alpha$ and $\bar{\rho}_G \leq \rho_G$. We can prove that $S \leq 0$ for any $\rho \in [0.5, 1]$ thus $\hat{\rho} = 1$. After some algebra, $S(\rho, \alpha) \geq 0$ becomes equivalent to

$$\sqrt{\alpha_G} - \sqrt{\alpha_B} \geq \frac{\alpha_B - \alpha_0}{1 - \sqrt{\alpha_B}}. \tag{A11}$$

By noting that $\alpha_B - \alpha_0 = \frac{\alpha - \alpha_B}{\bar{\rho}_G}$ and $\alpha_G - \alpha_B = \frac{\alpha - \alpha_B}{\rho_G}$, the above condition becomes $\bar{\rho}_G \geq \sqrt{\alpha_B} + \rho_G \sqrt{\alpha_G}$. However, since $\bar{\rho}_G = \bar{\rho}\alpha + \rho\bar{\alpha} \leq \frac{\bar{\rho}\alpha}{\bar{\rho}_G} + \rho\alpha = \alpha_B + \rho_G \alpha_G \leq \sqrt{\alpha_B} + \rho_G\sqrt{\alpha_G}$, (A11) does not hold. Therefore $S \leq 0$ whenever $\alpha \geq 0.5$ and $\hat{\rho} = 1$.

■

Proof of Corollary 1

By Lemma 2 (ii) and Theorem 1 (i), the variety reduction effect is evident when $\phi_0 \leq \underline{\phi}^*$. It is then sufficient to show that $\phi_0 = \frac{-\alpha_0}{1-\alpha_0} < \underline{\phi}^* = \min\left\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\right\} = \min\{\frac{\sqrt{\alpha_G} - \alpha_G}{1-\alpha_G}, \frac{\sqrt{\alpha_B} - \alpha_0}{1-\alpha_0}\}$. Since $\alpha_0 < \alpha_B < \alpha_G \leq 1$, there is $\frac{-\alpha_0}{1-\alpha_0} < \frac{\sqrt{\alpha_B} - \alpha_0}{1-\alpha_0}$. Further, $\frac{\partial \alpha_0}{\partial \rho} = \frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2} - \frac{2\alpha\bar{\alpha}}{\bar{\rho}_G^3} < 0$ and $\alpha_0|_{\rho=0.5} = \alpha > 0 > \alpha_0|_{\rho=1} = -\alpha$. Hence $\sqrt{\alpha_G} > -\alpha_0$; subsequently, $\frac{-\alpha_0}{1-\alpha_0} < \frac{\sqrt{\alpha_G}}{1+\sqrt{\alpha_G}} = \frac{\sqrt{\alpha_G} - \alpha_G}{1-\alpha_G}$. Therefore, $\phi_0 = \frac{-\alpha_0}{1-\alpha_0} < \underline{\phi}^*$ and there is variety reduction when $\frac{\theta_L}{\theta_H} \in [\phi_0, \underline{\phi}^*]$. ■

Proof of Proposition 1

Recall (A8), we first prove the following lemma which will be applied in the proof of the proposition.

LEMMA A3. *$\frac{\partial S(\rho,\alpha)}{\partial \rho}|_{\rho=0.5} \geq 0$ when $0 < \alpha \leq \frac{1}{9}$, and $\frac{\partial S(\rho,\alpha)}{\partial \rho}|_{\rho=0.5} \leq 0$ when $\frac{1}{9} \leq \alpha \leq \frac{1}{2}$;*

Proof. (i) $\frac{\partial S(\rho,\alpha)}{\partial \rho} = -\sqrt{\frac{\alpha_B}{\alpha_G}} \frac{\partial \alpha_G / \partial \rho}{2} - \sqrt{\frac{\alpha_G}{\alpha_B}} \frac{\partial \alpha_B / \partial \rho}{2} - \frac{\partial \alpha_B / \partial \rho}{2\sqrt{\alpha_B}} + \frac{\partial \alpha_G / \partial \rho}{2\sqrt{\alpha_G}} + \partial \alpha_0 / \partial \rho$. By (A10), it becomes $\frac{\partial S(\rho,\alpha)}{\partial \rho} = -\sqrt{\frac{\alpha_B}{\alpha_G}} \frac{\alpha\bar{\alpha}/\rho_G^2}{2} + \sqrt{\frac{\alpha_G}{\alpha_B}} \frac{\alpha\bar{\alpha}/\bar{\rho}_G^2}{2} + \frac{\alpha\bar{\alpha}/\bar{\rho}_G^2}{2\sqrt{\alpha_B}} + \frac{\alpha\bar{\alpha}/\rho_G^2}{2\sqrt{\alpha_G}} + \frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2} - \frac{2\alpha\bar{\alpha}}{\bar{\rho}_G^3}$. At $\rho \to 0.5$, there is $\alpha_G = \alpha_B = \alpha_0 = \alpha$ and $\rho_G = \rho_B = 0.5$. Replacing respective terms, there is:

$$\frac{\partial S(\rho,\alpha)}{\partial \rho}|_{\rho \to 0.5} = -\frac{\alpha\bar{\alpha}/\rho_G^2}{2} + \frac{\alpha\bar{\alpha}/\bar{\rho}_G^2}{2} + \frac{\alpha\bar{\alpha}/\bar{\rho}_G^2}{2\sqrt{\alpha}} + \frac{\alpha\bar{\alpha}/\rho_G^2}{2\sqrt{\alpha}} + \frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2} - \frac{2\alpha\bar{\alpha}}{\bar{\rho}_G^3} = 4\alpha\bar{\alpha}(\frac{1}{\sqrt{\alpha}} - 3).$$

Apparently, $\frac{\partial S(\rho,\alpha)}{\partial \rho}|_{\rho \to 0.5}$ is positive when $\alpha \leq \frac{1}{9}$ and negative otherwise. □

(i) When $0 < \alpha \leq \frac{1}{9}$, by Lemma A3 there $\frac{\partial S(\rho,\alpha)}{\partial \rho}|_{\rho \to 0.5} \geq 0$. Further, according to the latter half of Proof of Theorem 1, $S(0.5, \alpha) = 0$ and S convex in ρ. Therefore, $\frac{\partial S(\rho,\alpha)}{\partial \rho} \geq 0$ for any $\rho \in [0.5, 1]$ hence $\hat{\rho}(\alpha) = 0.5$.

(ii) When $\frac{1}{9} < \alpha < \frac{1}{2}$, by Lemma A3 there $\frac{\partial S(\rho,\alpha)}{\partial \rho}|_{\rho \to 0.5} < 0$. Due to $S(0.5, \alpha) = 0$ and S convex in ρ, for any fixed α there exists a $\rho_0 > 0.5$ such that $S(\rho_0, \alpha) = 0$. Therefore, $\hat{\rho}(\alpha) = \min\{\rho_0, 1\}$.

(iii) Follows immediately from the last bullet in the Proof of Theorem 1. ■

Proof of Proposition 4

To begin with, we identify the optimal expected profit of the firm under each market outcome as follows:

$$\Pi^{RR} = \frac{\alpha\theta_H^2}{2}$$
$$\Pi^{NR} = \frac{\rho_G\theta_G^2}{2} + \frac{\bar{\rho}_G\alpha_B\theta_H^2}{2}$$
$$\Pi^{RN} = \frac{\rho_G\alpha_G\theta_H^2}{2} + \frac{\bar{\rho}_G\theta_0^2}{2}$$
$$\Pi^{NN} = \frac{\rho_G\theta_G^2}{2} + \frac{\bar{\rho}_G\theta_0^2}{2}.$$

It can be observed that under market outcome **RR**, where all low-type customers will obtain the refund, the signal quality does not affect the expected profit. In all other market outcomes, the signal quality affects the firm's profit. We first prove the following lemma which is critical for the proof of the proposition:

LEMMA A4.
(i) $\frac{\partial\Pi^{RR}}{\partial\rho} = 0$
(ii) $\frac{\partial\Pi^{NR}}{\partial\rho} \leq 0$*;*
(iii) $\frac{\partial\Pi^{RN}}{\partial\rho} \geq 0$ *when* $\rho \to 1$ *and* $\frac{\theta_L}{\theta_H} \leq \alpha$ *or* $\frac{\theta_L}{\theta_H} \geq \frac{2\alpha+\alpha^2}{1+2\alpha^2}$*.*
(iv) $\frac{\partial\Pi^{NN}}{\partial\rho} > 0$ *when* $\rho \to 1$ *and* $\frac{4\alpha^2-2\alpha+1}{2\alpha^2+1} > \frac{\theta_L}{\theta_H}$*; and* $\frac{\partial\Pi^{NN}}{\partial\rho} \leq 0$ *when* $\rho \to 1$ *and* $\frac{4\alpha^2-2\alpha+1}{2\alpha^2+1} \leq \frac{\theta_L}{\theta_H}$*;* $\frac{\partial\Pi^{NN}}{\partial\rho} < 0$ *when* $\rho \to 0.5$*.*

Proof. We first summarize some common terms that will be used throughout the proof: $\frac{\partial\rho_G}{\partial\rho} = 2\alpha - 1$, $\frac{\partial\alpha_G}{\partial\rho} = \frac{\alpha\bar{\alpha}}{\rho_G^2}$, $\frac{\partial\alpha_B}{\partial\rho} = -\frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2}$, $\frac{\partial\alpha_0}{\partial\rho} = \frac{\alpha\bar{\alpha}}{\bar{\rho}_G^2} - \frac{2\alpha\bar{\alpha}}{\bar{\rho}_G^3} = -\frac{\alpha\bar{\alpha}(1+\rho_G)}{\bar{\rho}_G^3}$. Also, when $\rho \to 0.5$, there are $\rho_G \to 0.5$, $\bar{\rho}_G \to 0.5$, $\alpha_G \to \alpha$, $\alpha_B \to \alpha$ and $\alpha_0 \to \alpha$. When $\rho \to 1$, there are $\rho_G \to \alpha$, $\bar{\rho}_G \to \bar{\alpha}$, $\alpha_G \to 1$, $\alpha_B \to 0$ and $\alpha_0 \to -\alpha/\bar{\alpha}$.

(i) The first order derivative with Π^{RR} is apparent.

(ii) For Π^{NR}, it can be verified that

$$\frac{\partial\Pi^{NR}}{\partial\rho} \sim (2\alpha-1)(\theta_G^2 - \alpha_B\theta_H^2) + \frac{\alpha\bar{\alpha}}{\rho_G}\theta_G(\theta_H - \theta_L) - \frac{\alpha\bar{\alpha}}{\bar{\rho}_G}\theta_H^2.$$

Further, $\lim_{\rho\to 0.5}\frac{\partial\Pi^{NR}}{\partial\rho} \sim -\alpha\bar{\alpha}\theta_H^2 - \bar{\alpha}^2\theta_L^2 \leq 0$, $\lim_{\rho\to 0.5}\frac{\partial\Pi^{NR}}{\partial\rho} \sim -\bar{\alpha}\theta_H\theta_L \leq 0$, and $\frac{\partial^2\Pi^{NR}}{\partial\rho^2} \sim \frac{\alpha^2\bar{\alpha}^2(\theta_H-\theta_L)^2}{\rho_G^3} \geq 0$. Therefore, $\frac{\partial\Pi^{NR}}{\partial\rho} \leq 0$ regardless of the signal quality.

(iii) With the established terms at the beginning of the proof, there is $\frac{\partial\Pi^{RN}}{\partial\rho} = [(2\alpha-1)\alpha_G + \frac{\alpha\bar{\alpha}}{\rho_G}]\theta_H^2 + (1-2\alpha)\theta_0^2 - 2\frac{\alpha\bar{\alpha}(1+\rho_G)}{\bar{\rho}_G^2}\theta_0(\theta_H - \theta_L)$. When $\rho \to 1$, there are $\rho_G \to \alpha$, $\alpha_G \to 1$, and $\alpha_0 \to -\alpha/\bar{\alpha}$. After some algebra, it can be verified that $\lim_{\rho\to 1}\frac{\partial\Pi^{RN}}{\partial\rho} \sim (\alpha+\alpha^2+\alpha^3)\theta_H^2 - (4\alpha+2\alpha^3)\theta_H\theta_L + (1+$

$2\alpha^2)\theta_L^2 = (\alpha\theta_H - \theta_L)[(1+\alpha+\alpha^2)\theta_H - (1+2\alpha^2)\theta_L] + (1-\alpha)^2\theta_H\theta_L$ which is positive if $\theta_L/\theta_H \le \alpha$ or $\theta_L/\theta_H \ge \frac{2\alpha+\alpha^2}{1+2\alpha^2}$.

(iv) With the established terms at the beginning of the proof, there is

$$\begin{aligned}\frac{\partial \Pi^{NN}}{\partial \rho} &\sim \frac{\alpha-\bar{\alpha}}{2}(\theta_G^2 - \theta_0^2) + (\theta_H - \theta_L)\alpha\bar{\alpha}\left(\frac{\theta_G}{\rho_G} - \frac{\theta_0(1+\rho_G)}{\bar{\rho}_G^2}\right)\\ &= (\theta_H - \theta_L)\left[\theta_G\left(\frac{(\alpha-\bar{\alpha})(\alpha_G-\alpha_0)}{2} + \frac{\alpha\bar{\alpha}}{\rho_G}\right) + \theta_0\left(\frac{(\alpha-\bar{\alpha})(\alpha_G-\alpha_0)}{2} - \frac{\alpha\bar{\alpha}(1+\rho_G)}{\bar{\rho}_G^2}\right)\right]\end{aligned}$$

When $\rho \to 1$, it can be verified that

$$\begin{aligned}\lim_{\rho\to 1}\frac{\partial \Pi^{NN}}{\partial \rho} &= (\theta_H - \theta_L)\left[\theta_H\left(\frac{\alpha-\bar{\alpha}}{2\bar{\alpha}} + \bar{\alpha}\right) + \left(-\frac{\alpha}{\bar{\alpha}}\theta_H + \frac{1}{\bar{\alpha}}\theta_L\right)\left(\frac{\alpha-\bar{\alpha}}{2\bar{\alpha}} - \frac{\alpha(1+\alpha)}{\bar{\alpha}}\right)\right]\\ &= (\theta_H - \theta_L)\left[\theta_H\frac{1-2\alpha+2\alpha^2}{2\bar{\alpha}} + \frac{(\alpha\theta_H-\theta_L)(1+2\alpha^2)}{2\bar{\alpha}^2}\right]\\ &= (\theta_H - \theta_L)\frac{(4\alpha^2-2\alpha+1)\theta_H - (2\alpha^2+1)\theta_L}{2\bar{\alpha}^2}\end{aligned}$$

which is greater than 0 if any only if $\frac{4\alpha^2-2\alpha+1}{2\alpha^2+1} > \theta_L/\theta_H$. Thus for small θ_L/θ_H, Π^{NN} increases in ρ when it is close to 1; and for large θ_L/θ_H, Π^{NN} decreases in ρ when it is close to 1.

When $\rho \to 0.5$, there is

$$\lim_{\rho\to 0.5}\frac{\partial \Pi^{NN}}{\partial \rho} = (\theta_H - \theta_L)\left[2\theta_G\alpha\bar{\alpha} - 3\theta_0\alpha\bar{\alpha}\right] = -(\theta_H - \theta_L)(\alpha\theta_H + \bar{\alpha}\theta_L)\alpha\bar{\alpha} < 0$$

Therefore, Π^{NN} decreases in ρ when it is close to 0.5. □

(i) When the *aggregate* uncertainty level is high, the market outcome is primarily **RR**. According to Lemma A4 (i), firm's profit is insensitive to signal quality thus has little incentive to improve the signal.

(ii)When the *aggregate* uncertainty level is low, the market outcome is dominated by **NN**. According to Lemma A4 (iv), the firm benefits from weaker signals thus has the incentive to dilute it.

(iii) When the *aggregate* uncertainty level is low and signal quality low, according to Theorem 1 the market outcome is likely to be **NR**. By Lemma A4 (ii) the firm benefits from low quality signal thus may have incentive to dilute it. On the other hand, at moderate *aggregate* uncertainty level when signal quality is high, the market outcome is likely to be **RN**. By Lemma A4 (iii) the firm has the incentive to improve the signal if it is close to 1. ■

Proof of Theorem 2

To begin with, the same results hold for **NN** that

$$q_G^{NN}=\theta_G, p_G^{NN}=\theta_G(\theta_G-\theta_0)+\theta_0\theta_B, q_B^{NN}=\theta_0, p_B^{NN}=\theta_0\theta_B$$
$$\Pi^{NN}=\rho_G\frac{\theta_G^2}{2}+\bar{\rho}_G\frac{\theta_0^2}{2}.$$

For other market outcomes, the analysis of the optimal pricing at given quality levels is the same as in the proof of Theorem 1, summarized by:

For **RR**—

$$p_G^{RR}\in[\alpha_G\theta_H,\theta_H]q_G,\ \beta_G^{RR}=\frac{1}{\bar{\alpha}_G}-\frac{\alpha_G}{\bar{\alpha}_G}\frac{\theta_H q_G}{p_G},\ p_B^{RR}=\theta_H q_B,\ \beta_B^{RR}=1$$
$$\Pi^{RR}=\rho_G(\alpha_G\theta_H q_G-\alpha_G q_G^2/2-\bar{\alpha}_G\lambda q_G^2/2)+\bar{\rho}_G(\alpha_B\theta_H q_B-\alpha_B q_B^2/2-\bar{\alpha}_B\lambda q_B^2/2).$$

Therefore, the optimal design is

$$q_G^{RR}=\frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H,\quad q_B^{RR}=\frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H,$$
$$p_G^{RR}\in[\alpha_G,1]\theta_H q_G^{RR},\quad p_B^{RR}=\frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H^2,$$
$$\beta_G^{RR}=\frac{1}{\bar{\alpha}_G}-\frac{\alpha_G\theta_H}{\bar{\alpha}_G}\frac{q_G^{RR}}{p_G^{RR}},\beta_B^{RR}=1,$$
$$\Pi^{RR}=\rho_G\frac{\alpha_G^2\theta_H^2}{2(\alpha_G+\lambda\bar{\alpha}_G)}+\bar{\rho}_G\frac{\alpha_B^2\theta_H^2}{2(\alpha_B+\lambda\bar{\alpha}_B)}$$

For **NR**—

$$p_G^{NR}=\theta_G q_G,\ \beta_G^{NR}=0,\ p_B^{NR}=\theta_H q_B,\ \beta_B^{NR}=1.$$
$$\Pi^{NR}=\rho_G(\theta_G q_G-q_G^2/2)+\bar{\rho}_G(\alpha_B\theta_H q_B-\alpha_B q_B^2/2-\bar{\alpha}_B\lambda q_B^2/2)$$

Therefore, the optimal design is

$$q_G^{NR}=\theta_G,\quad q_B^{NR}=\frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H,$$
$$p_G^{NR}=\theta_G^2,\quad p_B^{NR}=\frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H^2,$$
$$\beta_G^{NR}=0,\quad \beta_B^{NR}=1,$$
$$\Pi^{NR}=\rho_G\frac{\theta_G^2}{2}+\bar{\rho}_G\frac{\alpha_B^2\theta_H^2}{2(\alpha_B+\lambda\bar{\alpha}_B)}$$

For **RN**—

$$p_G^{RN}\in\theta_B q_B+[\theta_G,\theta_H](q_G-q_B),\ \beta_G^{RN}=\frac{1}{\bar{\alpha}_G}-\frac{\alpha_G\theta_H q_G-(\theta_G-\theta_B)q_B}{\bar{\alpha}_G p_G^{RN}},p_B^{RN}=\theta_B q_B,\ \beta_B^{RN}=0.$$

$$\Pi^{RN} = \rho_G[\alpha_G\theta_H q_G - (\theta_G - \theta_B)q_B - \alpha_G q_G^2/2 - \bar{\alpha}_G\lambda q_G^2/2] + \bar{\rho}_G(\theta_B q_B - q_B^2/2)$$

Therefore, the optimal design is

$$\begin{aligned}
q_G^{RN} &= \frac{\alpha_G}{\alpha_G + \lambda\bar{\alpha}_G}\theta_H, \quad q_B^{RN} = \theta_0, \\
p_G^{RN} &\in \theta_0\theta_B + [\theta_G, \theta_H](\frac{\alpha_G}{\alpha_G + \lambda\bar{\alpha}_G}\theta_H - \theta_0), \quad p_B^{RN} = \theta_0\theta_B, \\
\beta_G^{RN} &= \frac{1}{\bar{\alpha}_G} - \frac{\frac{\alpha_G^2}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H^2 - (\theta_G - \theta_B)\theta_0}{\bar{\alpha}_G p_G^{RN}}, \quad \beta_B^{RN} = 0, \\
\Pi^{RN} &= \rho_G\frac{\alpha_G^2\theta_H^2}{2(\alpha_G + \lambda\bar{\alpha}_G)} + \bar{\rho}_G\frac{\theta_0^2}{2}
\end{aligned}$$

We can then compare the optimal profits under four kinds of products:

$$\begin{aligned}
\Pi^{RR} &= \rho_G\frac{\alpha_G^2\theta_H^2}{2(\alpha_G + \lambda\bar{\alpha}_G)} + \bar{\rho}_G\frac{\alpha_B^2\theta_H^2}{2(\alpha_B + \lambda\bar{\alpha}_B)} \\
\Pi^{NR} &= \rho_G\frac{\theta_G^2}{2} + \bar{\rho}_G\frac{\alpha_B^2\theta_H^2}{2(\alpha_B + \lambda\bar{\alpha}_B)} \\
\Pi^{RN} &= \rho_G\frac{\alpha_G^2\theta_H^2}{2(\alpha_G + \lambda\bar{\alpha}_G)} + \bar{\rho}_G\frac{\theta_0^2}{2} \\
\Pi^{NN} &= \rho_G\frac{\theta_G^2}{2} + \bar{\rho}_G\frac{\theta_0^2}{2}
\end{aligned}$$

By Lemma A1, **NN** is optimal product if $\frac{\theta_L}{\theta_H} \geq \max\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1), \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)\right\}$.
NR is optimal if $\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1) \leq \frac{\theta_L}{\theta_H} \leq \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)$.
RN is optimal if $\phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1) \leq \frac{\theta_L}{\theta_H} \leq \phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1)$.
RR is optimal if $\frac{\theta_L}{\theta_H} \leq \min\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1), \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)\right\}$.

■

Proof of Corollary 4

Both $\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}$ and $\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}$ decreases in λ. According to Lemma A1 (ii), both $\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1)$ and $\phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)$ decreases in λ. Thus the two thresholds between no refund and some refund, $\min\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1), \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)\right\}$, and between some refund and full refund, $\max\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1), \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)\right\}$, both decreases in λ. Therefore as λ increases, the optimal market outcome tends to offer less generous refund but more quality differentiation.

■

Table A1 Optimal Product Line under Partially Recoverable Quality Cost

Aggregate Uncertainty *Individual* Uncertainty	$High: \theta_L/\theta_H \leq \underline{\phi}^{**}$	Moderate: $\underline{\phi}^{**} < \theta_L/\theta_H \leq \bar{\phi}^{**}$ $High: \rho \leq \hat{\rho}$	 Low: $\rho > \hat{\rho}$	$Low: \theta_L/\theta_H > \bar{\phi}^{**}$
Quality (q_G^*, q_B^*)	$(\frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H, \frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H)$	$(\theta_G, \frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H)$	$(\frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H, \theta_0)$	(θ_G, θ_0)
Price (p_G^*, p_B^*)	$(\frac{\alpha_G^2}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H^2 \sim \frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H^2, \frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H^2)$	$(\theta_G^2, \frac{\alpha_B}{\alpha_B+\lambda\bar{\alpha}_B}\theta_H^2)$	$(\theta_G(\frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H - \theta_0) + \theta_0\theta_B \sim \theta_H(\frac{\alpha_G}{\alpha_G+\lambda\bar{\alpha}_G}\theta_H - \theta_0) + \theta_0\theta_B, \theta_0\theta_B)$	$(\theta_G(\theta_G - \theta_0) + \theta_0\theta_B, \theta_0\theta_B)$
Refund Rate (β_G^*, β_B^*)	$(\frac{1}{\bar{\alpha}_G} - \frac{\alpha_G\theta_H}{\bar{\alpha}_G}\frac{q_G^*}{p_G^*}, 1)$	$(0,1)$	$(\frac{1}{\bar{\alpha}_G} - \frac{\frac{\alpha_G^2\theta_H^2}{\alpha_G+\lambda\bar{\alpha}_G} - (\theta_G-\theta_B)\theta_0}{\bar{\alpha}_G p_G^*}, 0)$	$(0,0)$
Market Outcome	**RR**	**NR**	**RN**	**NN**

where $\underline{\phi}^{**} = \min\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1), \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)\right\}$ and $\bar{\phi}^{**} = \max\left\{\phi(\frac{\alpha_G}{\sqrt{\alpha_G+\lambda\bar{\alpha}_G}}, \alpha_G, 1), \phi(\frac{\alpha_B}{\sqrt{\alpha_B+\lambda\bar{\alpha}_B}}, \alpha_0, 1)\right\}$.

Proof of Theorem 3

First consider when only product B is upgradable, which entails $q_G > q_B$ and $\theta_H q_G - p_G - f > \theta_H q_B - p_B$. In this scenario, optimal design under each refund outcome can be analyzed as follows:

- $\mathbf{RR}^U$**.** When all low type customers will claim the refund *ex post*, there are $U_{GG} = \alpha_G(\theta_H q_G - p_G) - \bar{\alpha}_G\bar{\beta}_G p_G$, $U_{BB} = \alpha_B(\theta_H q_G - p_G - f) - \bar{\alpha}_B\bar{\beta}_B p_B$, $U_{GB} = \alpha_G(\theta_H q_G - p_G - f) - \bar{\alpha}_G\bar{\beta}_B p_B$, and $U_{BG} = \alpha_B(\theta_H q_G - p_G) - \bar{\alpha}_B\bar{\beta}_G p_G$. The problem is essentially:

$$\max_{p_G, p_B, f, \beta_G, \beta_B} \Pi^{RR} = \rho_G(\alpha_G p_G + \bar{\alpha}_G\bar{\beta}_G\rho_G) + \bar{\rho}_G(\alpha_B p_G + \alpha_B f + \bar{\alpha}_B\bar{\beta}_B p_B) - \alpha q_G^2/2.$$

$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_G & \leq & 1 & \\ \beta_B & \leq & 1 & \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G - p_G - f & \geq & \theta_H q_B - p_B & (SU) \end{array}$$

The IC constraints entail that $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$. It can be verified that to maximize Π^{RR} there should be $U_{BB} = 0$ and $U_{GG} = U_{GB}$. The problem then becomes

$$\max_{p_G, p_B, f, \beta_G, \beta_B} \Pi^{RR} = \rho_G(\alpha_G p_G + \alpha_G f + \bar{\alpha}_G\bar{\beta}_B p_B) + \bar{\rho}\alpha\theta_H q_G - \alpha q_G^2/2.$$

$$\text{s.t.}\quad \begin{array}{lcll} \alpha_B(p_G+f)+\bar{\alpha}_B\bar{\beta}_B p_B & = & \alpha_B\theta_H q_G & (IR_B) \\ \alpha_G f+\bar{\alpha}_G\bar{\beta}_B p_B & = & \bar{\alpha}_G\bar{\beta}_G p_G & (IC_G) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_G & \leq & 1 & \\ \beta_B & \leq & 1 & \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G - p_G - f & \geq & \theta_H q_B - p_B & (SU) \end{array}$$

The maximum is achieved at $\beta_B=1, q_B\in[0,\theta_H), p_B\in(\theta_H q_B,\theta_H^2)$; $q_G=\theta_H, p_G=\theta_H^2-f, \beta_G=1-\frac{\alpha_G f}{\bar{\alpha}_G p_G}$. Subsequently,

$$\Pi^{RR^U}=\alpha\frac{\theta_H^2}{2}.$$

- **RN^U.** When only customers purchasing product "G" will claim the refund, there are $U_{GG}=\alpha_G(\theta_H q_G-p_G)-\bar{\alpha}_G\bar{\beta}_G p_G$, $U_{BB}=\alpha_B(\theta_H q_G-p_G-f)+\bar{\alpha}_B(\theta_L q_B-p_B)$, $U_{GB}=\alpha_G(\theta_H q_G-p_G-f)+\bar{\alpha}_G(\theta_L q_B-p_B)$, and $U_{BG}=\alpha_B(\theta_H q_G-p_G)-\bar{\alpha}_B\bar{\beta}_G p_G$. The problem is to

$$\max_{p_G,p_B,f,\beta_G,\beta_B}\Pi^{RN}=\rho_G(\alpha_G p_G+\bar{\alpha}_G\bar{\beta}_G p_G)+\bar{\rho}_G(\alpha_B p_G+\alpha_B f+\bar{\alpha}_B p_B)-\alpha q_G^2/2-\rho\bar{\alpha}q_B^2/2.$$

$$\text{s.t.}\quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_G & \leq & 1 & \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G-p_G-f & \geq & \theta_H q_B-p_B & (SU) \end{array}$$

Similarly, (IR_B) and (IC_G) should be binding and there is $U_{GG}=U_{GB}\geq U_{BB}=0\geq U_{BG}$. The problem then becomes

$$\max_{p_G,p_B,f,\beta_G,\beta_B}\Pi^{RN}=\rho_G(\alpha_G p_G+\bar{\alpha}_G\bar{\beta}_G p_G)+\bar{\rho}\alpha\theta_H q_G+\rho\bar{\alpha}\theta_L q_B-\alpha q_G^2/2-\rho\bar{\alpha}q_B^2/2.$$

$$\text{s.t.}\quad \begin{array}{lcll} \alpha_B(p_G+f)+\bar{\alpha}_B p_B & = & \alpha_B\theta_H q_G+\bar{\alpha}_B\theta_L q_B & (IR_B) \\ \alpha_G f+\bar{\alpha}_G p_B & = & \bar{\alpha}_G(\bar{\beta}_G p_G+\theta_L q_B) & (IC_G) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_G & \leq & 1 & \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G-p_G-f & \geq & \theta_H q_B-p_B & (SU) \end{array}$$

The maximum is achieved when the second (SU) is binding. Together with (IR_B), we have $p_G+f=\theta_H q_G-\bar{\alpha}_B(\theta_H-\theta_L)q_B$, $p_B=\theta_B q_B$, $\beta_G=\frac{\theta_H q_G-(\theta_H-\theta_L)q_B-f/\bar{\alpha}_G}{\theta_H q_G-\bar{\alpha}_B(\theta_H-\theta_L)q_B}$ and $f\leq\bar{\alpha}_G(\theta_H-\theta_L)(q_G-q_B)$. Subsequently, $q_G=\theta_H, q_B=\theta_1=\max\{-\frac{\rho\alpha-\rho_G\alpha_B}{\rho\bar{\alpha}}\theta_H+\frac{\rho-\rho_G\alpha_B}{\rho\bar{\alpha}}\theta_L,0\}$ and

$$\Pi^{RN^U}=\alpha\frac{\theta_H^2}{2}+\rho\bar{\alpha}\frac{\theta_1^2}{2}.$$

- **NR^U.** When only customers purchasing product "B" will claim the refund, there are $U_{GG} = \alpha_G(\theta_H q_G - p_G) + \bar{\alpha}_G(\theta_L q_G - p_G)$, $U_{BB} = \alpha_B(\theta_H q_G - p_G - f) - \bar{\alpha}_B \bar{\beta}_B p_B$, $U_{GB} = \alpha_G(\theta_H q_G - p_G - f) - \bar{\alpha}_G \bar{\beta}_B p_B$, and $U_{BG} = \alpha_B(\theta_H q_G - p_G) + \bar{\alpha}_B(\theta_L q_G - p_G)$. The problem is then

$$\max_{p_G,p_B,f,\beta_G,\beta_B} \Pi^{NR} = \rho_G p_G + \bar{\rho}_G(\alpha_B p_G + \alpha_B f + \bar{\alpha}_B \bar{\beta}_B p_B) - (\alpha + \bar{\alpha}\bar{\rho}) q_G^2/2.$$

$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_B & \leq & 1 & \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G - p_G - f & \geq & \theta_H q_B - p_B & (SU) \end{array}$$

Similarly, (IR_B) and (IC_G) should be binding and there is $U_{GG} = U_{GB} \geq U_{BB} = 0 \geq U_{BG}$. The problem becomes

$$\max_{p_G,p_B,f,\beta_G,\beta_B} \Pi^{NR} = \rho_G p_G + \bar{\rho}\alpha\theta_H q_G - (\alpha + \bar{\alpha}\bar{\rho}) q_G^2/2.$$

$$\text{s.t.} \quad \begin{array}{lcll} \alpha_B p_G + \alpha_B f + \bar{\alpha}_B \bar{\beta}_B p_B & = & \alpha_B \theta_H q_G & (IR_B) \\ p_G & = & \theta_L q_G + \bar{\beta}_B p_B + \alpha_G f/\bar{\alpha}_G & (IC_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_B & \leq & 1 & \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G - p_G - f & \geq & \theta_H q_B - p_B & (SU) \end{array}$$

The maximum is achieved when $p_G = \frac{(\alpha_G - \alpha_B)\theta_H + \bar{\alpha}_G \theta_B}{\bar{\alpha}_B} q_G$, $f = \frac{(\theta_H - \theta_B)\bar{\alpha}_G}{\bar{\alpha}_B} q_G$, $\beta_B = 1$, $p_B > \theta_H q_B$. The profit function then becomes $\Pi^{NR} = (\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L) q_G - (\alpha + \bar{\alpha}\bar{\rho}) q_G^2/2$. Subsequently, $q_G = \theta_2 = \frac{\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L}{\alpha + \bar{\alpha}\bar{\rho}}$, $q_B \in [0, q_G)$ and the profit is

$$\Pi^{NR^U} = (\alpha + \bar{\alpha}\bar{\rho}) \frac{\theta_2^2}{2}.$$

- **NN^U.** When no customer will claim the refund, it is suffice to consider $\beta_G = \beta_B = 0$.

$$\max_{p_G,p_B,f,\beta_G,\beta_B} \Pi^{NN} = \rho_G p_G + \bar{\rho}_G(\alpha_B p_G + \alpha_B f + \bar{\alpha}_B p_B) - (\alpha + \bar{\rho}\bar{\alpha}) q_G^2/2 - \rho\bar{\alpha} q_B^2/2.$$

$$\text{s.t.} \quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G - p_G - f & \geq & \theta_H q_B - p_B & (SU) \end{array}$$

There are $U_{GG} = \alpha_G(\theta_H q_G - p_G) + \bar{\alpha}_G(\theta_L q_G - p_G)$, $U_{BB} = \alpha_B(\theta_H q_G - p_G - f) + \bar{\alpha}_B(\theta_L q_B - p_B)$, $U_{GB} = \alpha_G(\theta_H q_G - p_G - f) + \bar{\alpha}_G(\theta_L q_B - p_B)$, $U_{BG} = \alpha_B(\theta_H q_G - p_G) + \bar{\alpha}_B(\theta_L q_G - p_G)$. Similarly, (IR_B) and (IC_G) should be binding. The problem then becomes

$$\max_{p_G,p_B,f} \Pi^{NN} = \rho_G p_G + \bar{\rho}\alpha\theta_H q_G - (\alpha + \bar{\rho}\bar{\alpha}) q_G^2/2 - \rho\bar{\alpha} q_B^2/2.$$

$$\text{s.t.}\quad \begin{array}{lcll} \alpha_B p_G + \alpha_B f + \bar{\alpha}_B p_B & = & \alpha_B \theta_H q_G & (IR_B) \\ \bar{\alpha}_G (p_G - p_B) - \alpha_G f & = & \bar{\alpha}_G \theta_L (q_G - q_B) & (IC_G) \\ q_G & \geq & q_B & (SU) \\ \theta_H q_G - p_G - f & \geq & \theta_H q_B - p_B & (SU) \end{array}$$

The maximum is achieved at $p_B = \alpha_B \theta_H q_B$, $p_G = \theta_G q_G - [(\alpha_G - \alpha_B)\theta_H + \bar{\alpha}_G \theta_L] q_B$ and $f = \bar{\alpha}_G(\theta_H - \theta_L)(q_G - q_B)$. The objective function then becomes $\Pi^{NN} = (\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L) q_G - \frac{\alpha + \bar{\rho}\bar{\alpha}}{2} q_G^2 - \rho_G[(\alpha_G - \alpha_B)\theta_H + \bar{\alpha}_G \theta_L] q_B - \frac{\rho\bar{\alpha}}{2} q_B^2$. Then, the optimal quality levels are $q_G = \theta_2 = \frac{\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L}{\alpha + \bar{\alpha}\bar{\rho}}, q_B = 0$ and the profit is

$$\Pi^{NN^U} = (\alpha + \bar{\alpha}\bar{\rho}) \frac{\theta_2^2}{2}.$$

Then consider when product G is upgradable, which entails $q_B > q_G$ and $\theta_H q_B - p_B - f > \theta_H q_G - p_G$.

- $\mathbf{R^U R.}$ When all low-type customers return and all high-type customers ultimately go for product "B":

$$\max_{p_G, p_B, f, \beta_G, \beta_B} \Pi^{RR} = \rho_G(\alpha_G p_B + \alpha_G f + \bar{\alpha}_G \bar{\beta}_G \rho_G) + \bar{\rho}_G(\alpha_B p_B + \bar{\alpha}_B \bar{\beta}_B p_B) - \alpha q_B^2/2.$$

$$\text{s.t.}\quad \begin{array}{lcll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_G & \leq & 1 & \\ \beta_B & \leq & 1 & \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

where $U_{GG} = \alpha_G(\theta_H q_B - p_B - f) - \bar{\alpha}_G \bar{\beta}_G p_G$, $U_{BB} = \alpha_B(\theta_H q_B - p_B) - \bar{\alpha}_B \bar{\beta}_B p_B$, $U_{GB} = \alpha_G(\theta_H q_B - p_B) - \bar{\alpha}_G \bar{\beta}_B p_B$, and $U_{BG} = \alpha_B(\theta_H q_B - p_B - f) - \bar{\alpha}_B \bar{\beta}_G p_G$. Similarly, $U_{GG} = U_{GB} \geq U_{BB} = 0 \geq U_{BG}$. The problem then becomes

$$\max_{p_G, p_B, f, \beta_G, \beta_B} \Pi^{RR} = \rho_G(\alpha_G p_B + \alpha_G f + \bar{\alpha}_G \bar{\beta}_G p_G) + \bar{\rho}\alpha\theta_H q_B - \alpha q_B^2/2.$$

$$\text{s.t.}\quad \begin{array}{lcll} \alpha_B p_B + \bar{\alpha}_B \bar{\beta}_B p_B & = & \alpha_B \theta_H q_B & (IR_B) \\ -\alpha_G f + \bar{\alpha}_G \bar{\beta}_B p_B & = & \bar{\alpha}_G \bar{\beta}_G p_G & (IC_G) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_G & \leq & 1 & \\ \beta_B & \leq & 1 & \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

The maximum is achieved at $\beta_B = 1, q_B = \theta_H, p_B = \theta_H^2, f = 0$; $q_G = p_G = 0$, whereas

$$\Pi^{R^U R} = \alpha \frac{\theta_H^2}{2}.$$

- $\mathbf{R^UN}$. When only customers purchasing product "G" will claim the refund, there are $U_{GG} = \alpha_G(\theta_H q_B - p_B - f) - \bar{\alpha}_G\bar{\beta}_G p_G$, $U_{BB} = \alpha_B(\theta_H q_B - p_B) + \bar{\alpha}_B(\theta_L q_B - p_B)$, $U_{GB} = \alpha_G(\theta_H q_B - p_B) + \bar{\alpha}_G(\theta_L q_B - p_B)$, and $U_{BG} = \alpha_B(\theta_H q_B - p_B - f) - \bar{\alpha}_B\bar{\beta}_G p_G$.

$$\max_{p_G,p_B,f,\beta_G,\beta_B} \Pi^{RN} = \rho_G(\alpha_G p_B + \alpha_G f + \bar{\alpha}_G\bar{\beta}_G p_G) + \bar{\rho}_G p_B - (\alpha+\rho\bar{\alpha})q_B^2/2.$$

$$\text{s.t.} \quad \begin{array}{llll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_G & \leq & 1 & \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

Similarly, (IR_B) and (IC_G) should be binding and there is $U_{GG} = U_{GB} \geq U_{BB} = 0 \geq U_{BG}$. The problem then becomes

$$\max_{p_G,p_B,f,\beta_G,\beta_B} \Pi^{RN} = \rho_G(\alpha_G p_B + \alpha_G f + \bar{\alpha}_G\bar{\beta}_G p_G) + \bar{\rho}_G \theta_B q_B - (\alpha+\rho\bar{\alpha})q_B^2/2$$

$$\text{s.t.} \quad \begin{array}{llll} p_B & = & \theta_B q_B & (IR_B) \\ \alpha_G f + \bar{\alpha}_G\bar{\beta}_G p_G & = & \bar{\alpha}_G(\theta_B - \theta_L)q_B & (IC_G) \\ \beta_G p_G & \geq & \theta_L q_G & (RR_G) \\ \beta_G & \leq & 1 & \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

The maximum is achieved at $q_B = \theta_3 = \frac{\alpha_B}{\alpha+\rho\bar{\alpha}}\theta_H + (1 - \frac{\alpha_B}{\alpha+\rho\bar{\alpha}})\theta_L, p_B = \theta_B\theta_4$, $q_G = p_G = 0, f = \bar{\alpha}_G(\theta_B - \theta_L)\theta_4/\alpha_G$ and

$$\Pi^{R^U N} = (\alpha+\rho\bar{\alpha})\frac{\theta_3^2}{2}.$$

- $\mathbf{N^UR}$. When only customers purchasing product "B" will claim the refund, there are $U_{GG} = \alpha_G(\theta_H q_B - p_B - f) + \bar{\alpha}_G(\theta_L q_G - p_G)$, $U_{BB} = \alpha_B(\theta_H q_B - p_B) - \bar{\alpha}_B\bar{\beta}_B p_B$, $U_{GB} = \alpha_G(\theta_H q_B - p_B) - \bar{\alpha}_G\bar{\beta}_B p_B$, and $U_{BG} = \alpha_B(\theta_H q_B - p_B - f) + \bar{\alpha}_B(\theta_L q_G - p_G)$.

$$\max_{p_G,p_B,f,\beta_G,\beta_B} \Pi^{NR} = \rho_G(\alpha_G p_B + \alpha_G f + \bar{\alpha}_G p_G) + \bar{\rho}_G(\alpha_B p_B + \bar{\alpha}_B\bar{\beta}_B p_B) - \bar{\alpha}\bar{\rho}q_G^2/2 - \alpha q_B^2/2.$$

$$\text{s.t.} \quad \begin{array}{llll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_B & \leq & 1 & \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

Similarly, (IR_B) and (IC_G) should be binding and there is $U_{GG} = U_{GB} \geq U_{BB} = 0 \geq U_{BG}$. The problem becomes

$$\max_{p_G,p_B,f,\beta_G,\beta_B} \Pi^{NR} = \rho_G(\alpha_G p_B + \alpha_G f + \bar{\alpha}_G p_G) + \bar{\rho}\alpha\theta_H q_B - \bar{\alpha}\bar{\rho}q_G^2/2 - \alpha q_B^2/2.$$

$$\text{s.t.} \quad \begin{array}{llll} \alpha_B p_B + \bar{\alpha}_B \bar{\beta}_B p_B & = & \alpha_B \theta_H q_B & (IR_B) \\ \bar{\alpha}_G p_G + \alpha_G f & = & \bar{\alpha}_G \theta_L q_G + \bar{\alpha}_G \bar{\beta}_B p_B & (IC_G) \\ \beta_B p_B & \geq & \theta_L q_B & (RR_B) \\ \beta_B & \leq & 1 & \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

The maximum is achieved when $q_G = \theta_4 = \max\{-\alpha_{\theta_4}\theta_H + (1+\alpha_{\theta_4})\theta_L, 0\}$, where $\alpha_{\theta_4} = \frac{\alpha_G - \alpha_B}{\alpha_B}\frac{\rho\alpha}{\bar{\rho}\bar{\alpha}}$. $q_B = \theta_H, p_G = \theta_G\theta_4, p_B = \theta_H^2 - \frac{\bar{\alpha}_B}{\alpha_B}(\theta_G - \theta_L)\theta_4, f = 0, \beta_B = \frac{\theta_H^2 - \frac{\bar{\alpha}_B}{\alpha_B}(\theta_G - \theta_L)\theta_4}{\theta_H^2 - \frac{1}{\alpha_B}(\theta_G - \theta_L)\theta_4}$ and the profit is

$$\Pi^{N^U R} = \alpha\frac{\theta_H^2}{2} + \bar{\rho}\bar{\alpha}\frac{\theta_4^2}{2}.$$

- $\mathbf{N^U N}$**.** When no customer will claim the refund, it is suffice to consider $\beta_G = \beta_B = 0$.

$$\max_{p_G, p_B, f, \beta_G, \beta_B} \Pi^{NN} = \rho_G(\alpha_G p_B + \alpha_G f + \bar{\alpha}_G p_G) + \bar{\rho}_G p_B - \bar{\rho}\bar{\alpha} q_G^2/2 - (\alpha + \rho\bar{\alpha}) q_B^2/2.$$

$$\text{s.t.} \quad \begin{array}{llll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & U_{GB} & (IC_G) \\ u_{BB} & \geq & U_{BG} & (IC_B) \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

There are $U_{GG} = \alpha_G(\theta_H q_B - p_B - f) + \bar{\alpha}_G(\theta_L q_G - p_G)$, $U_{BB} = \alpha_B(\theta_H q_B - p_B) + \bar{\alpha}_B(\theta_L q_B - p_B)$, $U_{GB} = \alpha_G(\theta_H q_B - p_B) + \bar{\alpha}_G(\theta_L q_B - p_B)$, $U_{BG} = \alpha_B(\theta_H q_B - p_B - f) + \bar{\alpha}_B(\theta_L q_G - p_G)$. Similarly, (IR_B) and (IC_G) should be binding. The problem then becomes

$$\max_{p_G, p_B, f} \Pi^{NN} = \rho_G[\bar{\alpha}_G\theta_L(q_G - q_B) + \theta_B q_B] + \bar{\rho}_G\theta_B q_B - \bar{\rho}\bar{\alpha} q_G^2/2 - (\alpha + \rho\bar{\alpha}) q_B^2/2.$$

$$\text{s.t.} \quad \begin{array}{llll} p_B & = & \theta_B q_B & (IR_B) \\ \bar{\alpha}_G(p_G - p_B) + \alpha_G f & = & \bar{\alpha}_G\theta_L(q_G - q_B) & (IC_G) \\ q_B & \geq & q_G & (US) \\ \theta_H q_B - p_B - f & \geq & \theta_H q_G - p_G & (US) \end{array}$$

The maximum is achieved at $q_G = \theta_L, q_B = \theta_3 = \frac{\alpha_B}{\alpha + \rho\bar{\alpha}}\theta_H + (1 - \frac{\alpha_B}{\alpha + \rho\bar{\alpha}})\theta_L, p_B = \theta_B\theta_3$, $p_G = \theta_B\theta_3 - (\theta_3 - \theta_L)\theta_L - \frac{\alpha_G}{\bar{\alpha}_G} f$ and $f \leq \bar{\alpha}_G(\theta_H - \theta_L)(\theta_3 - \theta_L)$. The profit is

$$\Pi^{N^U N} = (\alpha + \rho\bar{\alpha})\frac{\theta_3^2}{2} + \bar{\rho}\bar{\alpha}\frac{\theta_L^2}{2}.$$

Comparing the optimal profits under each design, there are $\Pi^{N^U R} = \alpha\frac{\theta_H^2}{2} + \bar{\rho}\bar{\alpha}\frac{\theta_L^2}{2} \geq = (\alpha + \bar{\rho}\bar{\alpha})\frac{\theta_2}{2} = \Pi^{NR^U} = \Pi^{NN^U}$, $\Pi^{RN^U} = \alpha\frac{\theta_H^2}{2} + \rho\bar{\alpha}\frac{\theta_1^2}{2} \geq \Pi^{RR^U} = \Pi^{R^U R} = \alpha\frac{\theta_H^2}{2}$, $\Pi^{N^U N} = (\alpha + \rho\bar{\alpha})\frac{\theta_3^2}{2} + \bar{\rho}\bar{\alpha}\frac{\theta_L^2}{2} \geq (\alpha + \rho\bar{\alpha})\frac{\theta_3^2}{2} = \Pi^{R^U N}$. By $\rho \geq 1/2$ it can also be verified that $\alpha_{\theta_4} > \alpha_{\theta_1}$ thus $\Pi^{RN^U} > \Pi^{N^U R}$. Therefore, the optimal design can only be among $\mathbf{RN^U}$ or $\mathbf{N^U N}$.

When $\frac{\theta_L}{\theta_H} \leq \frac{\alpha_{\theta_1}}{1 + \alpha_{\theta_1}}$, where $\alpha_{\theta_1} = \frac{\rho_G(\alpha_G - \alpha_B)}{\rho\bar{\alpha}}$, there is $\theta_1 = 0$. And $\Pi^{RN^U} = \alpha\frac{\theta_H^2}{2}$; $\Pi^{N^U N}$ is in the form of $\frac{1}{2}[\epsilon\theta_H + (1-\epsilon)\theta_L]^2$ where $\epsilon > 0$. By Lemma A1, the optimal design is $\mathbf{N^U N}$ if $\frac{\alpha - \epsilon}{1 - \epsilon} \leq \frac{\theta_L}{\theta_H} \leq \frac{\alpha_{\theta_1}}{1 + \alpha_{\theta_1}}$, or $\mathbf{RN^U}$ if $\frac{\theta_L}{\theta_H} \leq \min\{\frac{\alpha - \epsilon}{1 - \epsilon}, \frac{\alpha_{\theta_1}}{1 + \alpha_{\theta_1}}\}$.

Otherwise when $\frac{\theta_L}{\theta_H} \geq \frac{\alpha_{\theta_1}}{1+\alpha_{\theta_1}}$, it can be verified that Π^{RN^U} is in the form of $\frac{\alpha+\rho\bar{\alpha}}{2}[-\delta\theta_H+(1+\delta)\theta_L]^2$ where $\delta > 0$ and $-\delta\theta_H + (1+\delta)\theta_L > 0$. By the same reasoning in Lemma A1, there is constantly $\Pi^{RN^U} \leq \Pi^{N^U N}$. Hence $\mathbf{N^U N}$ is the optimal design.

Define $\phi^U = \min\{\frac{\alpha-\epsilon}{1-\epsilon}, \frac{\alpha_{\theta_1}}{1+\alpha_{\theta_1}}\}$. Overall the optimal design is $\mathbf{RN^U}$ when $\frac{\theta_L}{\theta_H} \leq \phi^U$, and $\mathbf{N^U N}$ when $\frac{\theta_L}{\theta_H} \geq \phi^U$. Specifically in the former case, there is $\theta_1 = 0$ hence $q_B = 0$ and $\beta_G = 1$, and $\mathbf{RN^U}$ yield the same revenue as **RR**.

Appendix B: Optimal Product Line Design with a Single Quality Level

Consider a situation where only a single quality level is possible (thereby denoted with the superscript "Q"). This happens, for example, when it is rather costly to provide differentiated service qualities. The firm then needs to design a single quality level $q = q_G = q_B$ and two products (q, p_G, β_G) and (q, p_B, β_B), targeting good- and bad-signal customers, respectively. The optimal single-quality design is characterized in the following proposition.

Proposition 5 **(Optimal Design with a Single Quality Level)**. *When only a single quality is offered ($q_G = q_B = q$), there exist $\bar{\phi}^Q \geq \underline{\phi}^Q \geq 0$ with*

$$\underline{\phi}^Q = \min\left\{\max\left\{\phi(\sqrt{\alpha}, \alpha_B, 1), \phi\left(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}}\right)\right\}, \phi\left(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1\right)\right\},$$

$$\bar{\phi}^Q = \max\left\{\phi(\sqrt{\alpha}, \alpha_B, 1), \phi\left(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}}\right), \phi\left(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1\right)\right\}$$

such that

*(i) if $\frac{\theta_L}{\theta_H} \leq \underline{\phi}^Q$, the market outcome is **RR**. The optimal product design is*

$$q^Q = \theta_H,\ p_G^Q \in [\alpha_G\theta_H^2, \theta_H^2],\ \beta_G^Q = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G\theta_H^2}{\bar{\alpha}_G p_G^Q} \in [0,1],\ (p_B^Q, \beta_B^Q) = (\theta_H^2, 1);$$

*(ii) if $\underline{\phi}^Q \leq \frac{\theta_L}{\theta_H} \leq \bar{\phi}^Q$, the market outcome is **NR**. The optimal product design is*

$$q^Q = \frac{\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L}{1-\rho\bar{\alpha}}, (p_G^Q, \beta_G^Q) = (\theta_G q^Q, 0), (p_B^Q, \beta_B^Q) = (\theta_H q^Q, 1);$$

*(iii) if $\frac{\theta_L}{\theta_H} \geq \bar{\phi}^Q$, the market outcome is **NN**. The optimal product design is characterized in Proposition 2(i).*

Proof of Proposition 5

- **RR.** If the menus are of RR, the problem for the firm can be formulated as follows

$$\mathbf{RR}: \quad \max_{p_G, p_B, \beta_G, \beta_B \geq 0} \Pi^{RR} = \rho_G(p_G - \bar{\alpha}_G\beta_G p_G) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B) - \alpha\frac{q^2}{2}$$

$$
\text{s.t.}\quad
\begin{array}{lcll}
u_{GG} & \geq & 0 & (IR_G) \\
u_{BB} & \geq & 0 & (IR_B) \\
u_{GG} & \geq & u_{GB} & (IC_G) \\
u_{BB} & \geq & u_{BG} & (IC_B) \\
\beta_G p_G & \geq & \theta_L q & (RR_G) \\
\beta_B p_B & \geq & \theta_L q & (RR_B) \\
\beta_G & \leq & 1 & \\
\beta_B & \leq & 1 &
\end{array}
$$

All good-signal customers will choose (p_G, β_G) and pay p_G upfront. However, a fraction $\bar{\alpha}_G$ of the good-signal customers will later find their type to be L and return the product, which costs the firm $\beta_G p_G$. The second term of the objective function can be explained similarly.

The problem then becomes

$$
\mathbf{RR}: \quad \max_{p_G, p_B, \beta_G, \beta_B} \quad \rho_G(p_G - \bar{\alpha}_G\beta_G p_G) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B) - \alpha\frac{q^2}{2}
$$

$$
\text{s.t.}\quad
\begin{array}{lcll}
p_G - \bar{\alpha}_G\beta_G p_G & \leq & \alpha_G\theta_H q & (IR_G) \\
p_B - \bar{\alpha}_B\beta_B p_B & \leq & \alpha_B\theta_H q & (IR_B) \\
p_G - \bar{\alpha}_G\beta_G p_G & \leq & p_B - \bar{\alpha}_G\beta_B p_B & (IC_G) \\
p_B - \bar{\alpha}_B\beta_B p_B & \leq & p_G - \bar{\alpha}_B\beta_G p_G & (IC_B) \\
\beta_G p_G & \geq & \theta_L q & (RR_G) \\
\beta_B p_B & \geq & \theta_L q & (RR_B) \\
\beta_G & \leq & 1 & \\
\beta_B & \leq & 1 &
\end{array}
$$

The two IC constraints can be written as

$$
\bar{\alpha}_B(\beta_G p_G - \beta_B p_B) \leq p_G - p_B \leq \bar{\alpha}_G(\beta_G p_G - \beta_B p_B).
$$

Recall that $\rho \in (1/2, 1]$, implying $\alpha_G > \alpha > \alpha_B$. Hence, we must have $p_G \leq p_B$ and $\beta_G p_G \leq \beta_B p_B$. Furthermore, the objective function is increasing in the left hand sides of the two IR constraints, implying the two IR constraints should be binding in optimum. We also comment that (IR_G) and (RR_G) implies that $\beta_G \geq \frac{\theta_L}{\alpha_G\theta_H + \bar{\alpha}_G\theta_L}$. Similarly, (IR_B) and (RR_B) implies that $\beta_B \geq \frac{\theta_L}{\alpha_B\theta_H + \bar{\alpha}_B\theta_L}$.

The optimum can be achieved by any of the following solution:

$$
p_G^* \in [\alpha_G\theta_H q, \theta_H q], \;\; \beta_G^* = \frac{1}{\bar{\alpha}_G} - \frac{\alpha_G}{\bar{\alpha}_G}\frac{\theta_H q}{p_G}, \;\; p_B^* = \theta_H q, \;\; \beta_B^* = 1 \tag{A12a}
$$

and

$$
\Pi^* = \alpha\theta_H q - \alpha\frac{q^2}{2}.
$$

The optimal quality is $q^{RR} = \theta_H$ and $\Pi^{RR} = \frac{\alpha\theta_H^2}{2}$. In particular, the solutions include the following price-return pairs:

$$
p_G^{RR} = \theta_H\theta_G, \;\; \beta_G^{RR} = \theta_L/\theta_G, \;\; p_B^{RR} = \frac{\theta_H^2}{2}, \;\; \beta_B^{RR} = 1, \;\; \textit{or}
$$

$$p_G^{RR} = \theta_H^2,\ \beta_G^{RR} = 1,\ p_B^{RR} = \frac{\theta_H^2}{2},\ \beta_B^{RR} = 1.$$

• **NR.** In the case when refund only goes to bad-signal customers, the refund rate for menu G can simply be set at 0. The problem can be formulated as follows

$$\mathbf{NR}: \quad \max_{p_G, p_B, \beta_B \geq 0} \Pi^{NR} = \rho_G p_G + \bar{\rho}_G (p_B - \bar{\alpha}_B \beta_B p_B) - (1 - \rho\bar{\alpha}) \frac{q^2}{2}$$
$$\text{s.t.} \quad \begin{array}{lll} u_{GG} & \geq 0 & (IR_G) \\ u_{BB} & \geq 0 & (IR_B) \\ u_{GG} & \geq u_{GB} & (IC_G) \\ u_{BB} & \geq u_{BG} & (IC_B) \\ \beta_B p_B & \geq \theta_L q & (NR_L) \\ \beta_B & \leq 1 & \end{array}$$

Or equivalently,

$$\mathbf{NR}: \quad \max_{p_G, p_B, \beta_B \geq 0} \rho_G p_G + \bar{\rho}_G (p_B - \bar{\alpha}_B \beta_B p_B) - (1 - \rho\bar{\alpha}) \frac{q^2}{2}$$
$$\text{s.t.} \quad \begin{array}{llll} p_G & \leq & \theta_G q & (IR_G) \\ p_B - \bar{\alpha}_B \beta_B p_B & \leq & \alpha_B \theta_H q & (IR_B) \\ p_G - \bar{\alpha}_G \theta_L q & \leq & p_B - \bar{\alpha}_G \beta_B p_B & (IC_G) \\ p_B - \bar{\alpha}_B \beta_B p_B & \leq & p_G - \bar{\alpha}_B \theta_L q & (IC_B) \\ \beta_B p_B & \geq & \theta_L q & (NR_B) \\ \beta_B & \leq & 1 & \end{array}$$

Followed by a similar analysis as in RR, the optimum can be obtained as follows:

$$p_G^* = \theta_G q,\ \beta_G^* = 0,\ p_B^* = \theta_H q,\ \beta_B^* = 1. \tag{A14a}$$
$$\Pi^* = \alpha\theta_H q + \bar{\rho}\bar{\alpha}\theta_L q - (1 - \rho\bar{\alpha}) \frac{q^2}{2} \tag{A14b}$$

Thus $q^{NR} = \frac{\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L}{1 - \rho\bar{\alpha}}$ and $\Pi^{NR} = \frac{(\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L)^2}{2(1 - \rho\bar{\alpha})}$.

Obviously, $\Pi^{NR} \geq \Pi^{RR}$.

• **RN.** When the refund only goes to good-signal customers, i.e., $\beta_B = 0$, the following problem needs to be solved:

$$\mathbf{RN}: \quad \max_{p_G, p_B, \beta_G \geq 0} \Pi^{RN} = \rho_G (p_G - \bar{\alpha}_G \beta_G p_G) + \bar{\rho}_G p_B - (1 - \bar{\rho}\bar{\alpha}) \frac{q^2}{2}$$
$$\text{s.t.} \quad \begin{array}{lll} u_{GG} & \geq 0 & (IR_G) \\ u_{BB} & \geq 0 & (IR_B) \\ u_{GG} & \geq u_{GB} & (IC_G) \\ u_{BB} & \geq u_{BG} & (IC_B) \\ \beta_G p_G & \geq \theta_L q & (RN_G) \\ \beta_G & \leq 1 & \end{array}$$

which becomes

$$\mathbf{RN}: \quad \max_{p_G, p_B, \beta_G \geq 0} \rho_G (p_G - \bar{\alpha}_G \beta_G p_G) + \bar{\rho}_G p_B - (1 - \bar{\rho}\bar{\alpha}) \frac{q^2}{2}$$

$$\text{s.t.}\quad \begin{array}{lcll} p_G - \bar{\alpha}_G\beta_G p_G & \leq & \alpha_G\theta_H q & (IR_G) \\ p_B & \leq & \theta_B q & (IR_B) \\ p_G - \bar{\alpha}_G\beta_G p_G & \leq & p_B - \bar{\alpha}_G\theta_L q & (IC_G) \\ p_B - \bar{\alpha}_B\theta_L q & \leq & p_G - \bar{\alpha}_B\beta_G p_G & (IC_B) \\ \beta_G p_G & \geq & \theta_L q & (RN_H) \\ \beta_G & \leq & 1 & \end{array}$$

The two IC's imply that $\bar{\alpha}_B(\beta_G p_G - \theta_L q) \leq p_G - p_B \leq \bar{\alpha}_G(\beta_G p_G - \theta_L q)$. However, this cannot hold given (RN_G) and $\bar{\alpha}_G < \bar{\alpha}_B$. Therefore, **RN** is infeasible.

- **NN.** When no customer will exercise the refund option, only one product $(p, q, 0)$ will be offered and the firm needs to decide whether it will serve only good-signal customers, or the entire market. The former calls for solving the problem:

$$\max_{p,q\geq 0}\ \rho_G\left(p - q^2/2\right)$$
$$\text{s.t. } \theta_G q \geq p \geq \theta_B q;$$

while the latter invokes the following problem:

$$\max_{p,q\geq 0}\ p - q^2/2$$
$$\text{s.t. } \theta_B q \geq p.$$

The optimal solution corresponds to the maximum between the two. Comparing optimal profits between the two yields the optimal single-quality design:

$$(q^*, p^*) = \begin{cases} (\theta_G, \theta_G^2), \text{ if } \frac{\theta_L}{\theta_H} \leq \phi^{NS}, \\ (\theta_B, \theta_B^2), \text{ otherwise,} \end{cases}$$

and the optimal profit is

$$\Pi^{NN} = \max\{\frac{\rho_G\theta_G^2}{2}, \frac{\theta_B^2}{2}\}.$$

We next compare the optimal profit among across all possible types of of products:

$$\begin{aligned} \Pi^{RR} &= \frac{\alpha\theta_H^2}{2} \\ \Pi^{NR} &= \frac{(\alpha\theta_H + \bar{\rho}\bar{\alpha}\theta_L)^2}{2(1-\rho\bar{\alpha})} \\ \Pi^{NN} &= \max\{\frac{\rho_G\theta_G^2}{2}, \frac{\theta_B^2}{2}\}. \end{aligned}$$

By Lemma A1, if $\frac{\theta_L}{\theta_H} \leq \phi(\sqrt{\rho_G}, \alpha_B, \alpha_G)$, $\Pi^{NN} = \frac{\rho_G\theta_G^2}{2}$. Note that $\alpha_G \leq \frac{\alpha}{1-\rho\bar{\alpha}} \leq 1$, then $\Pi^{NN} = \frac{\rho_G\theta_G^2}{2} \leq \Pi^{NR} = \frac{(1-\rho\bar{\alpha})}{2}\mathsf{E}_{\frac{\alpha}{1-\rho\bar{\alpha}}}[\theta]$. Thus,

- **NR** is optimal if $\phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1) \leq \frac{\theta_L}{\theta_H}$,

- **RR** is optimal if $\frac{\theta_L}{\theta_H} \le \phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1)$.

On the other hand, if $\frac{\theta_L}{\theta_H} \ge \phi(\sqrt{\rho_G}, \alpha_B, \alpha_G)$, $\Pi^{NN} = \frac{\theta_B^2}{2}$. In addition, $\alpha_B \le \frac{\alpha}{1-\rho\bar{\alpha}} \le 1$.

- **NN** is optimal if $\frac{\theta_L}{\theta_H} \ge \max\left\{\phi(\sqrt{\alpha}, \alpha_B, 1), \phi(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}})\right\}$,
- **NR** is optimal if $\phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1) \le \frac{\theta_L}{\theta_H} \le \phi(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}})$,
- **RR** is optimal if $\frac{\theta_L}{\theta_H} \le \min\left\{\phi(\sqrt{\frac{\alpha}{1-\rho\bar{\alpha}}}, \frac{\alpha}{1-\rho\bar{\alpha}}, 1), \phi(\sqrt{1-\rho\bar{\alpha}}, \alpha_B, \frac{\alpha}{1-\rho\bar{\alpha}})\right\}$.

■

Appendix C: Optimal Product Line Design under a Standard Refund Rate

Consider the situation in which the same refund rate (thereby denoted with the superscript “R”), or a standard refund rate, is offered across products. This situation arises when it is difficult to explain differentiated refund rates to customers, or there are regulations forbidding service providers from discriminating among customers by using cancellation terms.

Under a standard refund rate where $\beta_G = \beta_B = \beta$ for some $\beta \in [0,1]$, the firm designs products (q_G, p_G, β) and (q_B, p_B, β) for good- and bad-signal customers, respectively. The optimal standard refund design is characterized as follows:

Proposition 6 **(Optimal Product Line Design with a Standard Refund Rate)**. *When a standard refund rate is offered ($\beta_G = \beta_B$), there exist $\bar{\phi}^R \ge \underline{\phi}^R \ge 0$ with*

$$\underline{\phi}^R = \min\left\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\right\},$$
$$\bar{\phi}^R = \phi(\sqrt{\alpha_G}, \alpha_G, 1)$$

such that

*(i) if $\frac{\theta_L}{\theta_H} \le \underline{\phi}^R$, the market outcome is **RR**. The optimal product line is*

$$(q_G^R, p_G^R) = (q_B^R, p_B^R) = (\theta_H, \theta_H^2), \quad \beta^R = 1;$$

*(ii) if $\underline{\phi}^R \le \frac{\theta_L}{\theta_H} \le \bar{\phi}^R$, the market outcome is **RN**. The optimal product line is*

$$(q_G^R, p_G^R) = (\theta_H, \theta_H\theta_G - \theta_0\theta_G + \theta_0\theta_B), \ (q_B^R, p_B^R) = (\theta_0, \theta_0\theta_B), \ \beta^R = \theta_L q_G^R / p_G^R < 1.$$

*(iii) if $\frac{\theta_L}{\theta_H} \ge \bar{\phi}^R$, the market outcome is **NN**. The optimal product design is characterized in Proposition 2(i).*

Proof of Proposition 6

For each type of possible products, we first characterize optimal pricing and refund decisions (p_G, p_B, β) under given quality levels (q_G, q_B). The optimal quality design is analyzed thereafter.

• **RR.** When all customers will exercise the refund option, the problem for the firm can be formulated as follows

$$
\begin{array}{rrcll}
\max\limits_{p_G,p_B,\beta} \Pi^{RR} = & \multicolumn{4}{l}{\rho_G(p_G - \bar{\alpha}_G\beta p_G - \alpha_G\frac{q_G^2}{2}) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta p_B - \alpha_B\frac{q_B^2}{2})} \\
 & u_{GG} & \geq & 0 & (IR_G) \\
 & u_{BB} & \geq & 0 & (IR_B) \\
 & u_{GG} & \geq & u_{GB} & (IC_G) \\
\text{s.t.} & u_{BB} & \geq & u_{BG} & (IC_B) \\
 & \beta p_G & \geq & \theta_L q_G & (RR_G) \\
 & \beta p_B & \geq & \theta_L q_B & (RR_B) \\
 & \beta & \leq & 1 & ,
\end{array}
$$

where $U_{GG} = -p_G + \bar{\alpha}_G\beta p_G + \alpha_G\theta_H q_G$, $U_{BB} = -p_B + \bar{\alpha}_B\beta p_B + \alpha_B\theta_H q_B$, $U_{GB} = U_{GG} + (1-\bar{\alpha}_G\beta)(p_G - p_B) - \alpha_G\theta_H(q_G - q_B)$, and $U_{BG} = U_{BB} - (1-\bar{\alpha}_B\beta)(p_G - p_B) + \alpha_B\theta_H(q_G - q_B)$. The two IC constraints imply that $\frac{\alpha_B}{1-\bar{\alpha}_B\beta}\theta_H(q_G - q_B) \leq p_G - p_B \leq \frac{\alpha_G}{1-\bar{\alpha}_G\beta}\theta_H(q_G - q_B)$. As $\frac{\alpha_B}{1-\bar{\alpha}_B\beta} \leq \frac{\alpha_G}{1-\bar{\alpha}_G\beta}$, there should be $p_G \geq p_B$ and $q_G \geq q_B$. In addition, high-type customers are always better off than low-type customers under the same product, i.e., $U_{GB} \geq U_{BB}$ and $U_{GG} \geq U_{BG}$. Therefore, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$.

Without affecting the optimal solution, the objective function can be re-written as $\min_{p_G,p_B,\beta} \rho_G U_{GG} + \bar{\rho}_G U_{BB}$. Apparently, the optimum is achieved when (IR_B) and (IC_G) are binding. The problem can then be simplified as

$$
\begin{array}{rrcll}
\max\limits_{p_G,p_B,\beta} \Pi^{RR} = & \multicolumn{4}{l}{\rho_G(p_G - \bar{\alpha}_G\beta p_G - \alpha_G\frac{q_G^2}{2}) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta p_B - \alpha_B\frac{q_B^2}{2})} \\
 & p_B - \bar{\alpha}_B\beta p_B - \alpha_B\theta_H q_B & = & 0 & (IR_B) \\
 & p_G - \bar{\alpha}_G\beta p_G - \alpha_G\theta_H q_G & = & p_B - \bar{\alpha}_G\beta p_B - \alpha_G\theta_H q_B & (IC_G) \\
\text{s.t.} & \beta p_G & \geq & \theta_L q_G & (RR_G) \\
 & \beta p_B & \geq & \theta_L q_B & (RR_B) \\
 & \beta & \leq & 1 &
\end{array}
$$

By (IR_B), $p_B^* = \frac{\alpha_B}{1-\bar{\alpha}_B\beta^*}\theta_H q_B$. Substitute this into (IC_G), we have

$$p_G^* - \bar{\alpha}_G\beta^* p_G^* = \alpha_G\theta_H q_G - \alpha_G\theta_H q_B + \frac{1-\bar{\alpha}_G\beta^*}{1-\bar{\alpha}_B\beta^*}\alpha_B\theta_H q_B.$$

Since $\Pi^* = (\rho\alpha + \bar{\rho}\bar{\alpha})(p_G^* - \bar{\alpha}_G\beta^* p_G^* - \alpha_B\frac{q_G^2}{2}) + (\bar{\rho}\alpha + \rho\bar{\alpha})(\alpha_B\theta_H q_B - \alpha_B\frac{q_B^2}{2})$, we should like to maximize the RHS of the above equation. Due to (A2) and $\beta^* \leq 1$, this can be achieved when $\beta^* = 1$. Subsequently, the optimal solution (with superscript RR) is

$$p_G^* = \theta_H q_G,\ \ p_B^* = \theta_H q_B,\ \ \beta^* = 1 \tag{A15}$$

and $\Pi^* = \rho_G(\alpha_G\theta_H q_G - \alpha_G\frac{q_G^2}{2}) + \bar{\rho}_G(\alpha_B\theta_H q_B - \alpha_B\frac{q_B^2}{2})$. Therefore,

$$\begin{aligned}
&q_G^{RR} = \theta_H, \quad q_B^{RR} = \theta_H,\\
&p_G^{RR} = \theta_H^2, \quad p_B^{RR} = \theta_H^2,\\
&\beta^{RR} = 1,\\
&\Pi^{RR} = \rho_G\frac{\alpha_G\theta_H^2}{2} + \bar{\rho}_G\frac{\alpha_B\theta_H^2}{2}
\end{aligned}$$

- **NR.** When only the bad-signal customers will exercise the refund, the problem can be formulated as follows

$$\begin{array}{rrcll}
\max_{p_G,p_B,\beta} \Pi^{NR} & = & \multicolumn{3}{l}{\rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2)}\\
& u_{GG} & \geq & 0 & (IR_G)\\
& u_{BB} & \geq & 0 & (IR_B)\\
& u_{GG} & \geq & u_{GB} & (IC_G)\\
\text{s.t.} & u_{BB} & \geq & u_{BG} & (IC_B)\\
& \beta p_G & \leq & \theta_L q_G & (NR_G)\\
& \beta p_B & \geq & \theta_L q_B & (NR_B)\\
& \beta & \leq & 1 & ,
\end{array}$$

where $U_{GG} = -p_G + \theta_G q_G$, $U_{BB} = -p_B + \bar{\alpha}_B\beta p_B + \alpha_B\theta_H q_B$, $U_{GB} = -p_B + \bar{\alpha}_G\beta p_B + \alpha_G\theta_H q_B$, and $U_{BG} = -p_G + \theta_B q_G$. Followed by a similar analysis as in RR, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$. Thus, due to the two NR constraints, we can only confirm that (IR_B) should be binding. The problem can be simplified to

$$\begin{array}{rrcll}
\max_{p_G,p_B,\beta} \Pi^{NR} & = & \multicolumn{3}{l}{\rho_G(p_G - q_G^2/2) + \bar{\rho}_G(p_B - \bar{\alpha}_B\beta_B p_B - \alpha_B q_B^2/2)}\\
& p_B & = & \alpha_B\theta_H q_B + \bar{\alpha}_B\beta p_B & (IR_B)\\
& p_G & \leq & \theta_G q_G + p_B - \alpha_G\theta_H q_B - \bar{\alpha}_G\beta p_B & (IC_G)\\
\text{s.t.} & \beta p_G & \leq & \theta_L q_G & (NR_G)\\
& \beta p_B & \geq & \theta_L q_B & (NR_B\ )\\
& \beta & \leq & 1 &
\end{array}$$

By (IR_B) and (NR_B), we have $\beta \geq \frac{\theta_L}{\theta_B}$. Therefore, $p_G \leq \theta_B q_G$. It can be verified that (IC_G) is also satisfied at $p_G = \theta_B q_G$. Thus, the optimal solution is

$$p_G^* = \theta_B q_G, \ p_B^* = \frac{\alpha_B\theta_H}{1 - \bar{\alpha}_B\theta_L/\theta_B} q_B, \ \beta^* = \theta_L/\theta_B \tag{A16}$$

and the expected profit is $\Pi^* = \rho_G(\theta_B q_G - \frac{q_G^2}{2}) + \bar{\rho}_G\alpha_B(\alpha_B\theta_H q_B - \alpha_B\frac{q_B^2}{2})$. Therefore,

$$\begin{aligned}
&q_G^{NR} = \theta_B, \quad q_B^{NR} = \theta_H,\\
&p_G^{NR} = \theta_B^2, \quad p_B^{NR} = \frac{\alpha_B\theta_H^2}{1 - \bar{\alpha}_B\theta_L/\theta_B},\\
&\beta^{NR} = \theta_L/\theta_B,\\
&\Pi^{NR} = \rho_G\frac{\theta_B^2}{2} + \bar{\rho}_G\frac{\alpha_B\theta_H^2}{2}
\end{aligned}$$

- **NN.** When no customer will exercise the refund option, the analysis follows Lemma 2 and the optimal profit is

$$\Pi^{NN} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}.$$

- **RN.** When only the good-signal customers will exercise the refund, the problem can be formulated by

$$\max_{p_G, p_B, \beta} \Pi^{RN} = \rho_G(p_G - \bar{\alpha}_G \beta p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - q_B^2/2)$$

$$\text{s.t.} \quad \begin{array}{llll} u_{GG} & \geq & 0 & (IR_G) \\ u_{BB} & \geq & 0 & (IR_B) \\ u_{GG} & \geq & u_{GB} & (IC_G) \\ u_{BB} & \geq & u_{BG} & (IC_B) \\ \beta p_G & \geq & \theta_L q_G & (RN_G) \\ \beta p_B & \leq & \theta_L q_B & (RN_B) \\ \beta & \leq & 1 & , \end{array}$$

where $U_{GG} = -p_G + \alpha_G \theta_H q_G + \bar{\alpha}_G \beta p_G$, $U_{BB} = -p_B + \theta_B q_B$, $U_{GB} = -p_B + \theta_G q_B$, and $U_{BG} = -p_G + \alpha_B \theta_H q_G + \bar{\alpha}_B \beta p_G$. Followed by a similar argument as in other scenarios, $U_{GG} \geq U_{GB} \geq U_{BB} \geq U_{BG}$, and in this case, only (IC_G) can be confirmed binding due to the two RN constraints. The problem can be simplified to:

$$\max_{p_G, p_B, \beta} \Pi^{RN} = \rho_G(p_G - \bar{\alpha}_G \beta p_G - \alpha_G q_G^2/2) + \bar{\rho}_G(p_B - q_B^2/2)$$

$$\text{s.t.} \quad \begin{array}{llll} p_B & \leq & \theta_B q_B & (IR_B) \\ (1 - \bar{\alpha}_G \beta) p_G & = & \alpha_G \theta_H q_G - \theta_G q_B + p_B & (IC_G) \\ \beta p_G & \geq & \theta_L q_G & (RN_G) \\ \beta p_B & \leq & \theta_L q_B & (RN_B) \\ \beta & \leq & 1 & \end{array}$$

It can be verified that the optimal solution is achieved when (IR_B) and (RN_G) are binding, which gives rise to the following solution:

$$p_G^* = \theta_G q_G - (\theta_G - \theta_B) q_B, \;\; p_B^* = \theta_B q_B, \;\; \beta^* = \theta_L q_G / p_G^* \tag{A17}$$

The expected profit is $\Pi^* = \rho_G(\alpha_G \theta_H q_G - \theta_G q_B + \theta_B q_B - \alpha_G \frac{q_G^2}{2}) + \bar{\rho}_G(\theta_B q_B - \frac{q_B^2}{2})$. Thus,

$$q_G^{RN} = \theta_H, \quad q_B^{RN} = \theta_0,$$

$$p_G^{RN} = (\theta_H - \theta_0)\theta_G + \theta_0 \theta_B, \quad p_B^{RN} = \theta_0 \theta_B,$$

$$\beta^{RN} = \theta_L q_G / p_G^{RN},$$

$$\Pi^{RN} = \rho_G \frac{\alpha_G \theta_H^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2}$$

where $\theta_0 = \alpha_0 \theta_H + (1 - \alpha_0)\theta_L$ and $\alpha_0 = \frac{\alpha_B - \rho\alpha}{\bar{\rho}_G} \leq \alpha_B \leq \alpha_G$.

We next compare the optimal profit among across all four types of products:

$$\Pi^{RR} = \rho_G \frac{\alpha_G \theta_H^2}{2} + \bar{\rho}_G \frac{\alpha_B \theta_H^2}{2}$$
$$\Pi^{NR} = \rho_G \frac{\theta_B^2}{2} + \bar{\rho}_G \frac{\alpha_B \theta_H^2}{2}$$
$$\Pi^{NN} = \rho_G \frac{\theta_G^2}{2} + \bar{\rho}_G \frac{\theta_0^2}{2} = \frac{\theta_B^2}{2} + \frac{\rho_G}{2\bar{\rho}_G}(\theta_G - \theta_B)^2$$
$$\Pi^{RN} = \rho_G \frac{\alpha_G \theta_H^2}{2} + + \bar{\rho}_G \frac{\theta_0^2}{2}.$$

We find that

- if $\sqrt{\alpha_G} \le \frac{\theta_B}{\theta_H}$, $\Pi^{RR} \le \Pi^{NR}$ and $\Pi^{RN} \le \Pi^{NN}$. Moreover, $\Pi^{NR} \le \frac{\theta_B^2}{2} \le \Pi^{NN}$. Therefore, **NN** is optimal.
- if $\frac{\theta_B}{\theta_H} < \sqrt{\alpha_G} \le \frac{\theta_G}{\theta_H}$, there is $\Pi^{RR} > \Pi^{NR}$ and $\Pi^{RN} \le \Pi^{NN}$. In this scenario, however, there is $\Pi^{RR} \le \frac{\theta_G^2}{2} \le \Pi^{NN}$. Again, **NN** is optimal.
- if $\frac{\theta_G}{\theta_H} < \sqrt{\alpha_G}$, $\Pi^{RR} > \Pi^{NR}$ and $\Pi^{RN} > \Pi^{NN}$. We only need to consider **RR** and **RN**. Specifically, if $\frac{\theta_0}{\theta_H} < \sqrt{\alpha_B}$, **RR** is optimal; otherwise **RN** is optimal.

By Lemma A1, if $\frac{\theta_L}{\theta_H} \ge \phi(\sqrt{\alpha_G}, \alpha_G, 1)$, **NN** is optimal. If $\frac{\theta_L}{\theta_H} \le \phi(\sqrt{\alpha_G}, \alpha_G, 1)$, **RR** is optimal if $\frac{\theta_L}{\theta_H} \le \phi(\sqrt{\alpha_B}, \alpha_0, 1)$ and **RN** is optimal if $\frac{\theta_L}{\theta_H} > \phi(\sqrt{\alpha_B}, \alpha_0, 1)$. Hence $\bar{\phi}^R = \phi(\sqrt{\alpha_G}, \alpha_G, 1)$ and $\underline{\phi}^R = \min\{\phi(\sqrt{\alpha_G}, \alpha_G, 1), \phi(\sqrt{\alpha_B}, \alpha_0, 1)\}$.

■